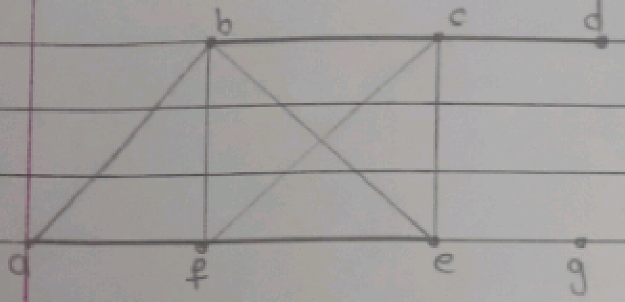
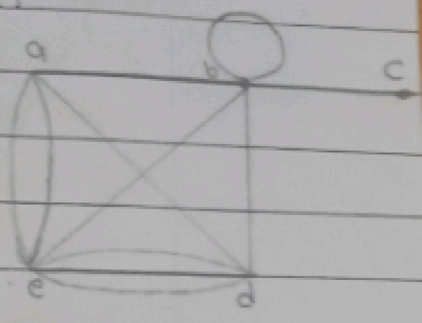
Unit 6. Graph.

Q1. What are the degrees and what are the neighborhods of the vertices in the graph G & H display in below figure.



G



H

Degree of graph G

deg(a) = 2
deg(b) = 4
deg(c) = 4
deg(d) = 1
deg(e) = 3
deg(f) = 4
deg(g) = 0

Degree of graph H

deg (a) = 4
deg (b) = 6
deg (c) = 1
deg (d) = 5
deg (e) = 6
deg (f) =

Graph G

vertex	neighbourhood
a	b, f
b	a, c, f, e
c	b, d, e, f
d	c
e	f, c, b
f	a, b, e, c
g	–

For Graph H

vertex	neighbourhood
a	b, e, d
b	a, c, d, e
c	b
d	e, b, a
e	a, d, b

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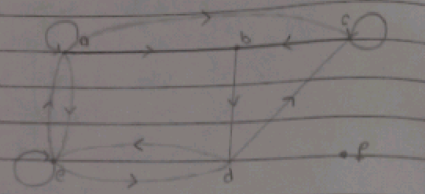
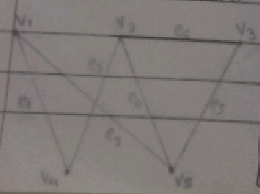
Q2. Find the in-degree and out-degree of each vertex in the graph G with directed edges show in figure.



Given graph is a directed graph hence it has both in-degree & out-degree

indegree	outdegree
$deg^-(a) = 2$	$deg^+(a) = 4$
$deg^-(b) = 2$	$deg^+(b) = 1$
$deg^-(c) = 3$	$deg^+(c) = 2$
$deg^-(d) = 2$	$deg^+(d) = 2$
$deg^-(e) = 3$	$deg^+(e) = 3$
$deg^-(f) = -$	$deg^+(f) = -$

Q4. Represent the graph shown in fig. below with an incidence matrix.



$$\begin{array}{c|cccccc} & e_1 & e_2 & e_3 & e_4 & e_5 & e_6 \\ \hline v_1 & 1 & 1 & 0 & 0 & 0 & 0 \\ v_2 & 0 & 0 & 1 & 1 & 0 & 1 \\ v_3 & 0 & 0 & 0 & 0 & 1 & 1 \\ v_4 & 1 & 0 & 1 & 0 & 0 & 0 \\ v_5 & 0 & 0 & 0 & 1 & 1 & 0 \end{array}$$

edges & loops. Multiple edges are represented in incidence matrix using column with identical entries because these edges are incident with same pair of vertices. ~~loops are represent using a column with exactly~~ one entry equal to 1 corresponding to the vertex i.e. incident with this loop.

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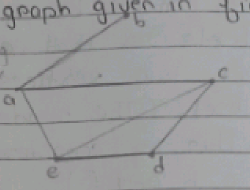
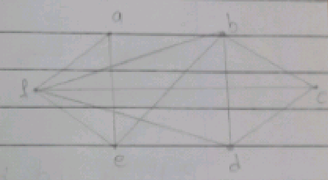
Q5. Use adjacency lists to describe the simple graph given in figure below



vertex	adjacency vertex
a	b, c, e
b	a
c	a, e, d
d	c, e
e	a, c, d

Q3. Are the graph G and H displayed in figure bipartile?



