Mobile Robots

1. Locomotion

Locomotion is the process of producing motion on an autonomous robot by applying forces to the robot.

1.1 Locomotion in Nature

Biological locomotion consists of a wide variety of movements such as walking, running, crawling and jumping.

1.2 Locomotion in Robots

- Many locomotion concepts in robotics are inspired by nature.
- Most natural locomotion concepts are difficult to produce technically.
- Rolling, which is NOT found in nature, is most efficient.

1.3 Wheeled Mobile Robots (WMR)

Wheel is a man-made mechanism which is popular in realizing locomotion in mobile robotics and vehicles.

Yamabico

MagellanPro

Sojourner

ATRV-2

Hilare 2-Bis

Koy

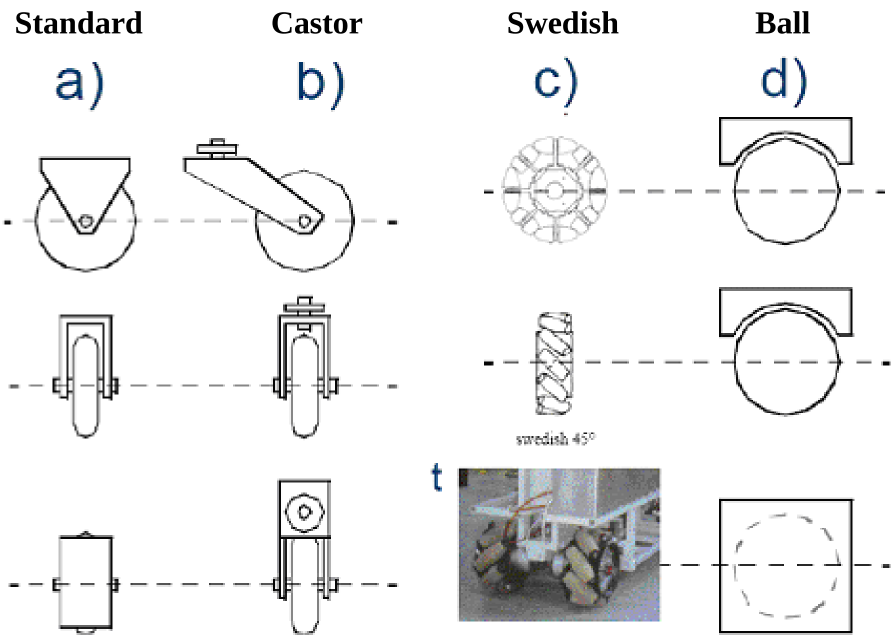
Stability is naturally ensured with 3 wheels, and can be improved with more.

1.3.1 Wheels

Wheel types

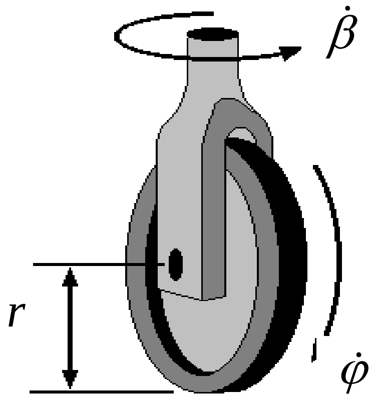
There are four basic wheel types: a) Standard wheel, b) Castor wheel, c) Swedish wheel, d) Ball or spherical wheel

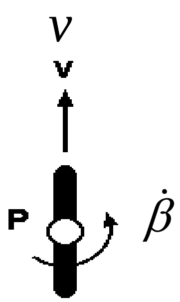
The choice of the wheel type has a large effect on the kinematics of the mobile robot.

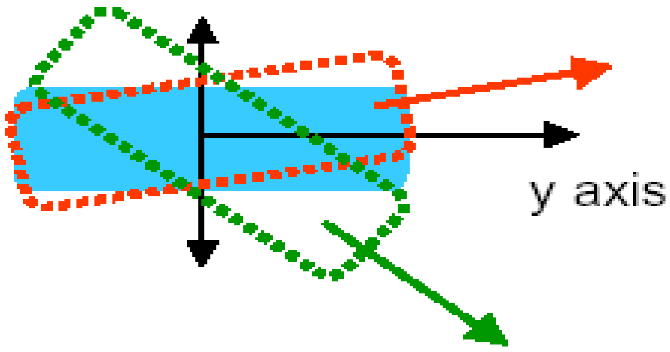


Steered wheel

In this case, the orientation of the rotation axis of the wheel can be controlled (i.e. steered)







- Wheel parameters:

  r = wheel radius

  v = wheel linear velocity

  $\dot{\varphi}$ = wheel angular velocity

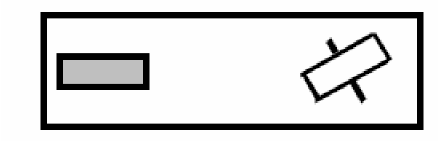
  $\dot{\beta}$ = steering velocity

1.3.2 Wheel configurations

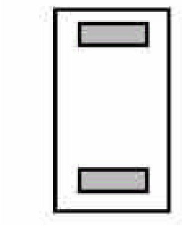
There are many possibilities of wheel configurations or wheel arrangements.

