Problem Set #2

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Part 1: Theory

Part 2: Applied: Returns to Scale in Electricity Supply

First, load our OLS function created in Problem Set #1. We're including a built in t-test this time around.

```r
ols <- function(data, y_data, X_data, intercept = T, H0 = 0, two_tail = T, alpha = 0.05) {
  # Function setup ----
    # Require the 'dplyr' package
    require(dplyr)
    # Function to convert tibble, data.frame, or tbl_df to matrix
    to_matrix <- function(the_df, vars) {
      # Create a matrix from variables in var
      new_mat <- the_df %>%
        #Select the columns given in 'vars'
        select_(.dots = vars) %>%
        # Convert to matrix
        as.matrix()
      # Return 'new_mat'
      return(new_mat)
    }

  # Create dependent and independent variable matrices ----
    # y matrix
    y <- to_matrix (the_df = data, vars = y_data)
    # X matrix
    X <- to_matrix (the_df = data, vars = X_data)
      # If 'intercept' is TRUE, then add a column of ones
      if (intercept == T) {
      X <- cbind(1,X)
      colnames(X) <- c("intercept", X_data)
      }

  # Calculate b, y_hat, and residuals ----
    b <- solve(t(X) %*% X) %*% t(X) %*% y
    y_hat <- X %*% b
    e <- y - y_hat

  # Useful -----
    n <- nrow(X) # number of observations
    k <- ncol(X) # number of independent variables
    dof <- n - k # degrees of freedom
    i <- rep(1,n) # column of ones for demeaning matrix
    A <- diag(i) - (1 / n) * i %*% t(i) # demeaning matrix
    y_star <- A %*% y # for SST
    X_star <- A %*% X # for SSM
    SST <- drop(t(y_star) %*% y_star)
```

```r
  SSM <- drop(t(b) %*% t(X_star) %*% X_star %*% b)
  SSR <- drop(t(e) %*% e)

# Measures of fit and estimated variance ----
  R2uc <- drop((t(y_hat) %*% y_hat)/(t(y) %*% y)) # Uncentered R^2
  R2 <- 1 - SSR/SST # Uncentered R^2
  R2adj <- 1 - (n-1)/dof * (1 - R2) # Adjusted R^2
  AIC <- log(SSR/n) + 2*k/n # AIC
  SIC <- log(SSR/n) + k/n*log(n) # SIC
  s2 <- SSR/dof # s^2

# Measures of fit table ----
  mof_table_df <- data.frame(R2uc, R2, R2adj, SIC, AIC, SSR, s2)
  mof_table_col_names <- c("$R^2_\\text{uc}$", "$R^2$",
                           "$R^2_\\text{adj}$",
                           "SIC", "AIC", "SSR", "$s^2$")
  mof_table <-  mof_table_df %>% knitr::kable(
    row.names = F,
    col.names = mof_table_col_names,
    format.args = list(scientific = F, digits = 4),
    booktabs = T,
    escape = F
  )

# t-test----
  # Standard error
  se <- as.vector(sqrt(s2 * diag(solve(t(X) %*% X))))
  # Vector of _t_ statistics
  t_stats <- (b - H0) / se
  # Calculate the p-values
  if (two_tail == T) {
  p_values <- pt(q = abs(t_stats), df = dof, lower.tail = F) * 2
  } else {
    p_values <- pt(q = abs(t_stats), df = dof, lower.tail = F)
  }
  # Do we (fail to) reject?
  reject <- ifelse(p_values < alpha, reject <- "Reject", reject <- "Fail to Reject")

  # Nice table (data.frame) of results
  ttest_df <- data.frame(
    # The rows have the coef. names
    effect = rownames(b),
    # Estimated coefficients
    coef = as.vector(b) %>% round(3),
    # Standard errors
    std_error = as.vector(se) %>% round(3),
    # t statistics
    t_stat = as.vector(t_stats) %>% round(3),
    # p-values
    p_value = as.vector(p_values) %>% round(4),
    # reject null?
    significance = as.character(reject)
    )
```

```r
    ttest_table <-  ttest_df %>% knitr::kable(
      booktabs = T,
      format.args = list(scientific = F),
      escape = F
    )

  # Data frame for exporting for y, y_hat, X, and e vectors ----
    export_df <- data.frame(y, y_hat, e, X) %>% tbl_df()
    colnames(export_df) <- c("y","y_hat","e",colnames(X))

  # Return ----
    return(list(n=n, dof=dof, b=b, vars=export_df, R2uc=R2uc,R2=R2,
                R2adj=R2adj, AIC=AIC, SIC=SIC, s2=s2, SST=SST, SSR=SSR,
                mof_table=mof_table, ttest=ttest_table))
}
```

