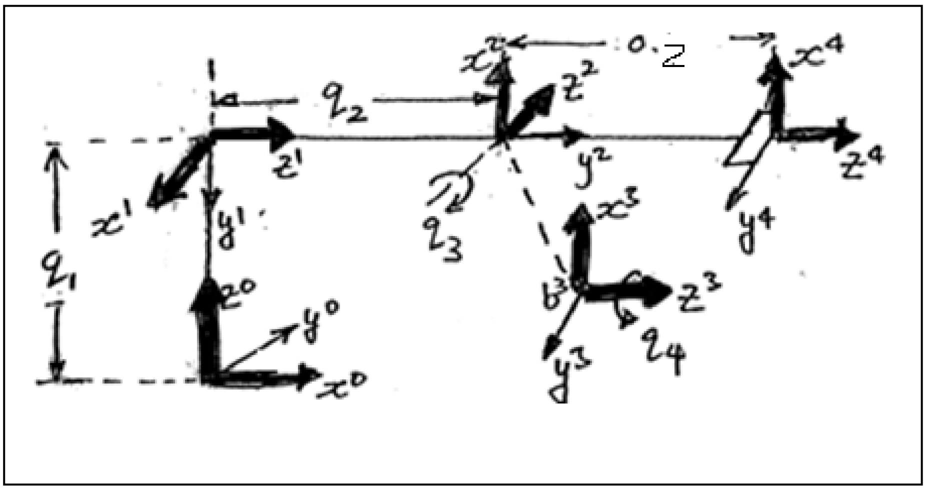
<u>Solution of Exercise: EE6221</u>

- *Please note that this is not the only solution as DH assignment is not unique.*

1. Using DH algorithm to obtain the link-coordinate diagram:



axis	θ (rad)	d (m)	a (m)	α (rad)
1	$-\frac{\pi}{2}$	q_1	0	$-\frac{\pi}{2}$
2	$-\frac{\pi}{2}$	q_2	0	$\frac{\pi}{2}$
3	q_3	0	0	$-\frac{\pi}{2}$
4	q_4	0.20	0	0

The link-coordinate homogeneous transformation matrices are:

$$\boldsymbol{T}_0^1 = \begin{bmatrix} 0 & 0 & 1 & 0 \\ -1 & 0 & 0 & 0 \\ 0 & -1 & 0 & q_1 \\ 0 & 0 & 0 & 1 \end{bmatrix}, \quad \boldsymbol{T}_1^2 = \begin{bmatrix} 0 & 0 & -1 & 0 \\ -1 & 0 & 0 & 0 \\ 0 & 1 & 0 & q_2 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$\boldsymbol{T}_2^3 = \begin{bmatrix} c_3 & 0 & -s_3 & 0 \\ s_3 & 0 & c_3 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}, \quad \boldsymbol{T}_3^4 = \begin{bmatrix} c_4 & -s_4 & 0 & 0 \\ s_4 & c_4 & 0 & 0 \\ 0 & 0 & 1 & 0.2 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$\boldsymbol{T}_{base}^{tool} = \boldsymbol{T}_0^1\, \boldsymbol{T}_1^2\, \boldsymbol{T}_2^3\, \boldsymbol{T}_3^4$$

$$= \begin{bmatrix} s_3c_4 & -s_3s_4 & c_3 & q_2 + 0.2c_3 \\ -s_4 & -c_4 & 0 & 0 \\ c_3c_4 & -c_3s_4 & -s_3 & q_1 - 0.2s_3 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

The tool configuration vector is:

$$\boldsymbol{w} = [q_2 + 0.2c_3,\ 0,\ q_1 - 0.2s_3,\ e^{\frac{q_4}{\pi}} c_3,\ 0,\ -e^{\frac{q_4}{\pi}} s_3]^T$$

Hence

$w_1 = q_2 + 0.2c_3$ -----(1)

$w_3 = q_1 - 0.2s_3$ ---- (2)

$w_4 = e^{\frac{q_4}{\pi}} c_3$ ------ (3)

$w_6 = -e^{\frac{q_4}{\pi}} s_3$ ------(4)

From (3) and (4)

$$\tan(q_3) = \frac{-w_6}{w_4}$$

⇨ $q_3 = \text{arctan2}\left(\frac{-w_6}{w_4}\right)$

Since q3 has been calculated, from (1) and (2)

⇨ $q_1 = w_3 + 0.2s_3$

⇨ $q_2 = w_1 - 0.2c_3$

From (3) and (4)

⇨ $q_4 = \pi \ln (w_4^2 + w_6^2)^{\frac{1}{2}}$

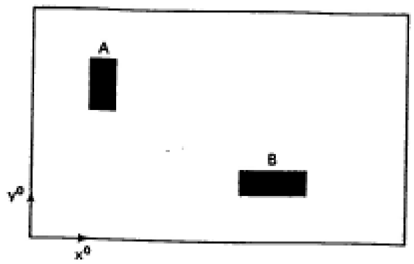
The tool configuration Jacobian is obtained from partial differentiation of:

$$\boldsymbol{w} = [q_2 + 0.2c_3,\ 0,\ q_1 - 0.2s_3,\ e^{\frac{q_4}{\pi}} c_3,\ 0,\ -e^{\frac{q_4}{\pi}} s_3]^T$$

that is

$$\begin{vmatrix} 0 & 1 & -0.2s_3 & 0 \\ 0 & 0 & 0 & 0 \\ 1 & 0 & -0.2c_3 & 0 \\ 0 & 0 & -e^{\frac{q_4}{\pi}} s_3 & \frac{1}{\pi} e^{\frac{q_4}{\pi}} c_3 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & -e^{\frac{q_4}{\pi}} c_3 & -\frac{1}{\pi} e^{\frac{q_4}{\pi}} s_3 \end{vmatrix}$$

2. $P_A = [6, 12, 2]^T$ and $P_B = [10, 5, 1]^T$.



To grasp along the long side of A is to align the sliding vector of end effector with the xo axis. Expressing with respect to the base frame, we have

$$T_{base}^{pick} = \begin{bmatrix} 0 & 1 & 0 & 6 \\ 1 & 0 & 0 & 12 \\ 0 & 0 & -1 & 2 \\ 0 & 0 & 0 & 1 \end{bmatrix} \text{ or } \begin{bmatrix} 0 & -1 & 0 & 6 \\ -1 & 0 & 0 & 12 \\ 0 & 0 & -1 & 2 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Part B has a height of 2 units and hence the centroid of part A should be 2 units above it i.e. 4 units. Therefore

$$T_{base}^{place} = \begin{bmatrix} 1 & 0 & 0 & 10 \\ 0 & -1 & 0 & 5 \\ 0 & 0 & -1 & 4 \\ 0 & 0 & 0 & 1 \end{bmatrix} \text{ or } \begin{bmatrix} -1 & 0 & 0 & 10 \\ 0 & 1 & 0 & 5 \\ 0 & 0 & -1 & 4 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$