- Stability is guaranteed with at least 3 wheels

Two-wheel configurations

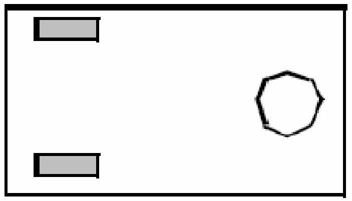


One steering wheel and one traction wheel

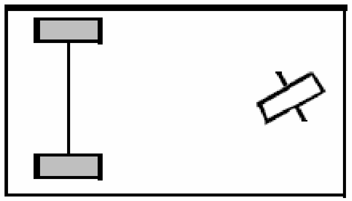


Two-wheel differential drive
(independent actuators for each wheel)

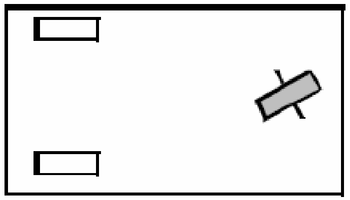
Three-wheel configurations



Two-wheel differential drive
and one unpowered
omnidirectional wheel

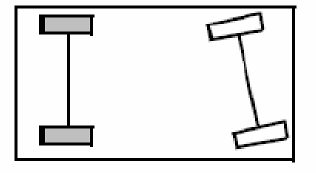


Two connected traction wheels
and one steering wheel

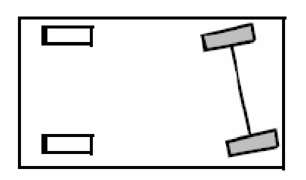


Two free wheels and one
steered traction wheel

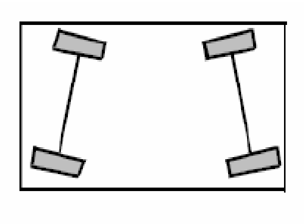
Four-wheel configurations



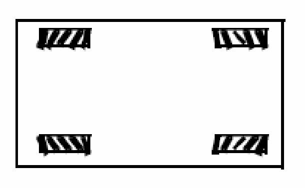
Two connected traction wheels and two steering wheels



Two free wheels and two connected steered traction wheels



Four connected steered traction wheels

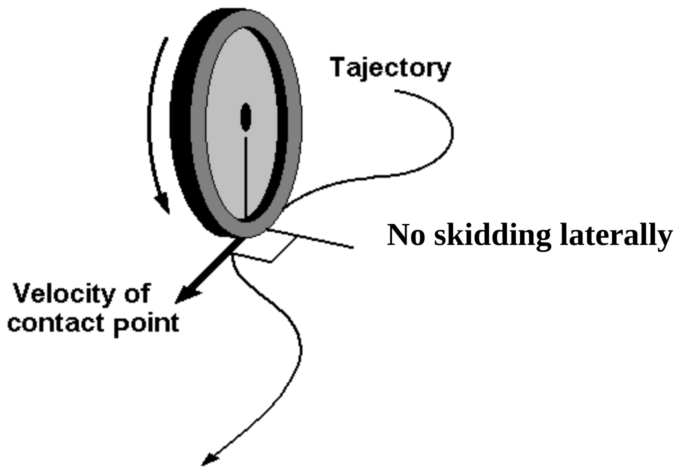


Four Swedish wheels

2. Mobile Robot Kinematics

The modelling and understanding of how mobile robot systems move or behave.

Each wheel plays a role in the **movement** but may also impose **constraints** on the robot; for example, unable to move or skid laterally.



Assumptions:

1. The robot is a **rigid mechanism**.
2. **No slipping** in the orthogonal direction of rolling (non-sliding constraint).
3. **No translational slip** occurs between the wheel and the floor (pure rolling).
4. All steering axes are perpendicular to the floor.

2.1 Kinematic Models and Constraints

Kinematic models describe how the mobile robot as a whole, moves as a function of its geometry and individual wheel behavior.

Deriving a model for the whole robot's motion is a bottom-up process.

- Each *individual* wheel contributes to the robot's motion
- The constraints of all wheels are combined to form the constraints of the mobile robot.

