1. $u = \alpha v + \beta$

$\alpha = 3\theta$

$\beta = 5\dot{\theta}^2 - 2\cos\theta$

2. $(1+\sqrt{3}\theta)\ddot{\theta} + \dot{\theta} + 2\cos\theta + \sqrt{2\theta} = \tau$

$\tau = \alpha v + \beta$

$\alpha = 1+\sqrt{3}\theta$

$\beta = \dot{\theta} + 2\cos\theta + \sqrt{2\theta}$

3) $q = \begin{bmatrix} \theta \\ h \\ r \end{bmatrix}$ $\tau = \begin{bmatrix} \tau_1 \\ \tau_2 \\ \tau_3 \end{bmatrix}$

$$\begin{bmatrix} m_2 r^2 & 0 & 0 \\ 0 & m_1+m_2 & 0 \\ 0 & 0 & m_2 \end{bmatrix} \begin{bmatrix} \ddot{\theta} \\ \ddot{h} \\ \ddot{r} \end{bmatrix} + \begin{bmatrix} 2m_2 r \dot{r} \dot{\theta} \\ 0 \\ -m_2 r \dot{\theta}^2 \end{bmatrix} + \begin{bmatrix} 0 \\ (m_1+m_2) g \\ 0 \end{bmatrix}$$

$$= \begin{bmatrix} \tau_1 \\ \tau_2 \\ \tau_3 \end{bmatrix}$$

$$\Rightarrow M(q)\ddot{q} + C(q,\dot{q})\dot{q} + g(q) = \tau$$

$$\tau = \alpha V + \beta$$

$$\alpha = \begin{bmatrix} m_2 r^2 & 0 & 0 \\ 0 & m_1+m_2 & 0 \\ 0 & 0 & m_2 \end{bmatrix} \quad \text{i.e. } M(q)$$

$$\beta = \begin{bmatrix} 2m_2 r \dot{r} \dot{\theta} \\ (m_1+m_2) g \\ -m_2 r \dot{\theta}^2 \end{bmatrix} \quad \text{i.e. } C(q,\dot{q})\dot{q} + g(q)$$

$$\Rightarrow \ddot{q} = V \quad \text{or} \quad \begin{bmatrix} \ddot{\theta} \\ \ddot{h} \\ \ddot{r} \end{bmatrix} = \begin{bmatrix} v_1 \\ v_2 \\ v_3 \end{bmatrix}$$

$$\Rightarrow \quad \ddot{\theta} = v_1$$

$$\ddot{h} = v_2$$

$$\ddot{r} = v_3$$

Servo portion

$$v_1 = \ddot{\theta}_d + k_{v1}\dot{e}_1 + k_{p1}e_1$$
$$v_2 = \ddot{h}_d + k_{v2}\dot{e}_2 + k_{p2}e_2$$
$$v_3 = \ddot{r}_d + k_{v3}\dot{e}_3 + k_{p3}e_3$$

$$\Rightarrow \quad \ddot{e}_1 + k_{v1}\dot{e}_1 + k_{p1}e_1 = 0$$

$$\ddot{e}_2 + k_{v2}\dot{e}_2 + k_{p2}e_2 = 0$$

$$\ddot{e}_3 + k_{v3}\dot{e}_3 + k_{p3}e_3 = 0$$

or

$$\ddot{e}_i + k_{vi}\dot{e}_i + k_{pi}e_i = 0 \quad , \quad i = 1, 2, 3$$

characteristic equations

$$s^2 + k_{vi}s + k_{pi} = 0$$

Performance specifications

i) $\zeta = 1$

ii) $\omega_{res} = 2\pi \times 14.5\ Hz = 91.1\ rad\ s^{-1}$

$\omega_n \leq 0.5\ \omega_{res} = 0.5 \times 91.1 = 45.55\ rad\ s^{-1}$

choose $\omega_n = 45$

Characteristic equation

$$s^2 + 2\zeta\omega_n s + \omega_n^2$$

$$= s^2 + 90s + 2025$$

Hence

$$K_{v1} = K_{v2} = K_{v3} = 90$$

$$K_{p1} = K_{p2} = K_{p3} = 2025$$

$$\tau = \alpha v + \beta$$

$$\begin{bmatrix} \tau_1 \\ \tau_2 \\ \tau_3 \end{bmatrix} = \begin{bmatrix} m_2 r^2 & 0 & 0 \\ 0 & m_1+m_2 & 0 \\ 0 & 0 & m_2 \end{bmatrix} \begin{bmatrix} \ddot{\theta}_d + 90\dot{e}_1 + 2025 e_1 \\ \ddot{r}_d + 90\dot{e}_2 + 2025 e_2 \\ \ddot{r}_d + 90\dot{e}_3 + 2025 e_3 \end{bmatrix}$$

$$+ \begin{bmatrix} 2m_2 r \dot{r} \dot{\theta} \\ (m_1+m_2) g h \\ -m_2 r \dot{\theta}^2 \end{bmatrix}$$

Sensory information

q measurement

$$\dot{q} = \frac{dq}{dt}$$