Calibration of a planar 2-axis robot using SVD

Consider a planar 2-axis robot with the following relations between the joint angles θ_1, θ_2 and the "tool-position" (x, y):

$$\begin{aligned} x &= x_0 + a\cos\theta_1 + b\cos(\theta_1+\theta_2) \\ y &= y_0 + a\sin\theta_1 + b\sin(\theta_1+\theta_2) \end{aligned}$$

where (x_0, y_0) is the position of the "robot-base" and a, b denote the lengths of the two robot parts. Now we wish to experimentally determine x_0, y_0, a, b by measuring coordinated values $(\theta_1^{(i)}, \theta_2^{(i)}, x^{(i)}, y^{(i)}) \quad i = 1, \ldots, N$. Hereby, we may establish $2M$ linear equations in the four unknowns x_0, y_0, a, b:

$$\begin{aligned} x^{(1)} &= x_0 + a\cos\theta_1^{(1)} + b\cos(\theta_1^{(1)}+\theta_2^{(1)}) \\ y^{(1)} &= y_0 + a\sin\theta_1^{(1)} + b\sin(\theta_1^{(1)}+\theta_2^{(1)}) \\ x^{(2)} &= x_0 + a\cos\theta_1^{(2)} + b\cos(\theta_1^{(2)}+\theta_2^{(2)}) \\ y^{(2)} &= y_0 + a\sin\theta_1^{(2)} + b\sin(\theta_1^{(2)}+\theta_2^{(2)}) \\ \ldots &= \ldots \\ \ldots &= \ldots \\ x^{(N)} &= x_0 + a\cos\theta_1^{(N)} + b\cos(\theta_1^{(N)}+\theta_2^{(N)}) \\ y^{(N)} &= y_0 + a\sin\theta_1^{(N)} + b\sin(\theta_1^{(N)}+\theta_2^{(N)}) \end{aligned}$$

1. Find expressions for the elements in the matrix A and the right-hand side, say z, for the associated systems of linear equations for the parameters $q = (x_0, y_0, a, b)$.
2. Read the files with values, insert them in A and compute U, W og V using the method from NR. State (with arguments based on the SVD matrices) if there are linear dependencies between the parameters x_0, y_0, a, b.
3. Estimate the parameters $q = (x_0, y_0, a, b)$ and state your results. State also the residual error $\|Aq - z\|$.
4. The values of the $x^{(i)}, y^{(i)}$'s are measured with a camera, and there is a measurement uncertainty that is estimated to 1mm on each coordinate. Estimate the resulting errors on the found parameters using NR, Eq. (15.4.19).