We'll also need a function to do an F-test for this Problem Set.

```r
# Joint test function (from Ed's notes). Could also write a more complex functions that takes R and r.
F_test <- function(data, y_var, X_vars) {
  # Turn data into matrices
  y <- to_matrix(data, y_var)
  X <- to_matrix(data, X_vars)
  # Add intercept
  X <- cbind(1, X)
  # Name the new column "intercept"
  colnames(X) <- c("intercept", X_vars)
  # Calculate n and k for degrees of freedom
  n <- nrow(X)
  k <- ncol(X)
  # J is k-1
  J <- k - 1
  # Create the R matrix: bind a column of zeros
  # onto a J-by-J identity matrix
  R <- cbind(0, diag(J))

  # Estimate coefficients
  b <- b_ols(data, y_var, X_vars)
  # Calculate OLS residuals
  e <- y - X %*% b
  # Calculate s^2
  s2 <- (t(e) %*% e) / (n-k)
  # Force s2 to numeric
  s2 %<>% as.numeric()

  # Create the inner matrix R(X'X)^(-1)R'
  RXXR <- R %*% solve(t(X) %*% X) %*% t(R)
  # Calculate the F stat
  f_stat <- t(R %*% b) %*% solve(RXXR) %*% (R %*% b) / J / s2
  # Calculate the p-value
  p_value <- pf(q = f_stat, df1 = J, df2 = n-k, lower.tail = F)
  # Create a data.frame of the f stat. and p-value
  results <- data.frame(
    f_stat = f_stat %>% as.vector(),
```

```
    p_value = p_value %>% as.vector())
  return(results)

}
```

Question 1:

Read the data into R. Inspect it. Sort by size (Q (kwh)).

```
nerlove <- readxl::read_excel("nerlove.xls", col_names=T)

# Fix typo in 13th row (missing a decimal!)
# DO THIS MORE ELEGANTLY!
nerlove[13, "PL"] <- 1.81

nerlove %>% arrange(Q)
```

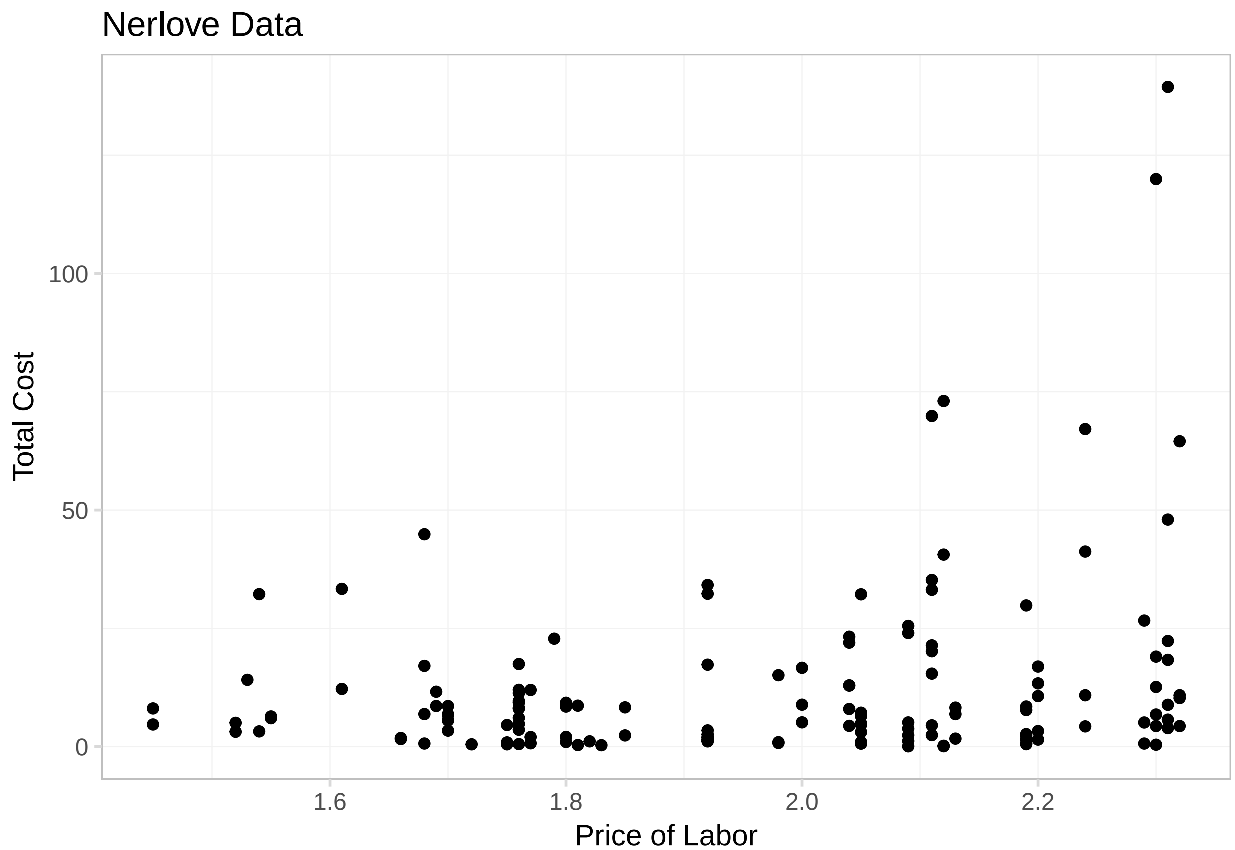
```
## # A tibble: 145 x 5
##        TC     Q    PL    PF    PK
##     <dbl> <dbl> <dbl> <dbl> <dbl>
##  1 0.0820    2.  2.09  17.9  183.
##  2 0.661     3.  2.05  35.1  174.
##  3 0.990     4.  2.05  35.1  171.
##  4 0.315     4.  1.83  32.2  166.
##  5 0.197     5.  2.12  28.6  233.
##  6 0.0980    9.  2.12  28.6  195.
##  7 0.949    11.  1.98  35.5  206.
##  8 0.675    13.  2.05  35.1  150.
##  9 0.525    13.  2.19  29.1  155.
## 10 0.501    22.  1.72  15.0  188.
## # ... with 135 more rows
```

```
head(nerlove)
```

```
## # A tibble: 6 x 5
##       TC     Q    PL    PF    PK
##    <dbl> <dbl> <dbl> <dbl> <dbl>
## 1 0.0820    2.  2.09  17.9  183.
## 2 0.661     3.  2.05  35.1  174.
## 3 0.990     4.  2.05  35.1  171.
## 4 0.315     4.  1.83  32.2  166.
## 5 0.197     5.  2.12  28.6  233.
## 6 0.0980    9.  2.12  28.6  195.
```

