Problem Set #2

Anaya Hall & Christian Miller

Part 1: Theory

(for practice only)

Part 2: Applied - Returns to Scale in Electricity Supply

First, load our OLS function created in Problem Set #1. We're including a built in t-test this time around.

```
ols <- function(data, y_data, X_data, intercept = T, H0 = 0, two_tail = T, alpha = 0.05) {
  # Function setup ----
    # Require the 'dplyr' package
    require(dplyr)
    # Function to convert tibble, data.frame, or tbl_df to matrix
    to_matrix <- function(the_df, vars) {
      # Create a matrix from variables in var
      new_mat <- the_df %>%
        #Select the columns given in 'vars'
        select_(.dots = vars) %>%
        # Convert to matrix
        as.matrix()
      # Return 'new_mat'
      return(new_mat)
    }

  # Create dependent and independent variable matrices ----
    # y matrix
    y <- to_matrix (the_df = data, vars = y_data)
    # X matrix
    X <- to_matrix (the_df = data, vars = X_data)
      # If 'intercept' is TRUE, then add a column of ones
      if (intercept == T) {
      X <- cbind(1,X)
      colnames(X) <- c("intercept", X_data)
      }

  # Calculate b, y_hat, and residuals ----
    b <- solve(t(X) %*% X) %*% t(X) %*% y
    y_hat <- X %*% b
    e <- y - y_hat

  # Useful -----
    n <- nrow(X) # number of observations
    k <- ncol(X) # number of independent variables
    dof <- n - k # degrees of freedom
```

```r
  i <- rep(1,n) # column of ones for demeaning matrix
  A <- diag(i) - (1 / n) * i %*% t(i) # demeaning matrix
  y_star <- A %*% y # for SST
  X_star <- A %*% X # for SSM
  SST <- drop(t(y_star) %*% y_star)
  SSM <- drop(t(b) %*% t(X_star) %*% X_star %*% b)
  SSR <- drop(t(e) %*% e)

# Measures of fit and estimated variance ----
  R2uc <- drop((t(y_hat) %*% y_hat)/(t(y) %*% y)) # Uncentered R^2
  R2 <- 1 - SSR/SST # Uncentered R^2
  R2adj <- 1 - (n-1)/dof * (1 - R2) # Adjusted R^2
  AIC <- log(SSR/n) + 2*k/n # AIC
  SIC <- log(SSR/n) + k/n*log(n) # SIC
  s2 <- SSR/dof # s^2

# Measures of fit table ----
  mof_table_df <- data.frame(R2uc, R2, R2adj, SIC, AIC, SSR, s2)
  mof_table_col_names <- c("$R^2_\\text{uc}$", "$R^2$",
                           "$R^2_\\text{adj}$",
                           "SIC", "AIC", "SSR", "$s^2$")
  mof_table <-  mof_table_df %>% knitr::kable(
    row.names = F,
    col.names = mof_table_col_names,
    format.args = list(scientific = F, digits = 4),
    booktabs = T,
    escape = F
  )

# t-test----
  # Standard error
  se <- as.vector(sqrt(s2 * diag(solve(t(X) %*% X))))
  # Vector of _t_ statistics
  t_stats <- (b - H0) / se
  # Calculate the p-values
  if (two_tail == T) {
  p_values <- pt(q = abs(t_stats), df = dof, lower.tail = F) * 2
  } else {
    p_values <- pt(q = abs(t_stats), df = dof, lower.tail = F)
  }
  # Do we (fail to) reject?
  reject <- ifelse(p_values < alpha, reject <- "Reject", reject <- "Fail to Reject")

  # Nice table (data.frame) of results
  ttest_df <- data.frame(
    # The rows have the coef. names
    effect = rownames(b),
    # Estimated coefficients
    coef = as.vector(b) %>% round(3),
    # Standard errors
```

```
      std_error = as.vector(se) %>% round(3),
      # t statistics
      t_stat = as.vector(t_stats) %>% round(3),
      # p-values
      p_value = as.vector(p_values) %>% round(4),
      # reject null?
      significance = as.character(reject)
      )

    ttest_table <-  ttest_df %>% knitr::kable(
      booktabs = T,
      format.args = list(scientific = F),
      escape = F
    )

  # Data frame for exporting for y, y_hat, X, and e vectors ----
    export_df <- data.frame(y, y_hat, e, X) %>% tbl_df()
    colnames(export_df) <- c("y","y_hat","e",colnames(X))

  # Return ----
    return(list(n=n, dof=dof, b=b, vars=export_df, R2uc=R2uc,R2=R2,
                R2adj=R2adj, AIC=AIC, SIC=SIC, s2=s2, SST=SST, SSR=SSR,
                mof_table=mof_table, ttest=ttest_table))
}
```

