**NANYANG TECHNOLOGICAL UNIVERSITY**

**SEMESTER 2 EXAMINATION 2018-2019**

**EE6221 – ROBOTICS AND INTELLIGENT SENSORS**

April / May 2019 Time Allowed: 3 hours

**INSTRUCTIONS**

1. This paper contains 5 questions and comprises 6 pages.
2. Answer ALL questions.
3. All questions carry equal marks.
4. This is a closed book examination.
5. Unless specifically stated, all symbols have their usual meanings.

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1. A robotic manipulator with six joints is shown in Figure 1.

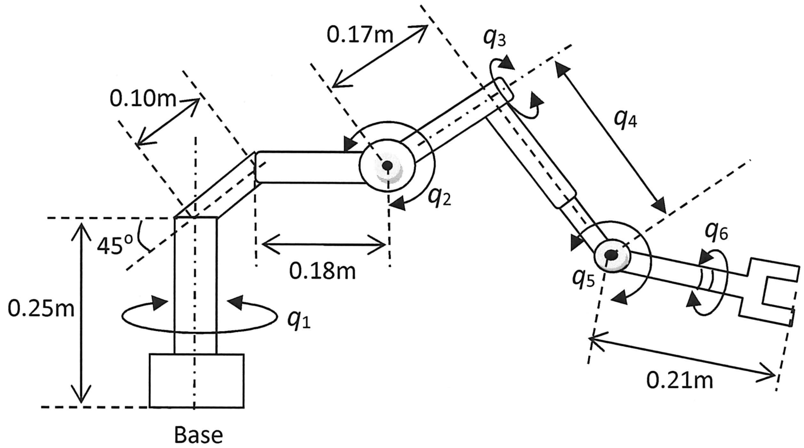



**Figure 1**

(a) Use the Denavit-Hartenberg (D-H) algorithm to obtain the link coordinate diagram. Derive the kinematic parameters of the robot.

(16 Marks)

Note: Question No. 1 continues on page 2.

(b) Assume that the arm matrix of the robot manipulator in part 1(a) has been derived as $T(q)_{base}^{tool}$, and the robot is now mounted on a platform as shown in Figure 2. The origin of the new reference frame is indicated in Figure 2. Discuss how the transformation matrix of the tool frame with respect to the new reference frame can be obtained.

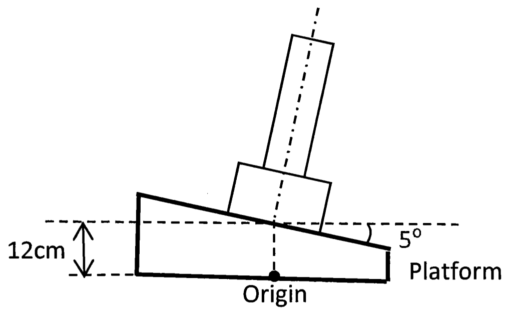



**Figure 2**

(4 Marks)

2. The dynamic equations of a Cartesian robot with three joint variables $x_1, x_2, x_3$ and three control inputs $u_1, u_2, u_3$ are given as follows:

$$5\ddot{x}_1 + 7\dot{x}_1 = u_1$$

$$2\ddot{x}_2 + 9\dot{x}_2 = u_2$$

$$10\ddot{x}_3 + 8\dot{x}_3 + 98 = u_3$$

(a) Design a hybrid position and force controller for the robot when the end effector is in contact with the environment such that the contact forces are given as $f_2 = 50(x_2 - 0.5)$ and $f_3 = 30(x_3 - 0.3)$. The system should be critically damped and does not excite all the unmodelled resonances at 10 Hz, 20 Hz and 30 Hz.

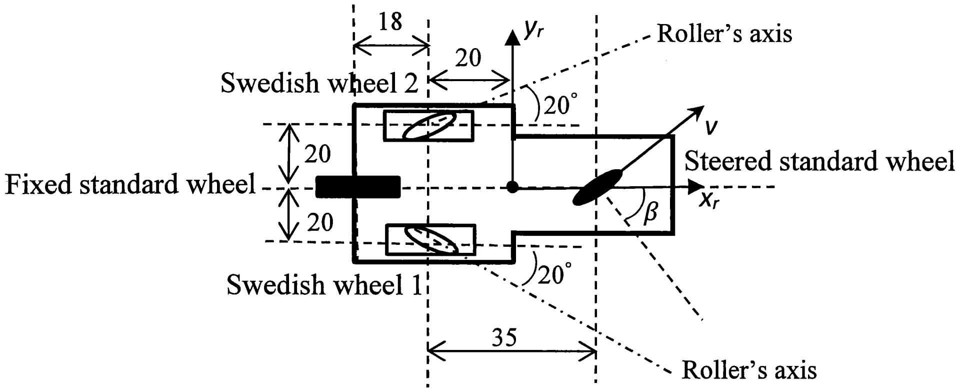
(12 Marks)

(b) The robot is now used to perform a task by using the controller designed in part 2(a). Derive the error equations if an unknown constant disturbance is acting on $x_1$ axis and the contact force along $x_3$ axis is estimated wrongly as $29(x_3 - 0.29)$.

