NANYANG TECHNOLOGICAL UNIVERSITY

SEMESTER 2 EXAMINATION 2020-2021

EE6221 – ROBOTICS AND INTELLIGENT SENSORS

April / May 2021 Time Allowed: 3 hours

INSTRUCTIONS

1. This paper contains 5 questions and comprises 6 pages.
2. Answer all 5 questions.
3. All questions carry equal marks.
4. This is a closed book examination.
5. Unless specifically stated, all symbols have their usual meanings.

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1. A robotic manipulator with six joints is shown in Figure 1.

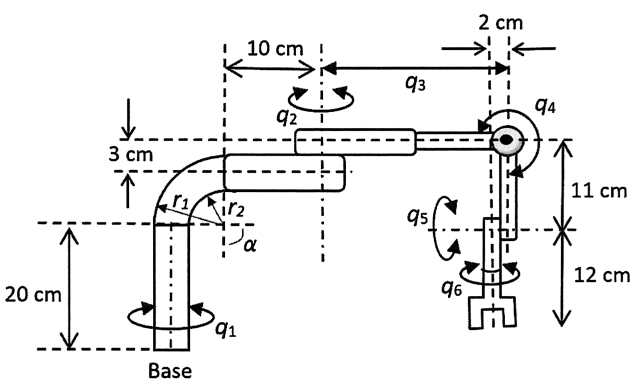


Figure 1

(a) Given that $r_1 = 8$ cm, $r_2 = 4$ cm and $\alpha = 90°$, obtain the link coordinate diagram by using the Denavit-Hartenberg (D-H) algorithm.

(10 Marks)

(b) Derive the kinematic parameters of the robot based on the coordinate diagram obtained in part (a).

(6 Marks)

Note: Question No. 1 continues on page 2.

(c) If $\alpha = 80°$, derive the kinematic parameters of the first two joints.

(4 Marks)

2. A Cartesian robot with two degrees of freedom is in contact with a frictionless surface as shown in Figure 2.

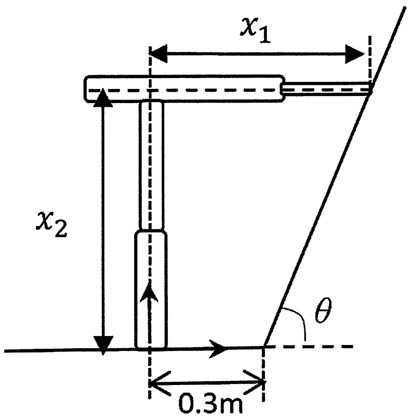


Figure 2

The dynamic equations of the robot with joint variables x_1, x_2 and control inputs u_1, u_2 are given as follows:

$$m_1\ddot{x}_1 + b_1\dot{x}_1 + f_1 = u_1$$

$$(m_1 + m_2)\ddot{x}_2 + b_2\dot{x}_2 + (m_1 + m_2)g + f_2 = u_2$$

where $m_1 = 2$ kg and $m_2 = 1$ kg are the masses of the links, $b_1 = 5$ and $b_2 = 7$ are the friction coefficients of the joints, f_1 and f_2 are the contact forces, $g = 9.8$ m/s^2 is the acceleration due to gravity, and u_1, u_2 are the control inputs. Since the surface is frictionless, the contact force is always perpendicular to the contact surface. Assume that the stiffness of the surface in the direction of the contact force is always equal to 10 N/m.

(a) If $\theta = 90°$ such that $f_2 = 0$, design a hybrid position and force controller for the robot. The system should be overdamped with a damping ratio of 1.1 and does not excite all the unmodelled resonances at 12 rad/s and 24 rad/s.

(10 Marks)

(b) The controller designed in part (a) is now implemented on the robot that is used to perform a task repeatedly in a spacecraft. If the spacecraft is travelling from earth to moon, explain the effects on the robot system by deriving the error equations.

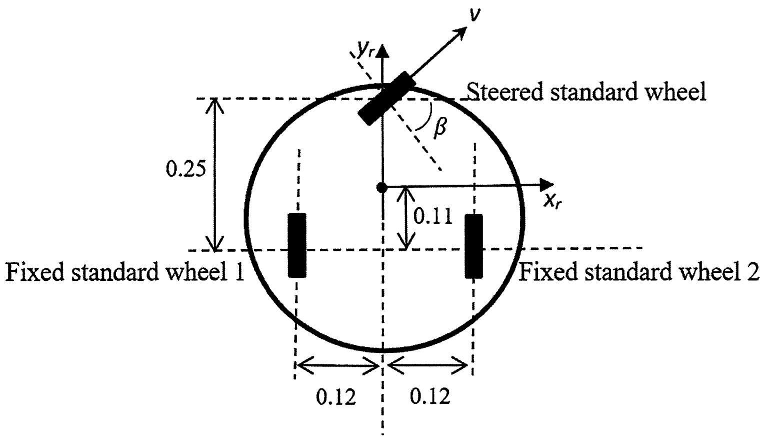
(5 Marks)

(c) If θ is less than 90°, design a hybrid position and force controller for the robot.