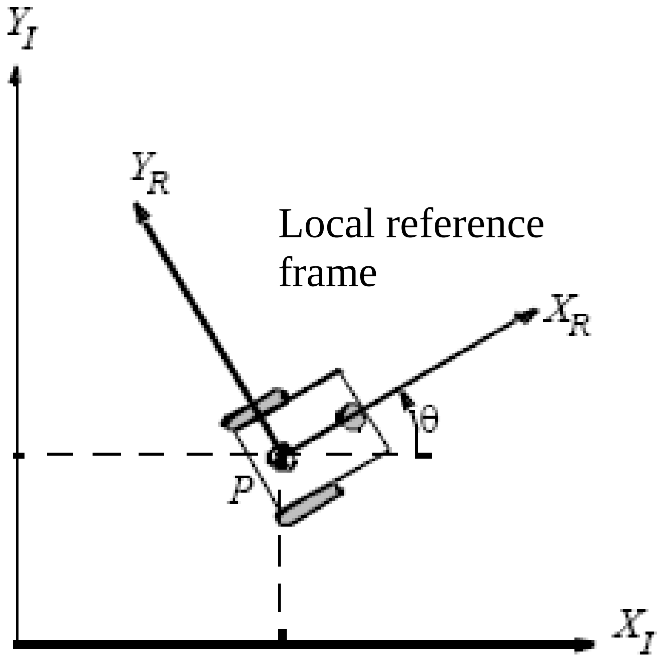
2.1.1 Position and Coordinate Frames

The mobile robot is modeled as a rigid body on wheels, operating on a **horizontal plane**.

Therefore, the total dimensionality of this robot on the plane is,

- two for position in the plane and
- one for orientation along the vertical axis, which is orthogonal to the plane.

To specify the position of the robot on the plane, a relationship between the **global reference frame** of the plane and the **local reference frame** of the robot is established.



Global reference frame: $\{\boldsymbol{X_I}, \boldsymbol{Y_I}\}$

Local reference frame: $\{\boldsymbol{X_R}, \boldsymbol{Y_R}\}$

A point P on the robot chassis is chosen as its position reference point.

- The **position** of P in the global reference frame is specified by coordinates $\boldsymbol{x}$ and $\boldsymbol{y}$
- The **heading angle** is the angular difference between the global and local reference frames that is given by $\boldsymbol{\theta}$.

That is,

$$\xi_I = \begin{bmatrix} x \\ y \\ \theta \end{bmatrix}$$

To describe robot motion in terms of component motions, it will be necessary to ***map motion*** *along the axes of the* ***global reference frame*** *to motion along the axes of the* ***robot's local reference frame***.

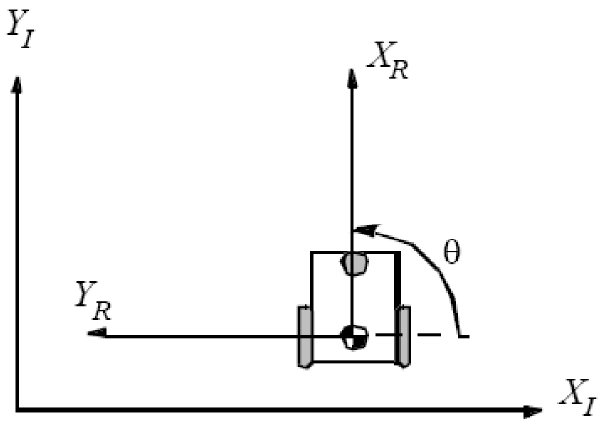
$$R(\theta) = \begin{bmatrix} \cos\theta & \sin\theta & 0 \\ -\sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

That is,

$$\dot{\xi}_R = R(\theta)\dot{\xi}_I = \begin{bmatrix} \cos\theta & \sin\theta & 0 \\ -\sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \dot{x} \\ \dot{y} \\ \dot{\theta} \end{bmatrix}$$

Example 1

Consider the robot in the following configuration.



Since $\theta = \frac{\pi}{2}$, the instantaneous rotation matrix R is

$$R(\frac{\pi}{2}) = \begin{bmatrix} 0 & 1 & 0 \\ -1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

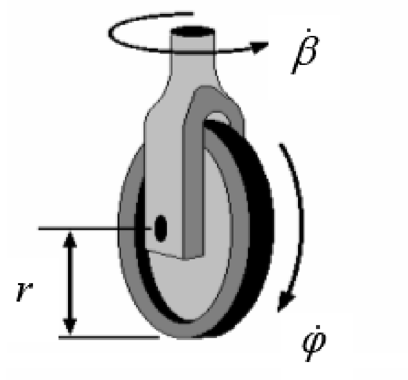
Given some velocity in the global reference frame we can compute the components of motion along this robot's local axes

$$\dot{\xi}_R = R(\frac{\pi}{2})\dot{\xi}_I = \begin{bmatrix} 0 & 1 & 0 \\ -1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \dot{x} \\ \dot{y} \\ \dot{\theta} \end{bmatrix} = \begin{bmatrix} \dot{y} \\ -\dot{x} \\ \dot{\theta} \end{bmatrix}$$

That is, motion along $\boldsymbol{X_R}$ is equal to $\dot{y}$ and motion along is $\boldsymbol{Y_R}$ is $-\dot{x}$.