Soln: Graph G is bipartile because its vertex set is the union of two disjoint set, {a, b, d} and {c, e, f, g} and each edge connects a vertex in one of these subsets to a vertex in the other subset. (Note that for G to be bipartile it is not necessary that every vertex in {a, b, d} be adjacent to every vertex in {c, e, f, g}. For instance, b and g are not adjacent.

mapping using adjacency of vertices & degree

$f(u_1) = v_6$ $f(u_4) = v_5$
$f(u_2) = v_3$ $f(u_5) = v_1$
$f(u_3) = v_4$ $f(u_6) = v_2$

one to one correspondence below vertex set of G & H

$f(u_1) = v_6$ $f(u_4) = v_5$
$f(u_2) = v_3$ $f(u_5) = v_1$
$f(u_3) = v_4$ $f(u_6) = v_2$

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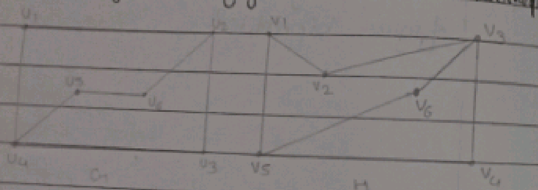
Graph H is not bipartile because its vertex set cannot be partitioned into two subsets so that edges do not connect two vertices from the same subset. (The reader should verify this by considering the vertices a, b and f.)

6. Determine whether the graphs G and H displayed in figure below are isomorphic



Both G and H have six vertices. and seven edges.

Degree sequence

For graph G
$deg(u_1) = 2$
$deg(u_2) = \cancel{2}3$
$deg(u_3) = 2$
$deg(u_4) = 3$
$deg(u_5) = 2$
$deg(u_6) = 2$
degree = 14

For graph H
$deg(v_1) = 2$
$deg(v_2) = 2$
$deg(v_3) = 3$
$deg(v_4) = 2$
$deg(v_5) = 3$
$deg(v_6) = 2$

we examine the adjacency matrix of G.

$$A_G = \begin{array}{c} \\ u_1 \\ u_2 \\ u_3 \\ u_4 \\ u_5 \\ u_6 \end{array} \begin{array}{c} \begin{array}{cccccc} u_1 & u_2 & u_3 & u_4 & u_5 & u_6 \end{array} \\ \begin{bmatrix} 0 & 1 & 0 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 & 1 \\ 0 & 1 & 0 & 0 & 1 & 0 \end{bmatrix} \end{array}$$

then,

we examine the adjacency matrix of H,

$$A_H = \begin{array}{c} \\ v_6 \\ v_5 \\ v_4 \\ v_3 \\ v_2 \\ v_1 \end{array} \begin{array}{c} \begin{array}{cccccc} v_6 & v_5 & v_4 & v_3 & v_2 & v_1 \end{array} \\ \begin{bmatrix} 0 & 1 & 0 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 & 1 \\ 0 & 1 & 0 & 0 & 1 & 0 \end{bmatrix} \end{array}$$

Because
$A_G = A_H$.
so,
The adjacency matrix is same
Hence,
it is isomorphic

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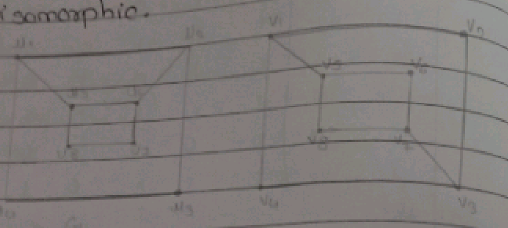
9. Determine whether the graph G and H displayed in figure below are isomorphic.



Soln: No. of edges of graph G is 10
No. of edges of graph H is 10

degree of G	degree of H
deg (v_1) = 3	deg (v_1) = 3
deg (v_2) = 3	deg (v_2) = 2
deg (v_3) = 2	deg (v_3) = 3
deg (v_4) = 2	deg (v_4) = 2
deg (v_5) = 3	deg (v_5) = 3
deg (v_6) = 3	deg (v_6) = 2
deg (v_7) = 2	deg (v_7) = 3
deg (v_8) = 2	deg (v_8) = 2

degree of G is 20 degree of H is 20
Degree of graph G and H are ~~not~~ isomorphic.