We'll also need a function to do an F-test for this Problem Set.

```
to_matrix <- function(the_df, vars) {
      # Create a matrix from variables in var
      new_mat <- the_df %>%
        #Select the columns given in 'vars'
        select_(.dots = vars) %>%
        # Convert to matrix
        as.matrix()
      # Return 'new_mat'
      return(new_mat)
    }


# Joint test function (from Ed's notes). Could also write a more complex functions that takes R an
F_test <- function(data, y_var, X_vars) {
  # Turn data into matrices
  y <- to_matrix(data, y_var)
  X <- to_matrix(data, X_vars)
  # Add intercept
  X <- cbind(1, X)
  # Name the new column "intercept"
  colnames(X) <- c("intercept", X_vars)
  # Calculate n and k for degrees of freedom
  n <- nrow(X)
```

```r
  k <- ncol(X)
  # J is k-1
  J <- k - 1
  # Create the R matrix: bind a column of zeros
  # onto a J-by-J identity matrix
  R <- cbind(0, diag(J))

  # Estimate coefficients
  b <- ols(data, y_var, X_vars)
  # Retrieve OLS residuals
  e <- b$vars$e
  # Retrieve s^2
  s2 <- b$s2 %<>% as.numeric()

  # Create the inner matrix R(X'X)^(-1)R'
  RXXR <- R %*% solve(t(X) %*% X) %*% t(R)
  # Calculate the F stat
  f_stat <- t(R %*% b$b) %*% solve(RXXR) %*% (R %*% b$b) / J / s2
  # Calculate the p-value;; why normal and not chi^2
  p_value <- pf(q = f_stat, df1 = J, df2 = n-k, lower.tail = F)
  # Create a data.frame of the f stat. and p-value
  results <- data.frame(
    f_stat = f_stat %>% as.vector(),
    p_value = p_value %>% as.vector())
  return(results)

}
```

Question 1:

Read the data into R. Inspect it. Sort by size (Q (kwh)).

```r
nerlove <- readxl::read_excel("nerlove.xls", col_names=T)
summary(nerlove)
```

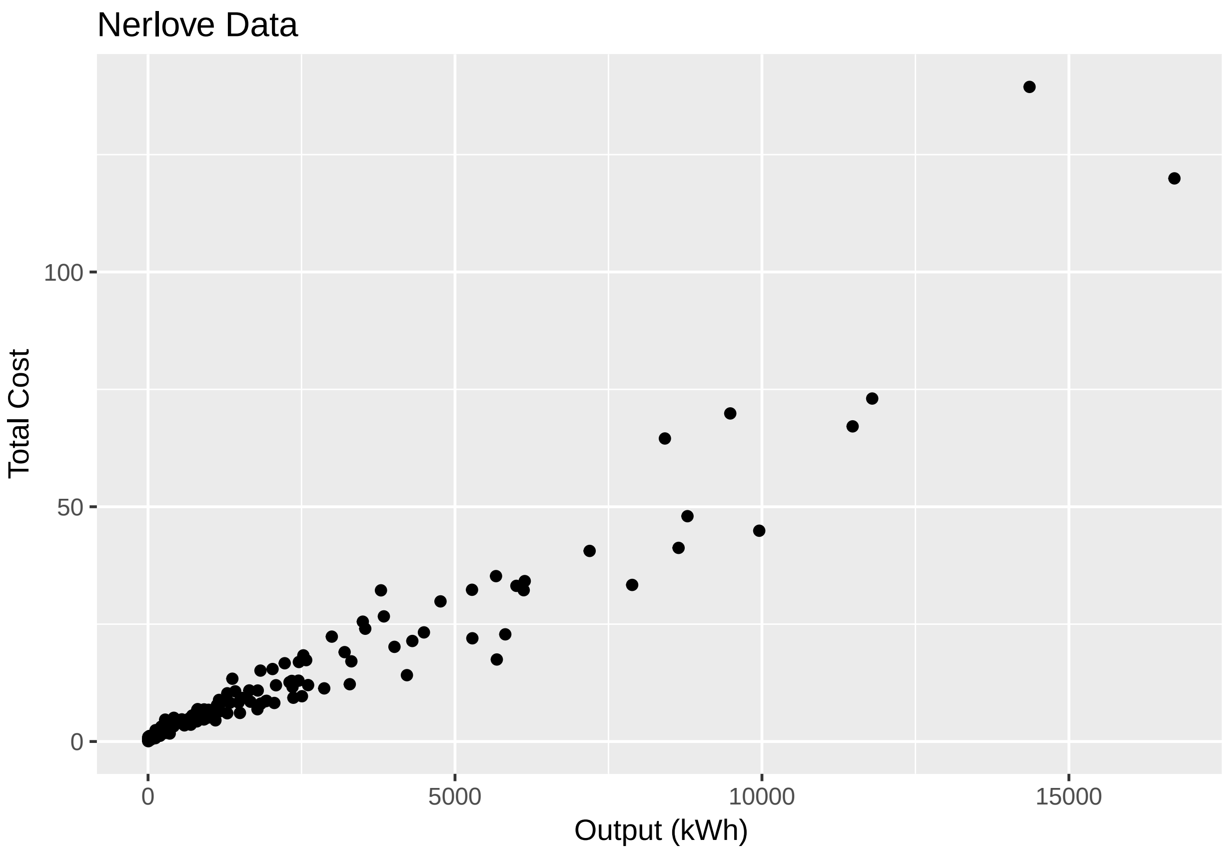
```
##        TC                Q               PL                PF
##  Min.   :  0.082   Min.   :    2   Min.   :  1.450   Min.   :10.30
##  1st Qu.:  2.382   1st Qu.:  279   1st Qu.:  1.760   1st Qu.:21.30
##  Median :  6.754   Median : 1109   Median :  2.040   Median :26.90
##  Mean   : 12.976   Mean   : 2133   Mean   :  3.208   Mean   :26.18
##  3rd Qu.: 14.132   3rd Qu.: 2507   3rd Qu.:  2.190   3rd Qu.:32.20
##  Max.   :139.422   Max.   :16719   Max.   :181.000   Max.   :42.80
##        PK
##  Min.   :138.0
##  1st Qu.:162.0
##  Median :170.0
##  Mean   :174.5
##  3rd Qu.:183.0
##  Max.   :233.0
```

```r
# Fix typo in 13th row (missing a decimal!)
# DO THIS MORE ELEGANTLY!
nerlove[13, "PL"] <- 1.81

# nerlove %>%
#   filter(PL > 100) %>%
#     mutate(PL = PL/100)

nerlove %>% arrange(Q)
```