(5 Marks)

3. (a) A mobile robot with two standard wheels and one steered standard wheel is shown in Figure 3. A local reference frame (x_r, y_r) and a steered angle β are assigned to the mobile robot as shown in Figure 3. The radius of each standard wheel is 5.0 cm and the radius of the steered standard wheel is 5.5 cm. If the rotational velocities of the steered standard wheel and the two standard wheels are denoted by $\dot{\varphi}_{ss}$, $\dot{\varphi}_{fs1}$, and $\dot{\varphi}_{fs2}$, respectively, derive the rolling and sliding constraints of the mobile robot.

(10 Marks)



Note: all lengths are in meters.

Figure 3

(b) A robot manipulator with four joint variables q_1, q_2, q_3, q_4 are mounted on a mobile robot. The link-coordinate homogeneous transformation matrix from the base coordinate to the tool coordinate of the robotic manipulator is given as:

$$T_{Base}^{Tool} = \begin{bmatrix} s_1s_4 + c_1c_4c_{23} & s_1c_4 - c_1s_4c_{23} & c_1s_{23} & 0.5c_1c_2 + c_1c_{23} \\ c_1s_4 + s_1c_4c_{23} & -c_1c_4 - s_1s_4c_{23} & s_1s_{23} & 0.5s_1c_2 + s_1c_{23} \\ c_4c_{23} & -s_4s_{23} & c_{23} & 0.2 + 0.5s_2 + s_{23} \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

where $s_1 = \sin(q_1)$, $s_2 = \sin(q_2)$, $s_4 = \sin(q_4)$, $c_1 = \cos(q_1)$, $c_2 = \cos(q_2)$, $c_4 = \cos(q_4)$, $c_{23} = \cos(q_2 + q_3)$ and $s_{23} = \sin(q_2 + q_3)$.

(i) Solve the inverse kinematic problem using an analytic method to express $(q_1, q_2, q_3)^T$ in terms of the position of the end effector $(x, y, z)^T$.

(ii) If an approach direction of the end effector should be specified as $(0, 0, -1)^T$, solve the inverse kinematic problem to express $(q_1, q_2, q_3)^T$ in terms of the position of the end effector $(x, y, z)^T$.

(10 Marks)

4. As shown in Figure 4, a moving camera takes three images of a fixed object at three poses. Three coordinate frames represented by *F*, *M* and *N* are attached to the projection centre of the camera at the three poses, respectively. Six coplanar feature points on the fixed object have been selected, and labeled by *P1, P2, P3, P4, P5* and *P6.* Any four of them are non-collinear.

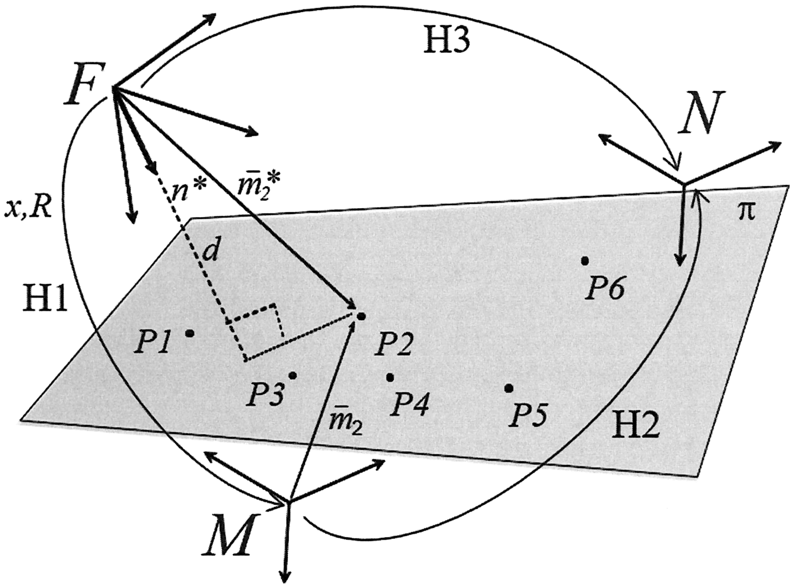


Figure 4

Four feature points *P1, P2, P3, P4* can be detected in the image taken at the pose attached to *F*. Their Euclidean coordinates in *F* are denoted as $\bar{m}_i^* = \left[x_i^*, y_i^*, z_i^*\right]^T$, $i = 1, 2, 3, 4$. Their corresponding normalized coordinates in *F* are given by $m_i^F = \dfrac{\bar{m}_i^*}{z_i^*} = \left[a_{ix}, a_{iy}, 1\right]^T$, $i = 1, 2, 3, 4$.

