NANYANG TECHNOLOGICAL UNIVERSITY

SEMESTER 2 EXAMINATION 2021-2022

EE6221 – ROBOTICS AND INTELLIGENT SENSORS

April / May 2022 Time Allowed: 3 hours

INSTRUCTIONS

1. This paper contains 5 questions and comprises 5 pages.
2. Answer all 5 questions.
3. All questions carry equal marks.
4. This is a closed book examination.
5. Unless specifically stated, all symbols have their usual meanings.

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1. A robotic manipulator with six joints is shown in Figure 1.

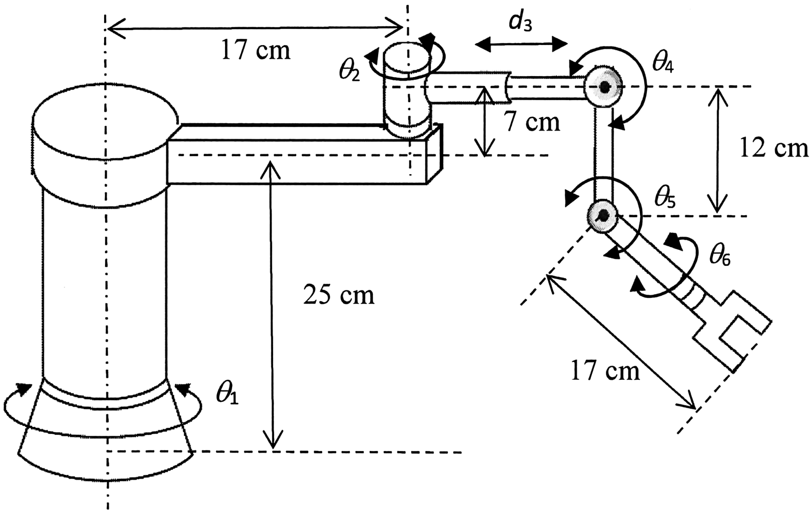


Figure 1

(a) Obtain the link coordinate diagram by using the Denavit-Hartenberg (D-H) algorithm.

(12 Marks)

Note: Question No. 1 continues on page 2.

(b) Derive the kinematic parameters of the robot based on the coordinate diagram obtained in part (a).

(8 Marks)

2. The dynamic equations of a Cartesian robot which is in contact with a frictionless surface are given as follows:

$$2\ddot{x}_1 + 5\,x_1\dot{x}_1 + f_1 = u_1$$

$$10\ddot{x}_2 + 7x_2\dot{x}_2 + f_2 = u_2$$

$$5\ddot{x}_3 + 3x_3\dot{x}_3 + 49 = u_3$$

where x_1, x_2, x_3 represent the position of the end effector in Cartesian coordinates, f_1 and f_2 are the contact forces, u_1, u_2, u_3 are the control inputs. The contact forces exerted on the environment are given by

$$\begin{bmatrix} f_1 \\ f_2 \end{bmatrix} = \begin{bmatrix} 20 & 0 \\ 0 & 30 \end{bmatrix} \begin{bmatrix} x_1 - 1 \\ x_2 - 1 \end{bmatrix}$$

The system possesses unmodelled resonances at 8 rad/s, 16 rad/s and 24 rad/s.

(a) Design a hybrid position and force controller for the robot. The system should be critically damped and does not excite all the unmodelled resonances.

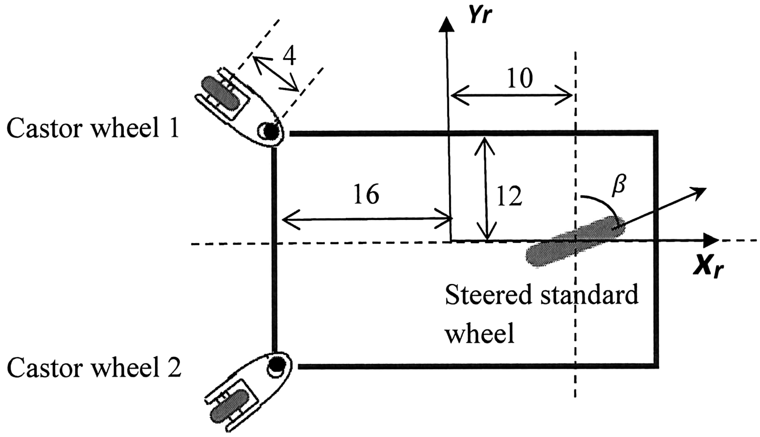
(12 Marks)