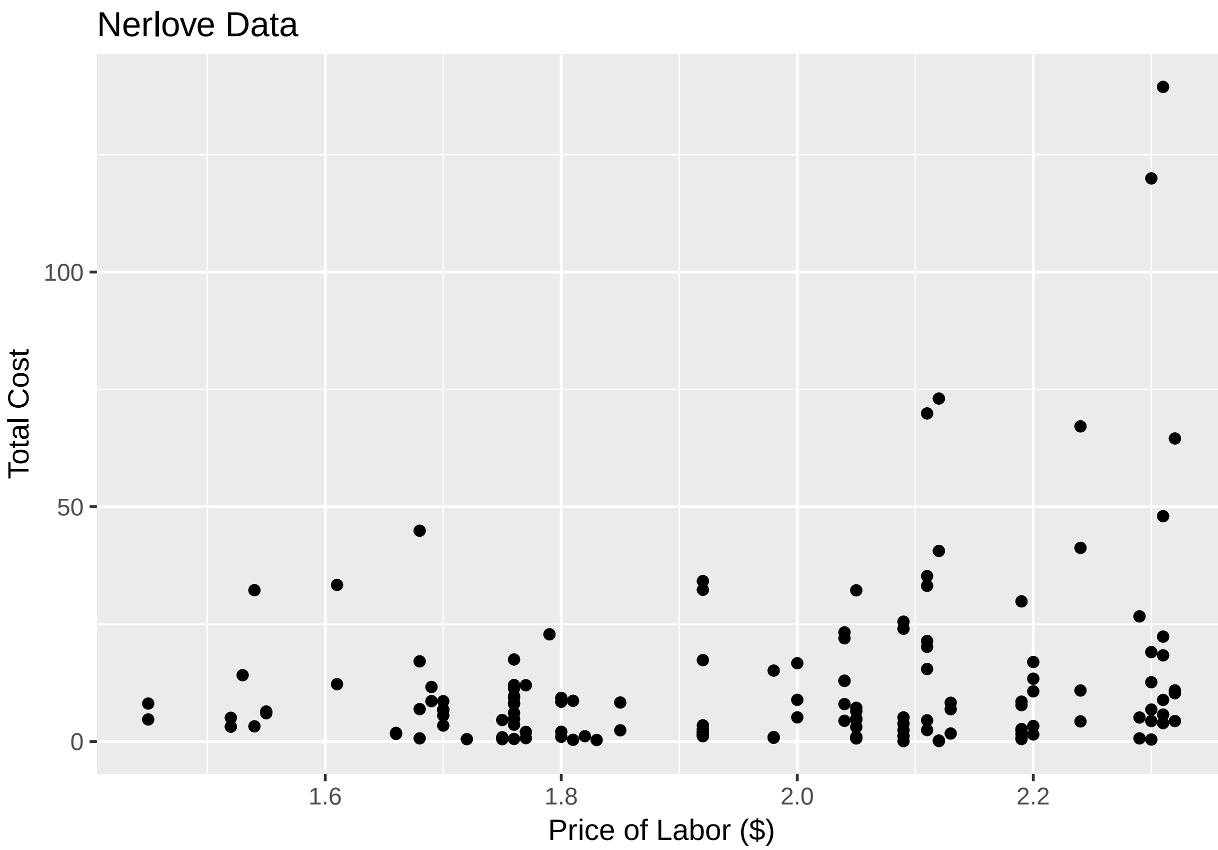
```
## # A tibble: 145 x 5
##        TC     Q    PL    PF    PK
##     <dbl> <dbl> <dbl> <dbl> <dbl>
##  1 0.0820    2.  2.09  17.9  183.
##  2 0.661     3.  2.05  35.1  174.
##  3 0.990     4.  2.05  35.1  171.
##  4 0.315     4.  1.83  32.2  166.
##  5 0.197     5.  2.12  28.6  233.
##  6 0.0980    9.  2.12  28.6  195.
##  7 0.949    11.  1.98  35.5  206.
##  8 0.675    13.  2.05  35.1  150.
##  9 0.525    13.  2.19  29.1  155.
## 10 0.501    22.  1.72  15.0  188.
## # ... with 135 more rows
```

