Calibration of a planar 2-axis robot using SVD

Question 1

$$A \cdot q = z \tag{1}$$

$$A = \begin{bmatrix} 1 & 0 & Cos\left(\theta_1^{(1)}\right) & Cos\left(\theta_1^{(1)} + \theta_2^{(1)}\right) \\ 0 & 1 & Sin\left(\theta_1^{(1)}\right) & Sin\left(\theta_1^{(1)} + \theta_2^{(1)}\right) \\ 1 & 0 & Cos\left(\theta_1^{(2)}\right) & Cos\left(\theta_1^{(2)} + \theta_2^{(2)}\right) \\ 0 & 1 & Sin\left(\theta_1^{(2)}\right) & Sin\left(\theta_1^{(2)} + \theta_2^{(2)}\right) \\ ... & ... & ... & ... \\ ... & ... & ... & ... \\ 1 & 0 & Cos\left(\theta_1^{(N)}\right) & Cos\left(\theta_1^{(N)} + \theta_2^{(N)}\right) \\ 0 & 1 & Sin\left(\theta_1^{(N)}\right) & Sin\left(\theta_1^{(N)} + \theta_2^{(N)}\right) \end{bmatrix} \tag{2}$$

$$q = \begin{bmatrix} x_0 \\ y_0 \\ a \\ b \end{bmatrix} \tag{3}$$

$$z = \begin{bmatrix} x^{(1)} \\ y^{(1)} \\ x^{(2)} \\ y^{(2)} \\ ... \\ ... \\ x^{(N)} \\ y^{(N)} \end{bmatrix} \tag{4}$$

Question 2

Linear dependencies can been seen the W vector. If any values is close to zero the parameter is irrelevant to the fit.

D1

$$W = \begin{bmatrix} 23.3788 \\ 22.6835 \\ 21.8248 \\ 21.5074 \end{bmatrix} \tag{5}$$

$$V = \begin{bmatrix} 0.589669 & -0.354287 & -0.351603 & 0.634938 \\ 0.311263 & 0.703527 & 0.509408 & 0.385577 \\ 0.493938 & -0.470162 & 0.62946 & -0.372496 \\ 0.55806 & 0.398095 & -0.469743 & -0.556265 \end{bmatrix} \tag{6}$$

As stated in eq. 5, there are no values close to zero, which means no linear dependencies between the parameters.

D2

$$W = \begin{bmatrix} 31.6427 \\ 22.7755 \\ 21.9092 \\ 0.0456291 \end{bmatrix} \tag{7}$$

$$V = \begin{bmatrix} 0.610775 & -0.340244 & -0.372019 & 0.610565 \\ 0.35454 & 0.633343 & 0.588192 & 0.356663 \\ 0.706218 & 0.0256017 & -0.0244777 & -0.707108 \\ 0.0501202 & -0.69459 & 0.717657 & 6.58184e-05 \end{bmatrix} \tag{8}$$

As stated in eq. 7, there are linear dependencies between the parameters.

Question 3

Residual error giving below (eq. 10 and eq. 12) is the average of all individual residual errors.

D1

$$q = \begin{bmatrix} 4.09826 \\ 6.09964 \\ 50.0933 \\ 40.0896 \end{bmatrix} \tag{9}$$

$$\|A \cdot q - z\| = 0.247414 \tag{10}$$

D2

$$q = \begin{bmatrix} 2.40792 \\ 5.11981 \\ 52.0294 \\ 40.0893 \end{bmatrix} \tag{11}$$

$$\|A \cdot q - z\| = 0.252372 \tag{12}$$

Question 4

D1

$$STD = \begin{bmatrix} 0.0448463 \\ 0.0447814 \\ 0.0448083 \\ 0.0448329 \end{bmatrix} \tag{13}$$

D2

$$STD = \begin{bmatrix} 13.3811 \\ 7.81668 \\ 15.4969 \\ 0.0448065 \end{bmatrix} \tag{14}$$