Five feature points *P1, P2, P3, P4, P5* can be detected in the image taken at the pose attached to M. The Euclidean coordinates in M are denoted as $\bar{m}_i = \left[x_i, y_i, z_i\right]^T$, $i = 1, 2, 3, 4, 5$. Their corresponding normalized coordinates in *M* are given by $m_i^M = \dfrac{\bar{m}_i}{z_i} = \left[b_{ix}, b_{iy}, 1\right]^T$, $i = 1, 2, 3, 4, 5$.

Five feature points *P2, P3, P4, P5, P6* can be detected in the image taken at the pose attached to *N*. Their corresponding normalized coordinates in *N* are given by $m_i^N = \left[c_{ix}, c_{iy}, 1\right]^T$, $i = 2, 3, 4, 5, 6$.

Note: Question No. 4 continues on page 5.

Let $H1$ denote the Euclidean homography matrix from F to M, satisfying $\bar{m}_i = H1\bar{m}_i^*$, $i = 1,2,3,4$. Let $H_n = \dfrac{H1}{h_{33}} = \begin{bmatrix} h_1 & h_2 & h_3 \\ h_4 & h_5 & h_6 \\ h_7 & h_8 & 1 \end{bmatrix}$, where h_{33} is the third row third column element of $H1$. The coordinate frame M is related to F by a rotation of R and a translation of x, so that $\bar{m}_i = R\bar{m}_i^* + x$, $i = 1,2,3,4$. The distance from the origin of F to the plane π is denoted as d, satisfying $d = \left(n^*\right)^T \bar{m}_2^*$, where n^* represents a unit direction vector.

Let $H2$ denote the Euclidean homography matrix from M to N, and let $H3$ denote the Euclidean homography matrix from F to N.

(a) Find an equation to relate m_2^F to m_2^M using R, x, d, n^*, z_2 and z_2^*.

(6 Marks)

(b) Derive a set of linear equations that can be used to compute H_n (i.e., the scaled homography matrix for $H1$).

(8 Marks)

(c) Can the scaled homography matrices for $H2$ and $H3$ be obtained? If yes, explain how to calculate them. If no, explain why.

(6 Marks)

5. Two sensors are used to measure the state x_k that is assumed to remain constant over time. That is, $x_{k+1} = x_k$.

The outputs of the two sensors are represented by z_{1k} and z_{2k}, respectively, and they are modelled by $z_{1k} = x_k + v_{1k}$ and $z_{2k} = x_k + v_{2k}$, respectively, where v_{1k} and v_{2k} are zero mean Gaussian noises with variances given by $4\sigma^2$ and σ^2, respectively.

(a) Denote $\hat{x}_k$ as the estimate of x_k. The predictor is designed as

$$\hat{x}_{k+1} = \hat{x}_k + L_k\left[\left(z_{1k} - \hat{x}_k\right) + \left(z_{2k} - \hat{x}_k\right)\right]$$

Let the estimation error be $\tilde{x}_{k+1} = x_{k+1} - \hat{x}_{k+1}$. Assume that the state estimation error and the noise terms v_{1k} and v_{2k} are uncorrelated, and that $E\left[\tilde{x}_{k+1}\right] = 0$. Let the estimation error variance be $p_{k+1} = E\left[\tilde{x}_{k+1}^2\right]$.

Note: Question No. 5 continues on page 6.

(i) Derive the update equation for the state estimation error variance p_{k+1}.

(ii) Design the update law for L_k to minimize the estimation error variance p_{k+1}.

(iii) If σ^2 is equal to zero, what would be the final state estimation error variance? Explain your answer.

(15 Marks)

(b) Can a predictor be designed to give a smaller estimation error variance in comparison with the predictor in part (a)? If yes, explain how to design. If no, explain why.

(5 Marks)

END OF PAPER

EE6221 ROBOTICS & INTELLIGENT SENSORS

Please read the following instructions carefully:

1. **Please do not turn over the question paper until you are told to do so. Disciplinary action may be taken against you if you do so.**

2. You are not allowed to leave the examination hall unless accompanied by an invigilator. You may raise your hand if you need to communicate with the invigilator.

3. Please write your Matriculation Number on the front of the answer book.

4. Please indicate clearly in the answer book (at the appropriate place) if you are continuing the answer to a question elsewhere in the book.