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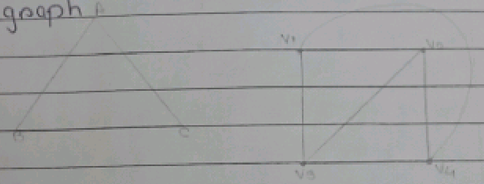
7. What is a planar graph? Explain with diagram.

Ans: A graph is called planar graph if it can be drawn in the plane without any edges crossing. Such a drawing is called as planar representation of the graph.

eg: The graph shown in fig is planar graph.



Consider a graph G = (V, E). A region is defined to be an area of the plane that is bounded by edges and cannot be further subdivided. A planar graph divides the plans into one or more regions. One of the regions will be infinite.

A graph is said to be planar if it can be drawn in a plane so that no edge cross.

We examine the adjacency matrix of G

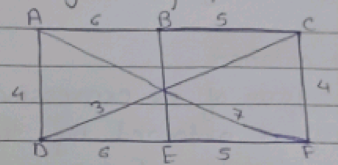
$$
A_G = \begin{array}{c} \\ v_1 \\ v_2 \\ v_3 \\ v_4 \\ v_5 \\ v_6 \\ v_7 \\ v_8 \end{array}
\begin{array}{c}
\begin{array}{cccccccc} v_1 & v_2 & v_3 & v_4 & v_5 & v_6 & v_7 & v_8 \end{array} \\
\begin{bmatrix}
0 & 1 & 0 & 1 & 1 & 0 & 0 & 0 \\
1 & 0 & 1 & 0 & 0 & 1 & 0 & 0 \\
0 & 1 & 0 & 1 & 0 & 0 & 0 & 0 \\
1 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\
1 & 0 & 0 & 0 & 0 & 1 & 0 & 1 \\
0 & 1 & 0 & 0 & 1 & 0 & 1 & 0 \\
0 & 0 & 0 & 0 & 0 & 1 & 0 & 1 \\
0 & 0 & 1 & 0 & 0 & 0 & 1 & 0
\end{bmatrix}
\end{array}
$$

We examine the ~~foll~~ adjacency matrix of H

$$
A_H = \begin{array}{c} \\ v_8 \\ v_7 \\ v_6 \\ v_5 \\ v_4 \\ v_3 \\ v_2 \\ v_1 \end{array}
\begin{array}{c}
\begin{array}{cccccccc} v_8 & v_7 & v_6 & v_5 & v_4 & v_3 & v_2 & v_1 \end{array} \\
\begin{bmatrix}
0 & 1 & 0 & 1 & 0 & 0 & 0 & 0 \\
1 & 0 & 1 & 0 & 0 & 1 & 0 & 0 \\
0 & 1 & 0 & 1 & 0 & 0 & 0 & 0 \\
1 & 0 & 1 & 0 & 0 & 0 & 0 & 1 \\
0 & 0 & 0 & 0 & 0 & 1 & 0 & 1 \\
0 & 1 & 0 & 0 & 1 & 0 & 1 & 0 \\
0 & 0 & 0 & 0 & 0 & 1 & 0 & 1 \\
0 & 0 & 0 & 1 & 1 & 0 & 1 & 0
\end{bmatrix}
\end{array}
$$

Tree.

Determine the minimum spanning tree of the weighted graph shown in figure



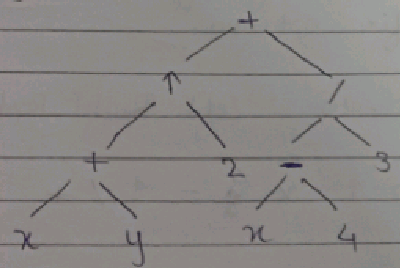
kruskals

Q4. Explain prefix, postfix and infix notation

Ans: Prefix Notation

prefix form of an expression is an expression obtained by traversing rooted three in preorder. Expression written in prefix form is said to be prefix / polish notation.

Eg: $((x+y)\uparrow 2) + ((x-4)/3)$



Prefix notation : Root left right

$+ \uparrow + x\ y\ 2\ /\ -\ x\ 4\ 3$

Infix notation.

Infix form of an expression is an expression obtained by traversing root tree in inorder. Expression written in infix form is said to be infix notation.