(8 Marks)

3. (a) A mobile robot with two Swedish wheels, one steered standard wheel and one fixed standard wheel, is shown in Figure 3. The radius of each Swedish wheel is 10 cm and the radius of each roller is 2.5 cm with the roller's axis shown in Figure 3. The radius of the fixed standard wheel is 7 cm and the radius of the steered standard wheel is 7.5 cm with the steered angle shown in Figure 3. The rotational velocities of the Swedish wheels and the rollers are denoted by $\dot{\varphi}_{S1}, \dot{\varphi}_{S2}, \dot{\varphi}_{r1}$ and $\dot{\varphi}_{r2}$, respectively. The rotational velocities of the steered standard wheel and the fixed standard wheel are denoted by $\dot{\varphi}_{ss}$ and $\dot{\varphi}_{fs}$, respectively. A local reference frame $(x_r, y_r)$ of the mobile robot is assigned as shown in Figure 3. Derive the rolling and sliding constraints of the mobile robot.

(10 Marks)




Note: all lengths are in centimeters.

**Figure 3**

(b) A robot manipulator with four joint variables $q_1, q_2, q_3, q_4$ are now mounted on the mobile robot. The link-coordinate homogeneous transformation matrices from the base coordinate to the tool coordinate of the robotic manipulator are given as:

$$T_{base}^{wrist} = \begin{bmatrix} c_1c_2 & -s_1 & c_1s_2 & 0.2s_1 + c_1s_2q_3 \\ s_1c_2 & c_1 & s_1s_2 & 0.2c_1 + s_1s_2q_3 \\ -s_2 & 0 & c_2 & c_2q_3 \\ 0 & 0 & 0 & 1 \end{bmatrix} \qquad T_{wrist}^{tool} = \begin{bmatrix} c_4 & s_4 & 0 & 0 \\ s_4 & -c_4 & 0 & 0 \\ 0 & 0 & 1 & 0.75 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

where $s_1 = \sin(q_1)$, $s_2 = \sin(q_2)$, $s_4 = \sin(q_4)$, $c_1 = \cos(q_1)$, $c_2 = \cos(q_2)$, and $c_4 = \cos(q_4)$. Determine the forward kinematic equations of this robot in the tool configuration form and the tool configuration Jacobian matrix. Formulate a resolve motion rate control problem.

Note: Calculation of inverse matrix is not required.

(10 Marks)

4. A camera takes three images of a fixed object at three poses as the camera moves. Three coordinate frames, represented by $F_1$, $F_2$ and $F_3$, respectively, are attached to the projection center of the camera at the three poses, respectively, as shown in Figure 4. Seven coplanar feature points on the fixed object have been selected and labeled as $O_1$, $O_2$, $O_3$, $O_4$, $O_5$, $O_6$ and $O_7$. Out of the seven feature points, any four of them are non-collinear.

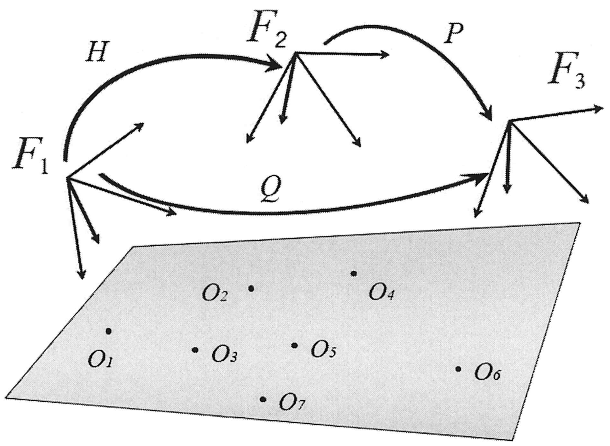



**Figure 4**

Five feature points $O_1$, $O_2$, $O_3$, $O_5$ and $O_7$ can be detected in the image taken at the pose attached to $F_1$. Their corresponding normalized coordinates in $F_1$ are given by

$$m_1^1 = [a_{1x}, a_{1y}, 1]^T, \qquad m_2^1 = [a_{2x}, a_{2y}, 1]^T, \qquad m_3^1 = [a_{3x}, a_{3y}, 1]^T$$
$$m_5^1 = [a_{5x}, a_{5y}, 1]^T, \qquad m_7^1 = [a_{7x}, a_{7y}, 1]^T.$$