(b) The controller designed in part (a) is now implemented on the robot but a constant steady state position error is found on x_3. Explain the possible effects and derive the error equations. Design a controller so that the steady state error can be eliminated and the same performance in part (a) is achieved.

(8 Marks)

3. (a) A three-wheeled mobile robot with two castor wheels and one steered standard wheel is shown in Figure 2 on page 3. A local reference frame (x_r, y_r) and a steered angle β are assigned to the mobile robot as shown in Figure 2. The radius of each wheel is 5 cm. If the rotational velocities of the steered standard wheel and the two castor wheels are denoted by $\dot{\varphi}_{ss}$, $\dot{\varphi}_{c1}$, and $\dot{\varphi}_{c2}$, respectively, derive the rolling and sliding constraints of the mobile robot.

(10 Marks)



Note: all lengths are in centimeters.

Figure 2

(b) A robot manipulator with four joint variables q_1, q_2, q_3, q_4 are mounted on a mobile robot. The link-coordinate homogeneous transformation matrix from the base coordinate to the tool coordinate of the robotic manipulator is given as:

$$T_{base}^{tool} = \begin{bmatrix} -C_1S_4 - S_1S_3C_4 & -C_1C_4 - S_1S_3S_4 & -S_1C_3 & -S_1(q_2 + 0.75C_3) \\ -S_1S_4 - C_1S_3C_4 & -S_1C_4 - C_1C_3S_4 & C_1C_3 & C_1(q_2 + 0.75C_3) \\ -C_3C_4 & -C_3S_4 & -S_3 & 0.1 - 0.75S_3 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

where $s_1 = \sin(q_1)$, $s_3 = \sin(q_3)$, $s_4 = \sin(q_4)$, $c_1 = \cos(q_1)$, $c_3 = \cos(q_3)$, $c_4 = \cos(q_4)$.

(i) Solve the inverse kinematic problem using an analytic method to express $(q_1, q_2, q_3)^T$ in terms of the position of the end effector $(x, y, z)^T$. (Note: orientation is not required).

(ii) Find the first column of the tool-configuration Jacobian matrix of this robot. (Note: only first column is required).

(10 Marks)

4. A fixed camera is used to measure a moving object. Two images are taken of the moving object at poses P_A and P_B. Four coplanar and non-collinear feature points can be detected on these two images. The pixel coordinates of the four feature points at pose P_A are denoted as $p_{1a} = [u_{1a}, v_{1a}, 1]^T$, $p_{2a} = [u_{2a}, v_{2a}, 1]^T$, $p_{3a} = [u_{3a}, v_{3a}, 1]^T$, and $p_{4a} = [u_{4a}, v_{4a}, 1]^T$, respectively. The pixel coordinates of the four feature points at pose P_B are denoted as $p_{1b} = [u_{1b}, v_{1b}, 1]^T$, $p_{2b} = [u_{2b}, v_{2b}, 1]^T$, $p_{3b} = [u_{3b}, v_{3b}, 1]^T$, and $p_{4b} = [u_{4b}, v_{4b}, 1]^T$, respectively. Let $H = \begin{bmatrix} h_{11} & h_{12} & h_{13} \\ h_{21} & h_{22} & h_{23} \\ h_{31} & h_{32} & h_{33} \end{bmatrix}$ denote the Euclidean homography matrix from pose P_A to pose P_B.