Plot the series and make sure your data are read in correctly.

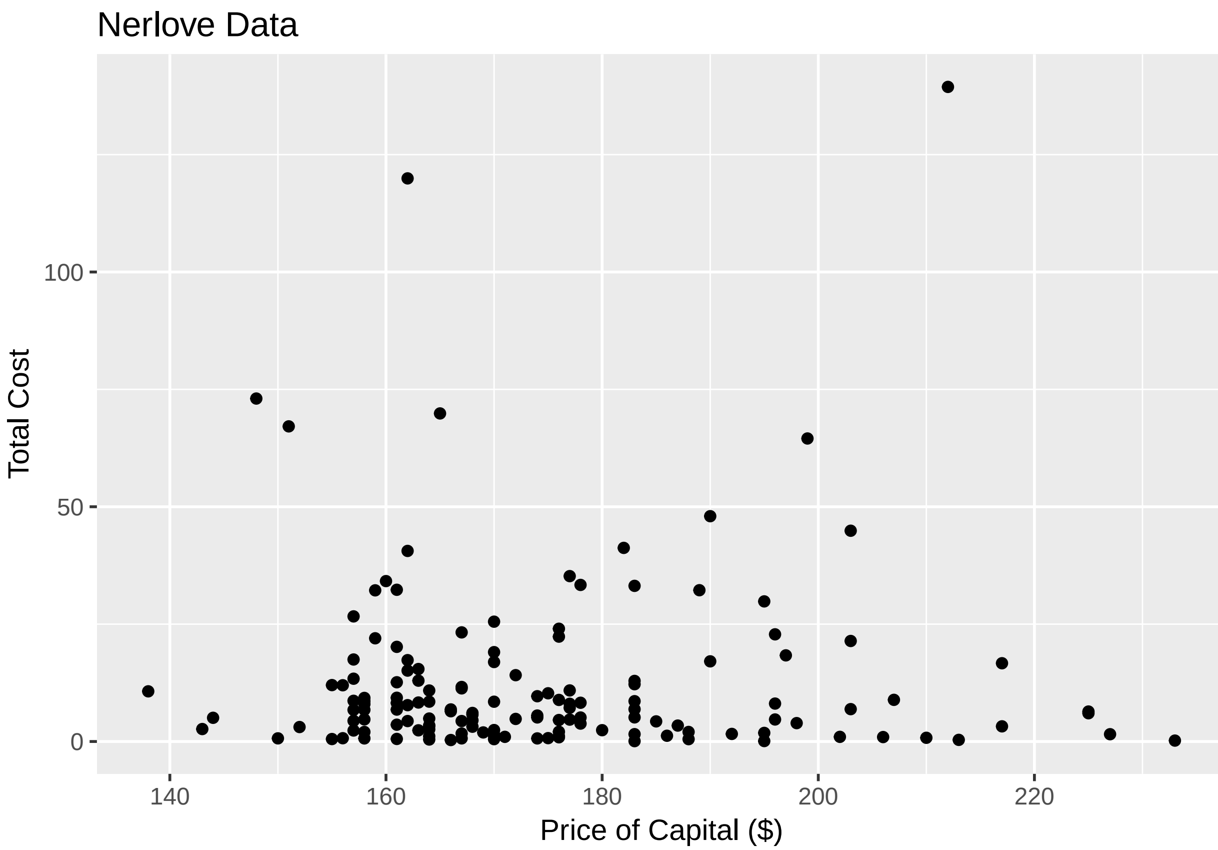
```r
ggplot(nerlove, aes(x=Q, y=TC)) +
  geom_point() +
  labs(title="Nerlove Data", x="Output (kWh)", y="Total Cost")
```