Five feature points $O_2$, $O_3$, $O_4$, $O_5$ and $O_7$ can be detected in the image taken at the pose attached to $F_2$. Their corresponding normalized coordinates in $F_2$ are given by

$$m_2^2 = [b_{2x}, b_{2y}, 1]^T, \qquad m_3^2 = [b_{3x}, b_{3y}, 1]^T, \qquad m_4^2 = [b_{4x}, b_{4y}, 1]^T$$
$$m_5^2 = [b_{5x}, b_{5y}, 1]^T, \qquad m_7^2 = [b_{7x}, b_{7y}, 1]^T.$$

Five feature points $O_3$, $O_4$, $O_5$, $O_6$ and $O_7$ can be detected in the image taken at the pose attached to $F_3$. Their corresponding normalized coordinates in $F_3$ are given by

$$m_3^3 = [c_{3x}, c_{3y}, 1]^T, \qquad m_4^3 = [c_{4x}, c_{4y}, 1]^T, \qquad m_5^3 = [c_{5x}, c_{5y}, 1]^T$$
$$m_6^3 = [c_{6x}, c_{6y}, 1]^T, \qquad m_7^3 = [c_{7x}, c_{7y}, 1]^T.$$

Note: Question No. 4 continues on page 5.

Let $H = \begin{bmatrix} h_{11} & h_{12} & h_{13} \\ h_{21} & h_{22} & h_{23} \\ h_{31} & h_{32} & h_{33} \end{bmatrix}$ denote the Euclidean homography matrix from $F_1$ to $F_2$.

Let $P = \begin{bmatrix} p_{11} & p_{12} & p_{13} \\ p_{21} & p_{22} & p_{23} \\ p_{31} & p_{32} & p_{33} \end{bmatrix}$ denote the Euclidean homography matrix from $F_2$ to $F_3$.

Let $Q = \begin{bmatrix} q_{11} & q_{12} & q_{13} \\ q_{21} & q_{22} & q_{23} \\ q_{31} & q_{32} & q_{33} \end{bmatrix}$ denote the Euclidean homography matrix from $F_1$ to $F_3$.

(a) Derive a set of linear equations that can be used to compute a scaled homography matrix $\dfrac{H}{h_{33}}$.

(7 Marks)

(b) Derive a set of linear equations that can be used to compute $\dfrac{P}{p_{33}}$.

(7 Marks)

(c) Can $\dfrac{Q}{q_{33}}$ be obtained? If yes, explain how to compute it. If no, explain why it cannot be obtained.

(6 Marks)

5. Three sensors are used to measure a parameter $x_k$ that can be modelled by $x_{k+1} = x_k$. The outputs of the three sensors are represented by $z_{1k}$, $z_{2k}$ and $z_{3k}$, respectively, and they are modelled by

$$\begin{aligned} z_{1k} &= a_1 x_k + v_{1k} \\ z_{2k} &= a_2 x_k + v_{2k} \\ z_{3k} &= a_3 x_k + v_{3k} \end{aligned}$$

where $v_{1k}$, $v_{2k}$ and $v_{3k}$ are zero mean Gaussian noises with variances given by $\sigma^2$, $4\sigma^2$ and $9\sigma^2$, respectively, and $a_1$, $a_2$ and $a_3$ are positive parameters of the sensors.

Note: Question No. 5 continues on page 6.

Denoting $\hat{x}_k$ as the estimate of $x_k$, the predictor is designed as:

$$\hat{x}_{k+1} = \hat{x}_k + L_k\left(z_{1k} - a_1\hat{x}_k\right) + M_k\left(z_{2k} - a_2\hat{x}_k\right) + N_k\left(z_{3k} - a_3\hat{x}_k\right)$$

Let the estimation error be denoted as $\tilde{x}_{k+1} = x_{k+1} - \hat{x}_{k+1}$. Assume that the estimation error and the noise terms $v_{1k}$, $v_{2k}$ and $v_{3k}$ are uncorrelated, and that $E\left[\tilde{x}_{k+1}\right] = 0$. Let the estimation error variance be $p_{k+1} = E\left[\tilde{x}_{k+1}^2\right]$.

(a) Find the update equation for the state estimation error variance $p_{k+1}$.

(7 Marks)

(b) Design the update laws for $L_k$, $M_k$ and $N_k$ to minimize the estimation error variance $p_{k+1}$.

(8 Marks)

(c) Under the designs in parts 5(a) and 5(b), select values for the parameters of the sensors $a_1$, $a_2$ and $a_3$ so that $p_{k+1}$ is independent of $\sigma^2$, if such values exist. If such values do not exist, explain why.

(5 Marks)

END OF PAPER

# EE6221 ROBOTICS & INTELLIGENT SENSORS

Please read the following instructions carefully:

1. **Please do not turn over the question paper until you are told to do so. Disciplinary action may be taken against you if you do so.**

2. You are not allowed to leave the examination hall unless accompanied by an invigilator. You may raise your hand if you need to communicate with the invigilator.

3. Please write your Matriculation Number on the front of the answer book.

4. Please indicate clearly in the answer book (at the appropriate place) if you are continuing the answer to a question elsewhere in the book.