Infix notation : Left Root Right

$x + y \uparrow 2 + x - 4 / 3$

Postfix Notation.

Postfix form of an expression is an expression obtained by traversing rooted tree in postorder. Expression written in postfix form is said to be postfix notation.

Postfix Notation : Left Right Root

$x\ y + 2 \uparrow x\ 4 - 3 / +$

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Unit 5. Tree

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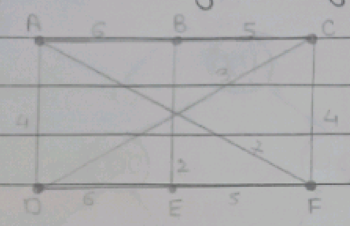
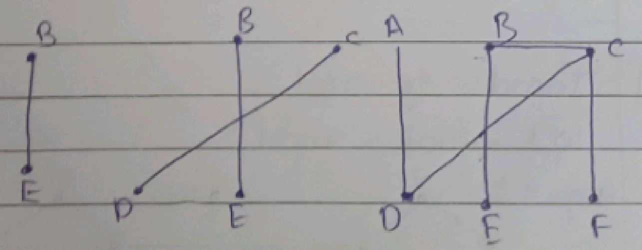
1. Determine the minimum spanning tree of the weighted graph shown in fig.

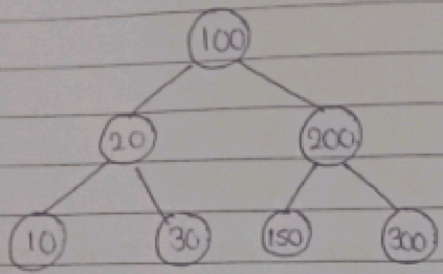


Soln: Given graph contain 6 nodes, so we

Choice	BE	
1	{B,E}	2
2	{C,D}	3
3	{C,F}	4
4	{A,D}	4
5	{B,C}	5
6	{	

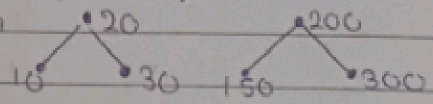


6. Find out preorder, inorder & postorder traversal in following tree.



1] preorder traversal
Root - left - Right

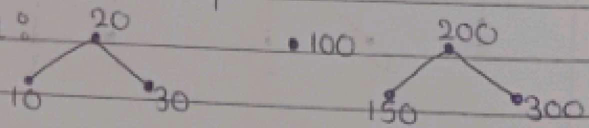
step 1: •100



step 2: •100 •200 •10 •30 •200 •150 •300

2] Inorder traversal
left - root - right

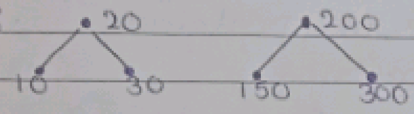
step 1:



step 2: 10 20 30 100 150 200 300

3. Postorder traversal
Left - right - root

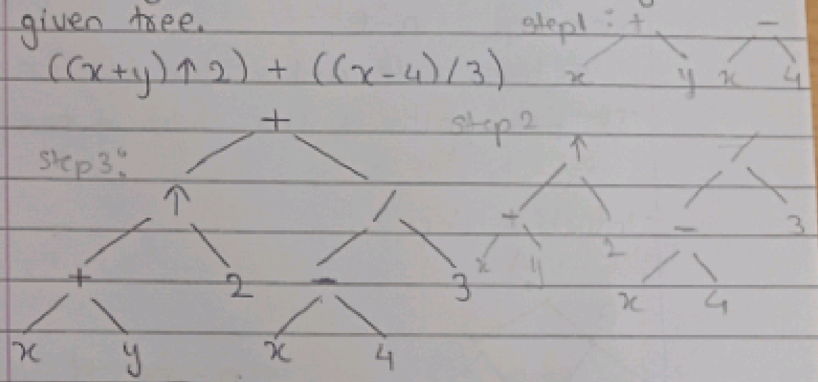
Step 1:



Step 2: •10 •30 •20 •150 •300 •200 •100

Q5. what is ordered rooted tree that represents the expression $((x+y)\uparrow 2) + ((x \div 4)/3)$? And write these expression in prefix and postfix notation of the given tree.