2.1.2 Forward kinematic models

How does the robot move, given its geometry and the speeds of its wheels?



More formally, forward kinematic model is to establish the *robot speed* $\dot{\xi}_I = [\,\dot{x}, \dot{y}, \dot{\theta}\,]^T$ as a function of the *wheel speeds* $\dot{\varphi}_1, \dot{\varphi}_2, \ldots, \dot{\varphi}_N$, *steering speeds* $\dot{\beta}_1, \dot{\beta}_2, \ldots, \dot{\beta}_M$ and the *geometric parameters* (e.g. radius of wheels, distance between wheels) of the robot.

$$\dot{\xi}_I = \begin{bmatrix} \dot{x} \\ \dot{y} \\ \dot{\theta} \end{bmatrix} = f(\dot{\varphi}_1, \dot{\varphi}_2, \ldots, \dot{\varphi}_N, \dot{\beta}_1, \dot{\beta}_2, \ldots, \dot{\beta}_M)$$

Forward kinematic model describes the motion of mobile robot in global reference frame as a function of wheel velocities. Since

$$\dot{\xi}_R = R(\theta)\dot{\xi}_I$$

It can be first expressed in the local frame and transform to the global frame:

$$\dot{\xi}_I = R^{-1}(\theta)\dot{\xi}_R$$

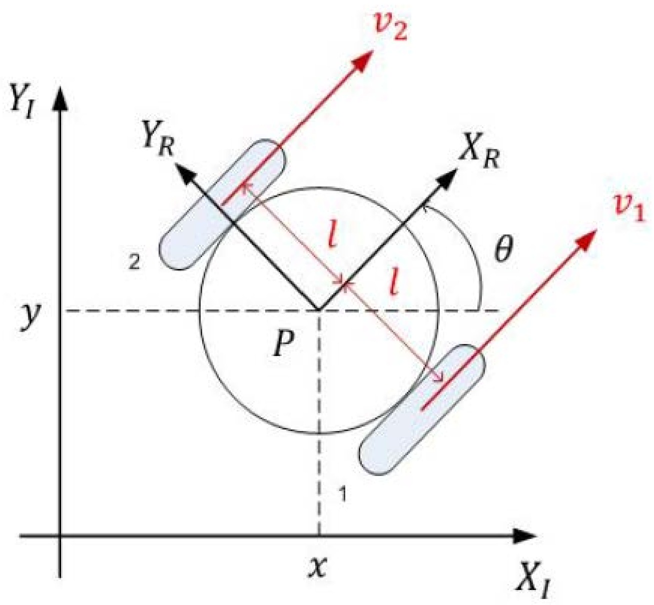
Differential Drive Robot

A differential drive robot has two wheels, each with diameter *r*. Each wheel is a distance *l* from *P*.

The forward kinematic problem is thus to establish the robot speed $\dot{\xi}_I = [\dot{x}, \dot{y}, \dot{\theta}]^T$ as a function of the two wheel speeds $\dot{\varphi}_1, \dot{\varphi}_2$, and the geometric parameters *r*. *l* of the robot.

$$\dot{\xi}_I = \begin{bmatrix} \dot{x} \\ \dot{y} \\ \dot{\theta} \end{bmatrix} = f(\dot{\varphi}_1, \dot{\varphi}_2)$$

We can compute the robot's motion in the global reference frame from motion in its local reference frame.



