
Calibration of a planar 2-axis robot using SVD

Question 1

$$A \cdot q = z \quad (1)$$

$$A = \begin{bmatrix} 1 & 0 & \cos(\theta_1^{(1)}) & \cos(\theta_1^{(1)} + \theta_2^{(1)}) \\ 0 & 1 & \sin(\theta_1^{(1)}) & \sin(\theta_1^{(1)} + \theta_2^{(1)}) \\ 1 & 0 & \cos(\theta_1^{(2)}) & \cos(\theta_1^{(2)} + \theta_2^{(2)}) \\ 0 & 1 & \sin(\theta_1^{(2)}) & \sin(\theta_1^{(2)} + \theta_2^{(2)}) \\ \dots & \dots & \dots & \dots \\ \dots & \dots & \dots & \dots \\ 1 & 0 & \cos(\theta_1^{(N)}) & \cos(\theta_1^{(N)} + \theta_2^{(N)}) \\ 0 & 1 & \sin(\theta_1^{(N)}) & \sin(\theta_1^{(N)} + \theta_2^{(N)}) \end{bmatrix} \quad (2)$$

$$q = \begin{bmatrix} x_0 \\ y_0 \\ a \\ b \end{bmatrix} \quad (3)$$

$$z = \begin{bmatrix} x^{(1)} \\ y^{(1)} \\ x^{(2)} \\ y^{(2)} \\ \dots \\ \dots \\ x^{(N)} \\ y^{(N)} \end{bmatrix} \quad (4)$$

Question 2

Linear dependencies can be seen in the W vector. If any value is close to zero, the parameter is irrelevant to the fit.

D1

$$W = \begin{bmatrix} 23.3788 \\ 22.6835 \\ 21.8248 \\ 21.5074 \end{bmatrix} \quad (5)$$

$$V = \begin{bmatrix} 0.589669 & -0.354287 & -0.351603 & 0.634938 \\ 0.311263 & 0.703527 & 0.509408 & 0.385577 \\ 0.493938 & -0.470162 & 0.62946 & -0.372496 \\ 0.55806 & 0.398095 & -0.469743 & -0.556265 \end{bmatrix} \quad (6)$$

As stated in eq. 5, there are no values close to zero, which means no linear dependencies between the parameters.

D2

$$W = \begin{bmatrix} 31.6427 \\ 22.7755 \\ 21.9092 \\ 0.0456291 \end{bmatrix} \quad (7)$$

$$V = \begin{bmatrix} 0.610775 & -0.340244 & -0.372019 & 0.610565 \\ 0.35454 & 0.633343 & 0.588192 & 0.356663 \\ 0.706218 & 0.0256017 & -0.0244777 & -0.707108 \\ 0.0501202 & -0.69459 & 0.717657 & 6.58184e - 05 \end{bmatrix} \quad (8)$$

As stated in eq. 7, it seems that there are linear dependencies between the parameters. By looking at the dataset it can be concluded that θ_1 has not been changed enough to excite the parameter.

Question 3

Residual error giving by eq. 10 and eq. 12.

D1

$$q = \begin{bmatrix} 4.09826 \\ 6.09964 \\ 50.0933 \\ 40.0896 \end{bmatrix} \quad (9)$$

$$\|A \cdot q - z\| = 8.96803 \quad (10)$$

D2

$$q = \begin{bmatrix} 2.40792 \\ 5.11981 \\ 52.0294 \\ 40.0893 \end{bmatrix} \quad (11)$$

$$\|A \cdot q - z\| = 9.18161 \quad (12)$$

Question 4

D1

$$STD = \begin{bmatrix} 0.0448463 \\ 0.0447814 \\ 0.0448083 \\ 0.0448329 \end{bmatrix} \quad (13)$$

D2

$$STD = \begin{bmatrix} 13.3811 \\ 7.81668 \\ 15.4969 \\ 0.0448065 \end{bmatrix} \quad (14)$$