(a) Let m_{1a}, m_{2a}, m_{3a} and m_{4a} denote the normalized coordinates of the four feature points in the camera coordinate frame, respectively, at pose P_A. Let m_{1b}, m_{2b}, m_{3b} and m_{4b} denote the normalized coordinates of the four feature points in the camera coordinate frame, respectively, at pose P_B. Let $\bar{m}_{1a}$, $\bar{m}_{2a}$, $\bar{m}_{3a}$ and $\bar{m}_{4a}$ denote the Euclidean coordinates of the four feature points in the camera coordinate frame, respectively, at pose P_A. Let $\bar{m}_{1b}$, $\bar{m}_{2b}$, $\bar{m}_{3b}$ and $\bar{m}_{4b}$ denote the Euclidean coordinates of the four feature points in the camera coordinate frame, respectively, at pose P_B. Let α_1, α_2, α_3 and α_4 denote the depth ratios associated with the four feature points, respectively. The matrix A represents the camera calibration matrix. Use two methods to find a set of linear equations, respectively, that can be used to compute the scaled homography matrix $\dfrac{H}{h_{33}}$.

(15 Marks)

(b) Describe a method that can be used to extract the rotation matrix information from the scaled homograph matrix. If there are multiple options for the rotation matrix, describe a method that can be used to find the correct solution.

(5 Marks)

5. Four sensors are used to measure a state variable x_k that can be modelled by $x_{k+1} = x_k + w_k$ where w_k represents a zero mean Gaussian noise with variance given by σ^2. The outputs of the sensors are z_{1k}, z_{2k}, z_{3k} and z_{4k}, which are governed by the models $z_{1k} = x_k + v_{1k}$, $z_{2k} = x_k + v_{2k}$, $z_{3k} = x_k + v_{3k}$ and $z_{4k} = x_k + v_{4k}$ where v_{1k}, v_{2k}, v_{3k} and v_{4k} are zero mean Gaussian sensor noises with variances given by a^2, a^2, b^2 and b^2, respectively.

Let $\hat{x}_k$ represent the estimate of x_k. Design an estimation predictor to achieve optimal estimation of x_k. Let the estimation error be $\tilde{x}_{k+1} = x_{k+1} - \hat{x}_{k+1}$. Assume that the estimation error $\tilde{x}_k$ and the noise terms w_k, v_{1k}, v_{2k}, v_{3k} and v_{4k} are uncorrelated, and that $E[\tilde{x}_{k+1}] = 0$. Let the estimation error variance be $p_{k+1} = E\left[\tilde{x}_{k+1}^2\right]$.

(a) Which one of the following designs will you adopt to design the optimal estimation predictor? Justify your answer.

(i) $\hat{x}_{k+1} = \hat{x}_k + L_k(z_{1k} - \hat{x}_k) + L_k(z_{2k} - \hat{x}_k) + L_k(z_{3k} - \hat{x}_k) + L_k(z_{4k} - \hat{x}_k)$

(ii) $\hat{x}_{k+1} = \hat{x}_k + L_{1k}(z_{1k} - \hat{x}_k) + L_{1k}(z_{2k} - \hat{x}_k) + L_{2k}(z_{3k} - \hat{x}_k) + L_{2k}(z_{4k} - \hat{x}_k)$

(iii) $\hat{x}_{k+1} = \hat{x}_k + L_{1k}(z_{1k} - \hat{x}_k) + L_{2k}(z_{2k} - \hat{x}_k) + L_{3k}(z_{3k} - \hat{x}_k) + L_{4k}(z_{4k} - \hat{x}_k)$

where L_k, L_{1k}, L_{2k}, L_{3k} and L_{4k} represent Kalman gains.

(4 Marks)

(b) Derive the update equation for the state estimation error variance p_{k+1}.

(8 Marks)

(c) Design the update laws for the Kalman gains to minimize the estimation error variance p_{k+1}.

(8 Marks)

END OF PAPER

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Please read the following instructions carefully:

1. **Please do not turn over the question paper until you are told to do so. Disciplinary action may be taken against you if you do so.**

2. You are not allowed to leave the examination hall unless accompanied by an invigilator. You may raise your hand if you need to communicate with the invigilator.

3. Please write your Matriculation Number on the front of the answer book.

4. Please indicate clearly in the answer book (at the appropriate place) if you are continuing the answer to a question elsewhere in the book.