Since

$$v_1 = (R + l)\omega$$

$$v_2 = (R - l)\omega$$

$$v = R\omega$$

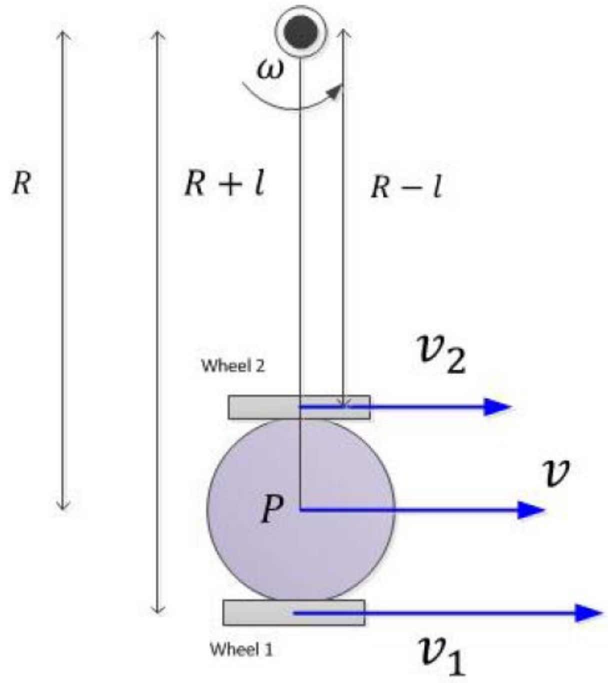
Therefore

$$v_1 + v_2 = 2R\omega = 2v$$

$$=> v = \frac{1}{2}(v_1 + v_2)$$

$$v_1 - v_2 = 2l\omega$$

$$=> \omega = \frac{1}{2l}(v_1 - v_2)$$

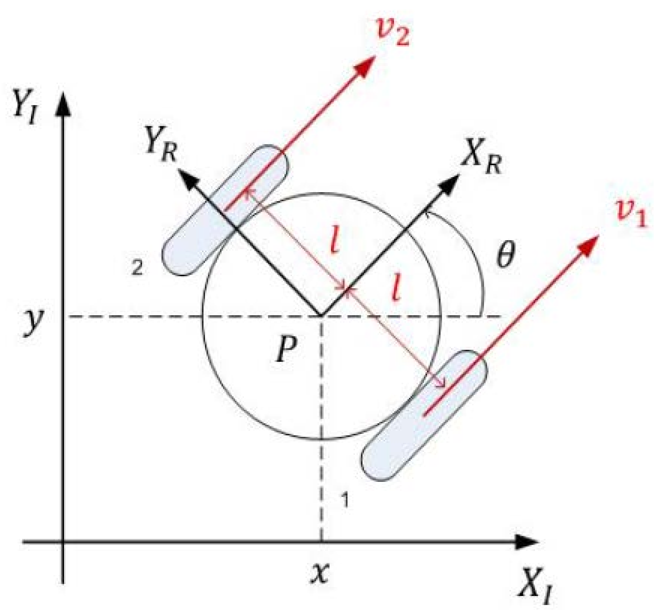


First, we express the velocities of the robot in its local reference frame,

$$\dot{X}_R = \frac{1}{2}(v_1 + v_2)$$

$$\dot{Y}_R = 0$$

$$\dot{\theta} = \frac{1}{2l}(v_1 - v_2)$$



where

$$v_1 = \dot{\varphi}_1 r$$

$$v_2 = \dot{\varphi}_2 r$$

The robot speed in local reference frame $\dot{\xi}_R$ can be expressed as a function of the two wheel speeds $\dot{\varphi}_1, \dot{\varphi}_2$, and the geometric parameters *r*. *l* of the robot as:

$$\dot{\xi}_R = \begin{bmatrix} \dot{X}_R \\ \dot{Y}_R \\ \dot{\theta} \end{bmatrix} = \begin{bmatrix} \frac{r\dot{\phi}_1}{2} + \frac{r\dot{\phi}_2}{2} \\ 0 \\ \frac{r\dot{\phi}_1}{2l} - \frac{r\dot{\phi}_2}{2l} \end{bmatrix}$$

We can now compute the robot's motion in the global reference frame as

$$\dot{\xi}_I = R^{-1}(\theta)\dot{\xi}_R$$

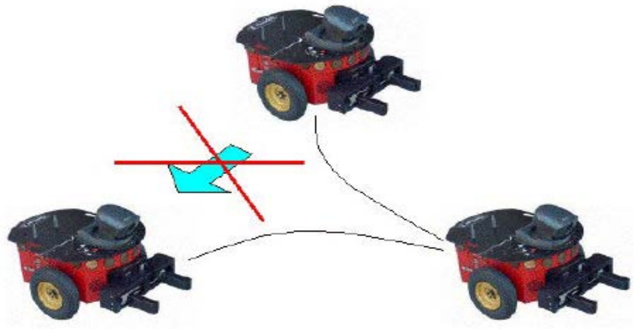
$$\begin{bmatrix} \dot{x} \\ \dot{y} \\ \dot{\theta} \end{bmatrix} = \begin{bmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \frac{r\dot{\varphi}_1}{2} + \frac{r\dot{\varphi}}{2} \\ 0 \\ \frac{r\dot{\varphi}_1}{2l} - \frac{r\dot{\varphi}}{2l} \end{bmatrix}$$

2.1.3 Wheel kinematic constraints

Some mobile robots can move in some directions and not other directions.

An important step in kinematic modeling is to express constraints on individual wheels.

The motions of individual wheels are combined to compute the motion of the robot as a whole.



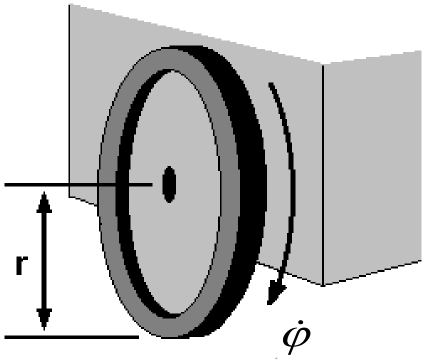
There are four basic wheel types and we begin by presenting sets of constraints specific to each wheel type.