→ $((x+y)\uparrow 2) + ((x-4)/3)$



prefix: Prefix form of an expression is an expression obtained by traversing rooted tree in pre order

∴ prefix expression

$+ \uparrow + x\ y\ 2\ /\ -\ x\ 4\ 3$

postfix : postfix expression is an expression obtained by traversing rooted tree in postorder.

postfix expression:

$x \; y \; + \; 2 \; \uparrow \; x \; 4 \; - \; 3 \; / \; +$

Q3. Describe properties and application of tree

Ans: Properties of Trees

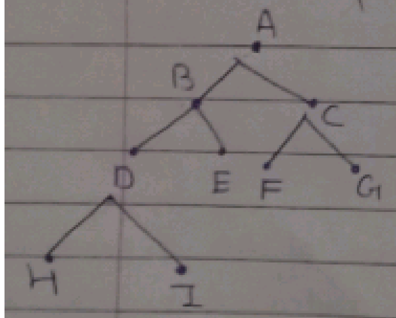
(1) A tree with n vertices has n−1 edges

basic step : n−1

a tree with n−1 vertex has no. edges

It ~~has~~ follows the theorem true for n−1

Inductive step:



n = no. of vertices

∴ n = 9.

then

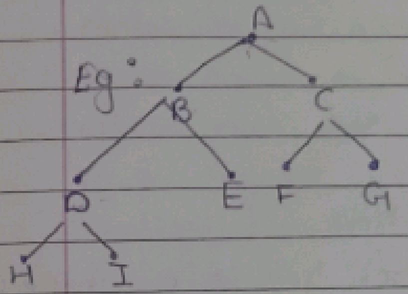
no. of edges = n−1

= 9−1

= 8 edges

(2) A full m-ary tree with internal vertices(i) contains $n = mi + 1$ vertices

Eg:



Internal vertices

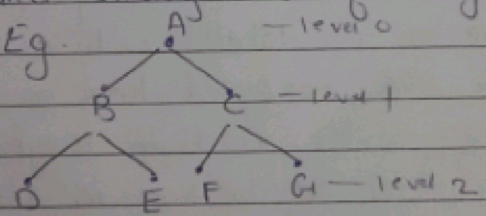
i = 4 (3 exception)

m = children

m = G

(3) There are at ~~moth~~ most 2^H leaves in the a binary tree of height H.

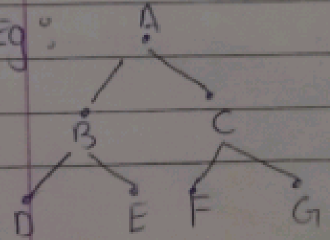
Eg.



$2^H = 2^2 = 4$ leaves

(4) If a binary tree with ~~capital~~ L leaves is full and balanced then its height is $H = \lceil \log_2 L \rceil$

Eg:



$H = \lceil \log_2 L \rceil$

$H = \lceil \log_2 4 \rceil$

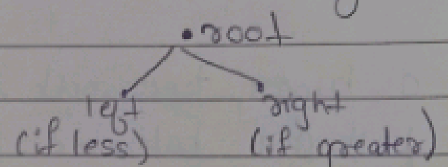
H = 2

Application of Tree

1. Binary search tree
2. Decision tree
3. Prefix code
4. Game tree

1) Binary Search tree

Binary search tree is in which each child of vertex is designated as right or left child, no vertex has more than one right child or left child and each vertex is labeled with a key, vertices are assigned keys so that the keys of vertex is both larger than the keys of all vertices in its left subtree and smaller than the keys of all vertices in its right subtree.

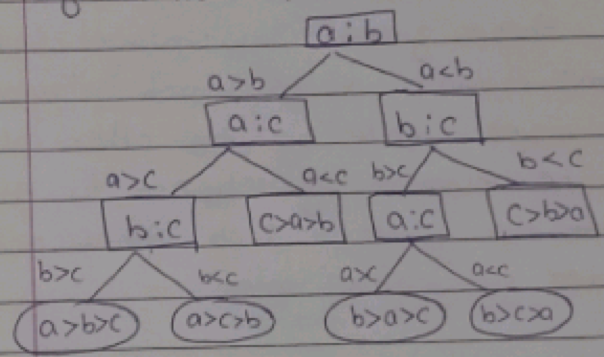


2) Decision tree.