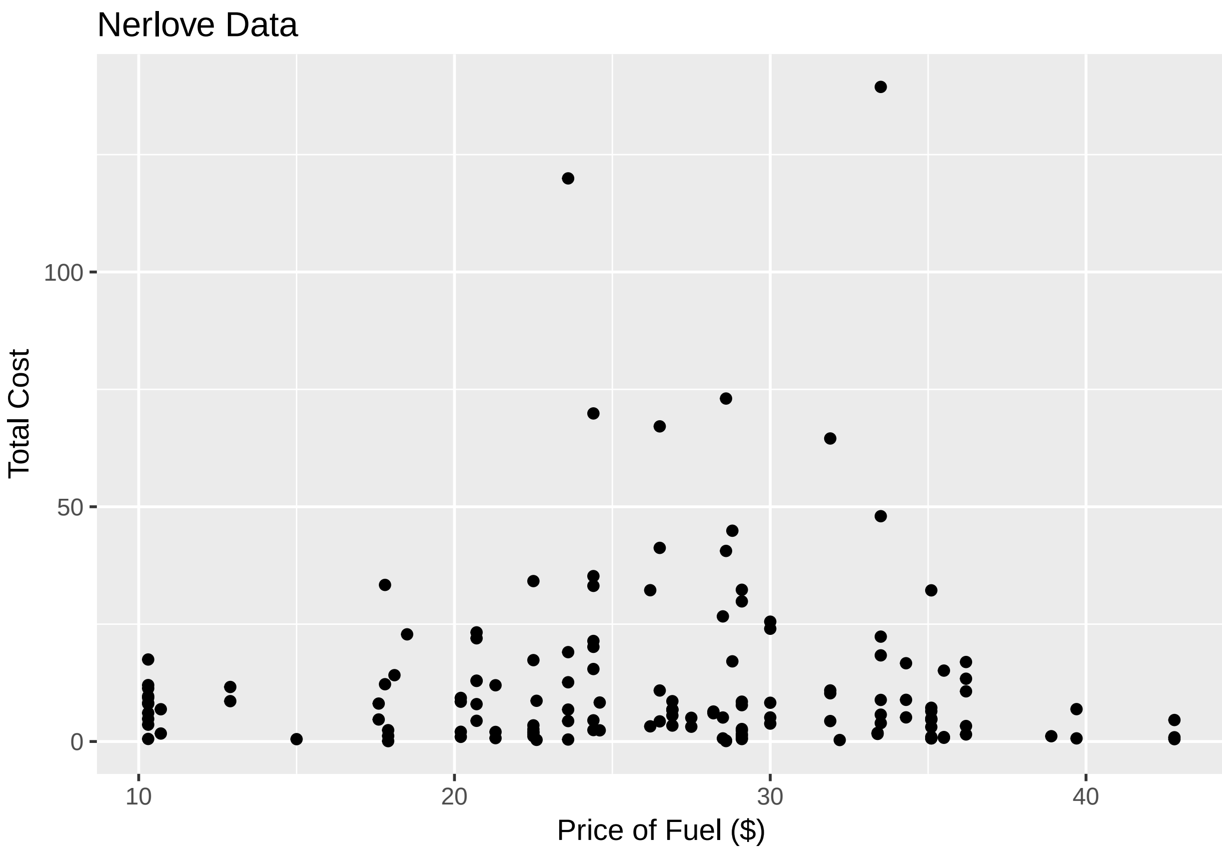

```r
ggplot(nerlove, aes(x=PL, y=TC)) +
  geom_point() +
  labs(title="Nerlove Data", x="Price of Labor ($)", y="Total Cost")
```




```r
ggplot(nerlove, aes(x=PK, y=TC)) +
  geom_point() +
  labs(title="Nerlove Data", x="Price of Capital ($)", y="Total Cost")
```




```r
ggplot(nerlove, aes(x=PF, y=TC)) +
  geom_point() +
  labs(title="Nerlove Data", x="Price of Fuel ($)", y="Total Cost")
```



Question 2:

Replicate regression I (page 176) in the paper.

Regression I:

$$log(TC) - log(P_F) = \beta_0 + \beta_1 Q + \beta_2 \Big(log(P_L) - log(P_F) \Big) + \beta_3 \Big(log(P_K) - log(P_F) \Big)$$

Equivalent to:

$$log(\frac{TC}{P_F}) = \beta_0 + \beta_1 Q + \beta_2 log(\frac{P_L}{P_F}) + \beta_3 log(\frac{P_K}{P_F})$$

Where:
TC = total production cost,

P_L = wage rate,

P_K = "price" of capital,

P_F = price of fuel,

Q = output (measured in kWh)

In generalized Cobb-Douglas form:

$\beta_1 = \frac{1}{r}$,

$\beta_2 = \frac{a_L}{r}$,

$\beta_3 = \frac{a_K}{r}$

Prepare variables for Regression I.

```r
# Create log variables
nerlove %<>% mutate(
  TClog = log(TC),
  Qlog = log(Q),
  PLlog = log(PL),
  PKlog = log(PK),
  PFlog = log(PF)
)

# Create PF scaled variables
nerlove %<>% mutate(
  TCscaled = TClog - PFlog,
  PLscaled = PLlog - PFlog,
  PKscaled = PKlog - PFlog
)
```

Variable names:

$log(\frac{TC}{P_F})$ = "TCscaled"