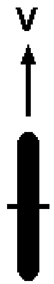
Two constraints for every wheel type will be shown.

- The first constraint enforces the concept of rolling contact — *that the wheel must roll when motion takes place in the appropriate direction.*
- The second constraint enforces the concept of no lateral slippage — *that the wheel must not slide orthogonal to the wheel plane.*

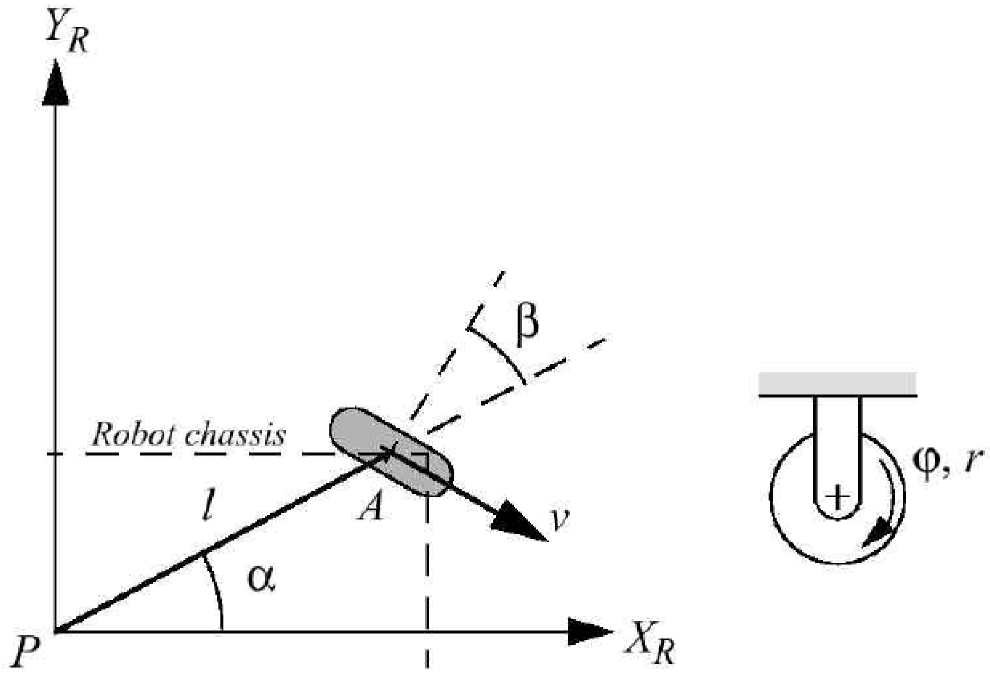
Fixed standard wheel

The *fixed* standard wheel has *no vertical axis of rotation for steering*. It is limited to motion back and forth along the wheel plane.





Consider a fixed standard wheel *A* and indicates its position pose relative to the robot's local reference frame **{X_R, Y_R}.**

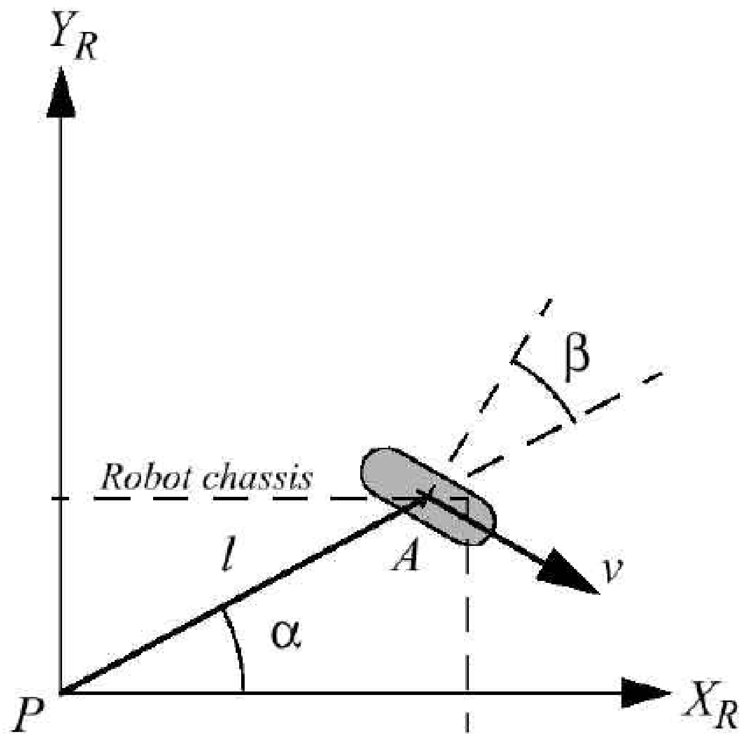


The position of ***A*** is expressed in polar coordinates by *distance* ***l*** and *angle* α.