Plot the series and make sure your data are read in correctly.

```
ggplot(nerlove, aes(x=PL, y=TC)) +
  geom_point() +
  labs(title="Nerlove Data", x="Price of Labor", y="Total Cost") +
  theme_are
```




```
### plot rest of independent vars too.
```

Question 2:

Replicate regression I (page 176) in the paper.

Regression I:

$log(TC) - log(P_F) = \beta_0 + \beta_1 Q + \beta_2 \Big(log(P_L) - log(P_F)\Big) + \beta_3 \Big(log(P_K) - log(P_F)\Big)$

Equivalent to:

$log(\frac{TC}{P_F}) = \beta_0 + \beta_1 Q + \beta_2 \Big(log(\frac{P_L}{P_F})\Big) + \beta_3 \Big(log(\frac{P_K}{P_F}\Big)$

Where:
TC = total production cost, P_L = wage rate, P_K = "price" of capital, P_F = price of fuel
Q = output (measured in kWh)

In generalized Cobb-Douglas form: $\beta_1 = \frac{1}{r}$, $\beta_2 = \frac{a_L}{r}$, $\beta_3 = \frac{a_K}{r}$

Prepare variables for Regression I.

```
# Create log variables
nerlove %<>% mutate(
  TClog = log10(TC),
  Qlog = log10(Q),
  PLlog = log10(PL),
  PKlog = log10(PK),
  PFlog = log10(PF)
```

```
)

# Create PF scaled variables
nerlove %<>% mutate(
  TCscaled = TClog - PFlog,
  PLscaled = PLlog - PFlog,
  PKscaled = PKlog - PFlog
)
```

Variable names: $log(\frac{TC}{P_F})$ = "TCscaled" $log(\frac{P_L}{P_F}$ = "PLscaled" $log(\frac{P_K}{P_F}$ = "PKscaled"

```
# Regression I:
# dep var = (log costs - log fuel price) = TCscaled
reg_I <- ols(data = nerlove,y_data = "TCscaled",
             X_data = c("Qlog","PLscaled","PKscaled"),
             intercept = T, H0 = 0, alpha = 0.05)

reg_I$ttest
```

effect	coef	std_error	t_stat	p_value	significance
intercept	-2.037	0.384	-5.301	0.0000	Reject
Qlog	0.721	0.017	41.334	0.0000	Reject
PLscaled	0.593	0.205	2.898	0.0044	Reject
PKscaled	-0.007	0.191	-0.039	0.9692	Fail to Reject

```
reg_I$mof_table
```

R^2_{uc}	R^2	R^2_{adj}	SIC	AIC	SSR	s^2
0.966	0.9316	0.9301	-3.433	-3.515	4.082	0.02895
Coefficients are p	retty clo	se to those in the	paper. $R	^2$ match	es!	

Question 3:

Conduct the hypothesis test using constant returns to scale (β_1 = 1) as your null hypothesis. What is the p- value associated with you test statistic? What is your point estimate of returns to scale? Constant? Increasing? Decreasing?

```
# Re-run regression with null hypothesis H0 = 1
reg_I <- ols(data = nerlove,y_data = "TCscaled",
             X_data = c("Qlog","PLscaled","PKscaled"),
             intercept = T, H0 = 1, alpha = 0.05)

reg_I$ttest
```

effect	coef	std_error	t_stat	p_value	significance
intercept	-2.037	0.384	-7.903	0.0000	Reject
Qlog	0.721	0.017	-16.020	0.0000	Reject
PLscaled	0.593	0.205	-1.990	0.0485	Reject
PKscaled	-0.007	0.191	-5.282	0.0000	Reject