$log(\frac{P_L}{P_F})$ = "PLscaled"

$log(\frac{P_K}{P_F})$ = "PKscaled"

```r
# Regression I:
# dep var = (log costs - log fuel price) = TCscaled
reg_I <- ols(data = nerlove,y_data = "TCscaled",
             X_data = c("Qlog","PLscaled","PKscaled"),
             intercept = T, H0 = 0, alpha = 0.05)

reg_I$ttest
```

effect	coef	std_error	t_stat	p_value	significance
intercept	-2.037	0.384	-5.301	0.0000	Reject
Qlog	0.721	0.017	41.334	0.0000	Reject
PLscaled	0.593	0.205	2.898	0.0044	Reject
PKscaled	-0.007	0.191	-0.039	0.9692	Fail to Reject

```r
reg_I$mof_table
```

R^2_{uc}	R^2	R^2_{adj}	SIC	AIC	SSR	s^2
0.966	0.9316	0.9301	-3.433	-3.515	4.082	0.02895

Coefficients are pretty close to those in the paper. R^2 matches!

Question 3:

Conduct the hypothesis test using constant returns to scale ($\beta_1 = 1$) as your null hypothesis.

```r
# Re-run regression with null hypothesis H0 = 1
reg_I <- ols(data = nerlove, y_data = "TCscaled",
             X_data = c("Qlog","PLscaled","PKscaled"),
             intercept = T, H0 = 1, alpha = 0.05)

reg_I$ttest
```

effect	coef	std_error	t_stat	p_value	significance
intercept	-2.037	0.384	-7.903	0.0000	Reject
Qlog	0.721	0.017	-16.020	0.0000	Reject
PLscaled	0.593	0.205	-1.990	0.0485	Reject
PKscaled	-0.007	0.191	-5.282	0.0000	Reject

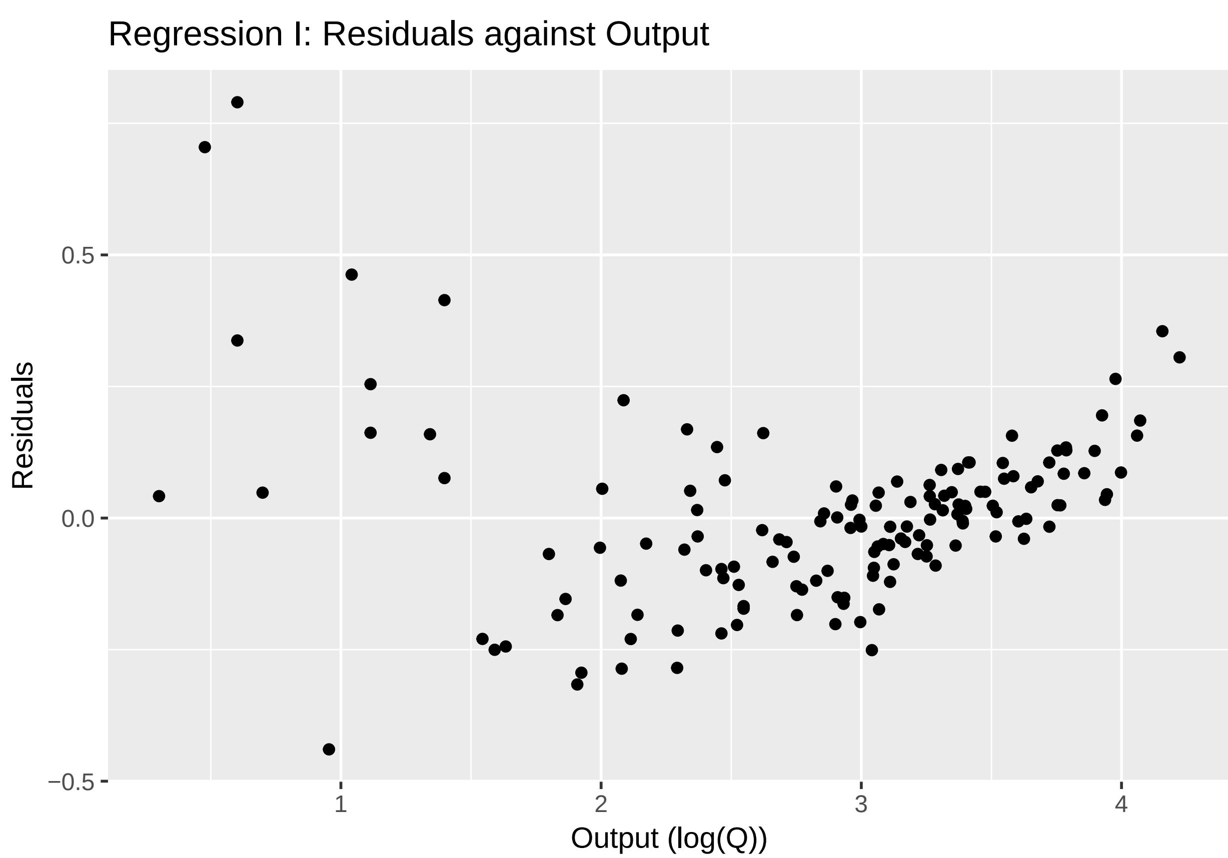
What is the p- value associated with you test statistic? What is your point estimate of returns to scale? Constant? Increasing? Decreasing?