The *angle of the wheel plane relative to the chassis* is denoted by β , which is fixed.

The wheel, which has radius ***r***, and its rotational position is a function of time ***t***: $\varphi(t)$.

The **rolling constraint** for this wheel is,



$$v = \dot{\varphi}\ r = \dot{x}_r\ sin(\alpha + \beta) - \dot{y}_r\ cos(\alpha + \beta) - \dot{\theta}\ l\cos\beta$$

$$= \left[sin(\alpha + \beta) \quad -cos(\alpha + \beta) \quad -l\cos\beta\right] \begin{bmatrix} \dot{x}_r \\ \dot{y}_r \\ \dot{\theta} \end{bmatrix}$$

The above *rolling constraint* can be written as:

$$\left[sin(\alpha + \beta) \quad -cos(\alpha + \beta) \quad -l\cos\beta\right] R(\theta)\dot{\xi}_I - r\dot{\varphi} = 0$$

or

$$j\ R(\theta)\dot{\xi}_I - r\dot{\varphi} = 0$$

where $j = \left[sin(\alpha + \beta) \quad -cos(\alpha + \beta) \quad -l\cos\beta\right]$.

The **sliding constraint** for this wheel is,

$$\dot{x}_r \cos(\alpha+\beta) + \dot{y}_r \sin(\alpha+\beta) + \dot{\theta}\ l \sin\beta = 0$$

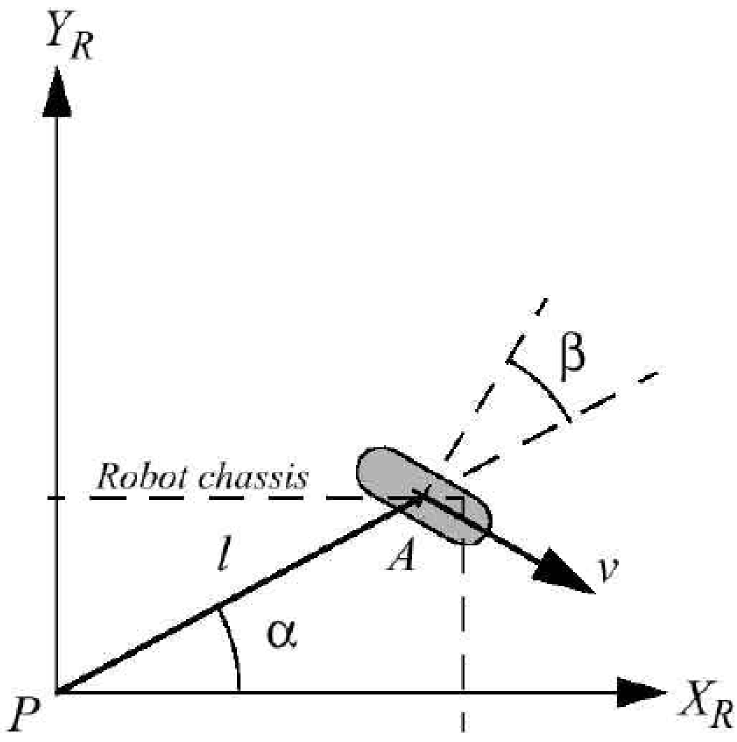
The above *sliding constraint* can be written as:

$$\left[\cos(\alpha+\beta) \quad \sin(\alpha+\beta) \quad l\sin\beta\right] R(\theta)\dot{\xi}_I = 0$$

or

$$c\ R(\theta)\dot{\xi}_I = 0$$

where $c = \left[\cos(\alpha+\beta) \quad \sin(\alpha+\beta) \quad l\sin\beta\right]$.



Example 2:

Derive the rolling and sliding constraints for wheel 1.

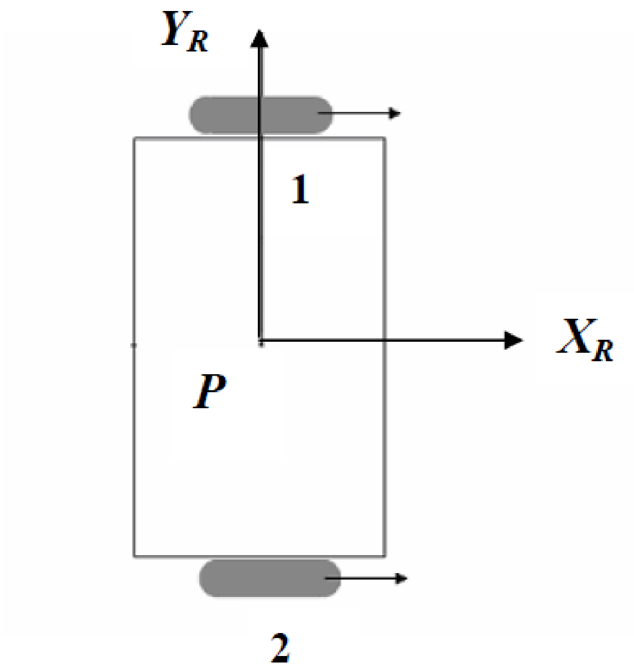
Note that wheel 1 is in a position such that $\alpha = \frac{\pi}{2}$, $\beta = 0$.