A Decision tree is a binary tree containing an algorithm to decide which courses action to take. A rooted tree in which each internal ~~code~~ vertex corresponds to a decision with a subtree at these vertices for each possible outcomes of the decision is called as Decision tree.

Eg:

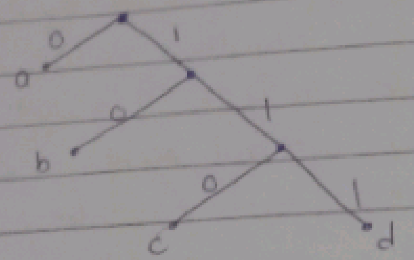
A decision tree that orders the elements of the list a, b, c



3) Prefix code

A set of sequence is said to prefix code if no sequence in the set is prefix for another sequence in set

Eg: {01, 10, 11, 000, 001}



a = 0
b = 10
c = 110
d = 111

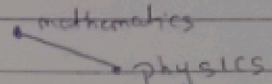
4. Game tree

Trees can be used to analyse certain types of games such as tic-tac-toe, nim, Checkers and chess. In each of these games, two playes takes turns making moves. Each player knows the moves made by the other players & no. element of chance to enters into the game. Such ~~game~~ model games uses game trees.

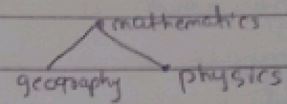
Q9. Ex: Form a binary search tree for the words mathematics, physics, geography, zoology, meteorology, geology, psychology, chemistry using alphabetical order.

Step 1: • mathematics

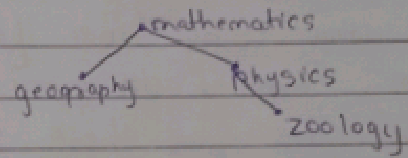
Step 2:



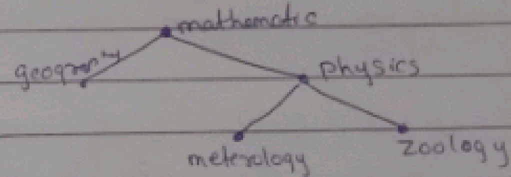
Step 3:



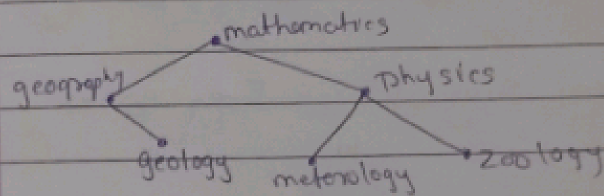
Step 4:



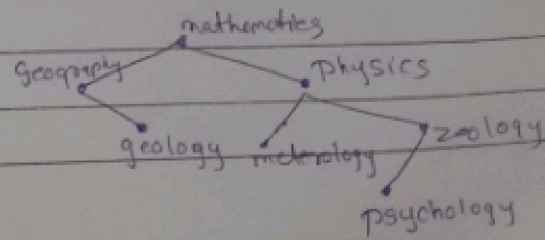
Step 5:



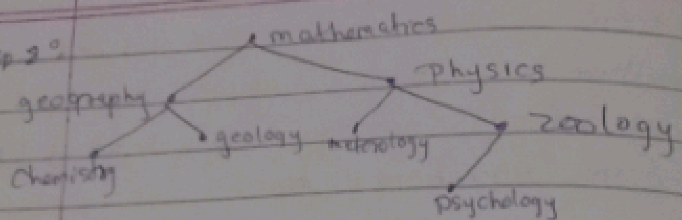
Step 6:



Step 7:



Step 3:

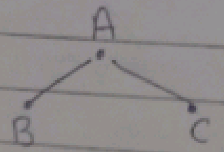


~~Step 4~~:

Q7)
→ An algorithm that takes as input the frequency of symbols in a string and produces as output a prefix code that encodes the string using the fewest possible bits, among all possible binary prefix codes for these symbols. This algorithm, known as Huffman coding

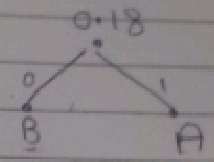
0.08	0.10	0.12	0.15	0.20
A	B	C	D	E

Given symbol & Frequencies



0.08	0.10	0.12	0.15	0.20	0.35
A	B	C	D	E	F

Step1: 0.12 C 0.15 D



0.20 E 0.35 F

Step2: