2nd Portfolio - Radiation Balance between parallel plates

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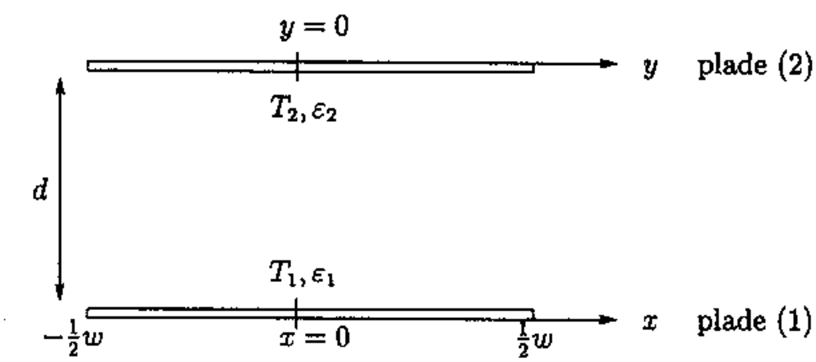
1 Introduction

Two infinitely long parallel plates of width w is placed in a distance d from one another, as shown in the figure below:



The plates radiates and reflects diffusely and has a uniform temperature of T_1 and T_2 respectively. Let $u(x)$ denote the radiation per unit time per unit area from plate (1) in distance x, from the plate's central axis, and let $v(y)$ denote the same function for plate (2). u and v is then solvable according to the two coupled integration equations:

$$u(x) = \varepsilon_1 \sigma T_1^4 + (1 - \varepsilon_1) \int_{-\frac{1}{2}w}^{\frac{1}{2}w} F(x, y, d) v(y) dy \tag{1}$$

$$v(y) = \varepsilon_2 \sigma T_2^4 + (1 - \varepsilon_2) \int_{-\frac{1}{2}w}^{\frac{1}{2}w} F(x, y, d) u(x) dx \tag{2}$$

where σ is the Stefan–Boltzmann constant, and F is given by:

$$F(x, y, d) = \frac{1}{2} \frac{d^2}{(d^2 + (x - y)^2)^{3/2}} \tag{3}$$

The integrals:

$$I_1(x) = \int_{-\frac{1}{2}w}^{\frac{1}{2}w} F(x,y,d)v(y)dy \tag{4}$$

$$I_2(y) = \int_{-\frac{1}{2}w}^{\frac{1}{2}w} F(x,y,d)u(x)dx \tag{5}$$

denotes the irradiance per unit time per unit area from the other plate. The total amount of heat which per unit time (and unit length) may be applied to each plate to keep the temperature constant, is given by:

$$Q_1(x) = \int_{-\frac{1}{2}w}^{\frac{1}{2}w} (u(x) - I_1(x))dx \tag{6}$$

$$Q_2(y) = \int_{-\frac{1}{2}w}^{\frac{1}{2}w} (v(y) - I_2(y))dy \tag{7}$$

Data: $T_1 = 1000,\ T_2 = 500,\ \varepsilon_1 = 0.80,\ \varepsilon_2 = 0.6,\ \sigma = 1.7212 \cdot 10^{-9},\ d = 1.0,\ w = 1.0$

(a) Use the Trapez Method to approximate the integrals in equations (1) and (2). Insert $(u_i)_{i=0}^N$ and $(v_i)_{i=0}^N$ for the values of u and v in the points $(x_i)_{i=0}^N$, which is contained in the quadrature formula. Then formulate the discrete problem corresponding to (1) and (2) as a linear system of equations in the variable $(u_i)_{i=0}^N$ and $(v_i)_{i=0}^N$.

(b) Write a program which establishes the system of equations in (a) and solves it for a given value of N. u and v should be specified for each N value in the chosen points (seen below). Q_1 and Q_2 should similarly be specified for this.
The chosen points are:

- $u(x)$ for $x = 0,\ x = \pm\frac{1}{4},\ x = \pm\frac{1}{2}$
- $v(y)$ for $y = 0,\ y = \pm\frac{1}{4},\ y = \pm\frac{1}{2}$

Choose $N = 4, 16, 32, ...$

(c) Determine Q_1 and Q_2 as accurate as possible from the previous results and specify a associated error of assessment.