The p-value is 0.000. The point estimate of returns to scale is $\frac{1}{\beta} = r = 1.3875639$, hence returns to scale is increasing.

Question 4:

Plot residuals against output.

```r
ggplot(reg_I$vars, aes(y=e, x=Qlog)) + geom_point() + labs(title="Regression I: Residuals against
```



What do you notice? What does this potentially tell you from an economic perspective?

Evidence of heteroskedasticity: residuals seem to track the log output through parabola. We may want to rethink our specification!

Compute the correlation coefficient of the residuals with output for the entire sample? What does this tell you?

```
# R = cov(xy)/var(x)var(y)
R_I <- (cov(x=reg_I$vars$e, y=reg_I$vars$Qlog)/(var(reg_I$vars$e)*var(reg_I$vars$Qlog))) %>% signi
R_I
```

```
## [1] "7.01e-14"
```

The correlation coefficient is extremely small: 7.01e-14.

Question 5:

Divide your sample into 5 subgroups of 29 firms each according to the level of output. Estimate the regression model again for each group separately.

```
# Divide sample into 5 subgroups
d <- split(nerlove,rep(1:5,each=29))

# Now we have a list, d, with five dataframes '1' through '5' for our five subgroups.
```

```
# Regression IIIA
reg_IIIA <- ols(data = d$`1`,y_data = "TCscaled",
             X_data = c("Qlog","PLscaled","PKscaled"),
             intercept = T, H0 = 1, alpha = 0.05)
reg_IIIA$ttest
```

effect	coef	std_error	t_stat	p_value	significance
intercept	-1.452	1.366	-1.795	0.0848	Fail to Reject
Qlog	0.400	0.084	-7.101	0.0000	Reject
PLscaled	0.615	0.729	-0.528	0.6024	Fail to Reject
PKscaled	-0.081	0.706	-1.531	0.1384	Fail to Reject

```
# Regression IIIB
reg_IIIB <- ols(data = d$`2`,y_data = "TCscaled",
             X_data = c("Qlog","PLscaled","PKscaled"),
             intercept = T, H0 = 1, alpha = 0.05)
reg_IIIB$ttest
```

effect	coef	std_error	t_stat	p_value	significance
intercept	-2.818	0.614	-6.222	0.0000	Reject
Qlog	0.658	0.116	-2.939	0.0070	Reject
PLscaled	0.094	0.274	-3.304	0.0029	Reject
PKscaled	0.378	0.277	-2.250	0.0335	Reject

```r
# Regression IIIC
reg_IIIC <- ols(data = d$`3`,y_data = "TCscaled",
            X_data = c("Qlog","PLscaled","PKscaled"),
            intercept = T, H0 = 1, alpha = 0.05)
reg_IIIC$ttest
```

effect	coef	std_error	t_stat	p_value	significance
intercept	-3.185	0.734	-5.705	0.0000	Reject
Qlog	0.938	0.198	-0.312	0.7578	Fail to Reject
PLscaled	0.402	0.199	-2.997	0.0061	Reject
PKscaled	0.250	0.187	-4.010	0.0005	Reject

```r
# Regression IIID
reg_IIID <- ols(data = d$`4`,y_data = "TCscaled",
            X_data = c("Qlog","PLscaled","PKscaled"),
            intercept = T, H0 = 1, alpha = 0.05)
reg_IIID$ttest
```

effect	coef	std_error	t_stat	p_value	significance
intercept	-2.843	0.506	-7.596	0.0000	Reject
Qlog	0.912	0.107	-0.818	0.4210	Fail to Reject
PLscaled	0.507	0.187	-2.630	0.0144	Reject
PKscaled	0.093	0.164	-5.525	0.0000	Reject

```r
# Regression IIIE
reg_IIIE <- ols(data = d$`5`,y_data = "TCscaled",
            X_data = c("Qlog","PLscaled","PKscaled"),
            intercept = T, H0 = 1, alpha = 0.05)
reg_IIIE$ttest
```

effect	coef	std_error	t_stat	p_value	significance
intercept	-2.916	0.454	-8.618	0.0000	Reject
Qlog	1.044	0.065	0.683	0.5008	Fail to Reject
PLscaled	0.603	0.197	-2.014	0.0549	Fail to Reject
PKscaled	-0.289	0.175	-7.374	0.0000	Reject

```
## May want to clean this up and run as for loop???
```

Can you replicate Equations IIIA - IIIE? Calculate the point estimates for returns to scale for each sample. Is there a pattern relating to size of output? Regression | Returns to Scale ————- | ————- IIIA | 2.4981863 IIIB | 1.5194085 IIIC | 1.0657807 IIID | 1.0964381 IIIE | 0.9574966

Coefficients roughly match the results of the paper!

Question 6:

Create "dummy variables" for each industry. Interact them with the output variable to create five "slope coefficients".

```
# create group categorical var
for (i in 1:5) {
  d[[i]] %<>% mutate(gvar=i)
}

# unsplit
df <- rbind(d[[1]], d[[2]], d[[3]], d[[4]], d[[5]])

# create dummies
df %<>%
    mutate(
      g1 = ifelse(gvar==1, 1, 0),
      g2 = ifelse(gvar==2, 1, 0),
      g3 = ifelse(gvar==3, 1, 0),
      g4 = ifelse(gvar==4, 1, 0),
      g5 = ifelse(gvar==5, 1, 0))

df %<>% select(-gvar)

# interact with output variable
df %<>%
    mutate(
      lQ_A = Qlog*g1,
      lQ_B = Qlog*g2,
      lQ_C = Qlog*g3,
      lQ_D = Qlog*g4,
      lQ_E = Qlog*g5)
```

Run a model, letting the intercept and slope coefficient on output differ across plants, but let the remainder of the coefficients be pooled across plants.

```
# I'm not sure this is the right model... but I'm gettin similar coeffs!



reg_IV <- ols(data = df, y_data = "TCscaled",
             X_data = c("lQ_A", "lQ_B", "lQ_C", "lQ_D", "lQ_E", "PLscaled","PKscaled", "g1",
                        "g2", "g3", "g4", "g5"),
             intercept = F, H0 = 0, alpha = 0.05)

# Can also omit one of the dummie vars (g) and include an intercept >> get the same coefs
reg_IV$ttest
```

effect	coef	std_error	t_stat	p_value	significance
lQ_A	0.397	0.043	9.214	0.0000	Reject
lQ_B	0.648	0.147	4.402	0.0000	Reject

effect	coef	std_error	t_stat	p_value	significance
lQ_C	0.885	0.297	2.976	0.0035	Reject
lQ_D	0.909	0.274	3.321	0.0012	Reject
lQ_E	1.063	0.131	8.091	0.0000	Reject
PLscaled	0.426	0.163	2.608	0.0101	Reject
PKscaled	0.104	0.152	0.681	0.4967	Fail to Reject
g1	-1.815	0.305	-5.952	0.0000	Reject
g2	-2.194	0.489	-4.491	0.0000	Reject
g3	-2.879	0.972	-2.963	0.0036	Reject
g4	-2.922	0.966	-3.024	0.0030	Reject
g5	-3.511	0.599	-5.857	0.0000	Reject

```r
reg_IV$mof_table
```

R^2_{uc}	R^2	R^2_{adj}	SIC	AIC	SSR	s^2
0.9802	0.9602	0.9569	-3.701	-3.947	2.372	0.01784

Are there any noticeable changes in returns to scale from the previous part?

yes. . . .

Question 7:

Conduct a statistical test comparing the first model you estimate to the last model you estimated. (Hint: Is one model a restricted version of the other?). Would separate t-test have given you the same results?

Regression I is unrestricted model, Regression IV is restricted model

$F = \frac{(SSR_R - SSR_U)/J}{SSR_U/(n-k)}$

```r
ssr_u <- reg_I$SSR
dof <- reg_I$dof

ssr_r <- reg_IV$SSR
j <- nrow(reg_IV$b)

f_statistic <- ((ssr_r - ssr_u) / j) / (ssr_u / dof)
f_statistic
```

```
## [1] -4.920922
```

```r
F_test(df, "TCscaled", c("lQ_A", "lQ_B", "lQ_C", "lQ_D", "lQ_E", "PLscaled","PKscaled", "g1", "g2'
```

```
##     f_stat      p_value
## 1 291.9844 2.238126e-87
```

blerrrrrg- these don't match. . . . I'm not sure which is correct though. . . .

Question 8:

To see whether returns to scale declined with output, Nerlove tested a nonlinear specification by including $ln(y)^2$ as a regressor. Conduct a statistical test you feel is appropriate to test this hypothesis.

```
df %<>% mutate(Qlog_sq = (Qlog)^2)

reg_nl <- ols(data = df, y_data = "TCscaled",
             X_data = c("Qlog","Qlog_sq", "PLscaled","PKscaled"),
             intercept = T, H0 = 0, alpha = 0.05)

reg_nl$ttest
```

effect	coef	std_error	t_stat	p_value	significance
intercept	-1.635	0.305	-5.365	0.0000	Reject
Qlog	0.153	0.062	2.466	0.0149	Reject
Qlog_sq	0.116	0.012	9.418	0.0000	Reject
PLscaled	0.481	0.161	2.984	0.0034	Reject
PKscaled	0.074	0.150	0.494	0.6218	Fail to Reject