The rolling constraint for wheel 1 reduces to

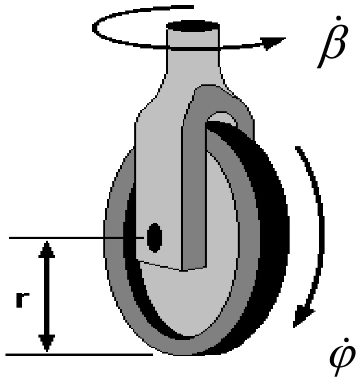
$$\dot{\phi}\ r = \dot{x}_r \sin(\frac{\pi}{2}) - \dot{y}_r \cos(\frac{\pi}{2}) - \dot{\theta}\ l \cos 0 => \begin{bmatrix} 1 & 0 & -l \end{bmatrix} \begin{bmatrix} \dot{x}_R \\ \dot{y}_R \\ \dot{\theta} \end{bmatrix} - r\dot{\phi} = 0$$

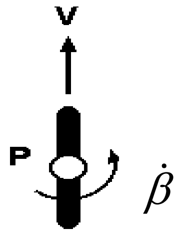
The sliding constraint for wheel *1* is

$$\dot{x}_r \cos(\frac{\pi}{2}) + \dot{y}_r \sin(\frac{\pi}{2}) + \dot{\theta}\ l \sin 0 = 0 => \begin{bmatrix} 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} \dot{x}_R \\ \dot{y}_R \\ \dot{\theta} \end{bmatrix} = 0$$



Steered standard wheel





The steered standard wheel differs from the fixed standard wheel only in that the wheel may rotate around a vertical axis passing through the center of the wheel and the ground contact point.

The equations of position for the steered standard wheel are identical to that of the fixed standard wheel but $\boldsymbol{\beta}$ instead varies as a function of time: $\boldsymbol{\beta}$ ***(t)***.

The rolling and sliding constraints are:

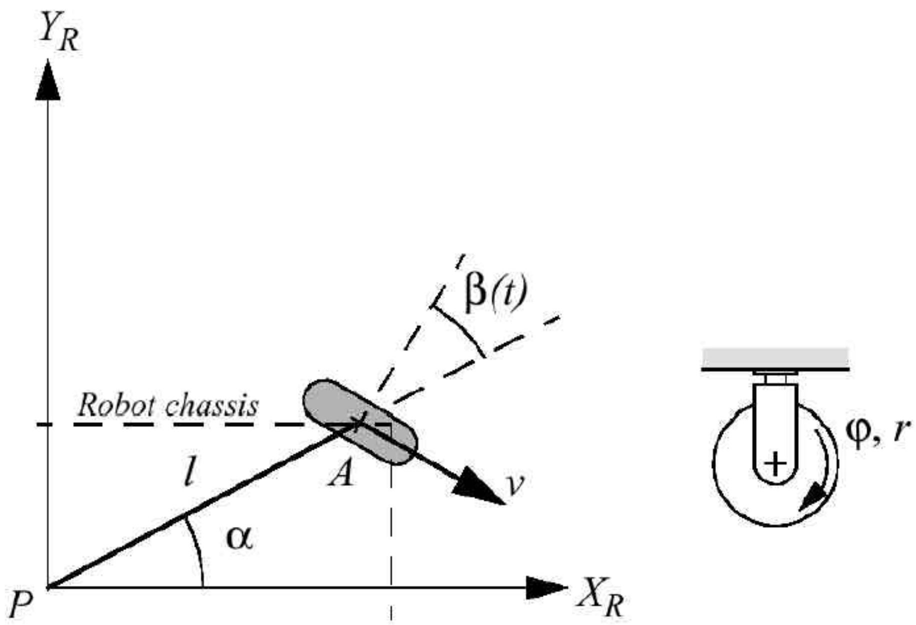
$$\left[\sin(\alpha+\beta) \quad -\cos(\alpha+\beta) \quad -l\cos\beta\right]R(\theta)\dot{\xi}_I - r\dot{\phi} = 0$$

$$\left[\cos(\alpha+\beta) \quad \sin(\alpha+\beta) \quad l\sin\beta\right]R(\theta)\dot{\xi}_I = 0$$

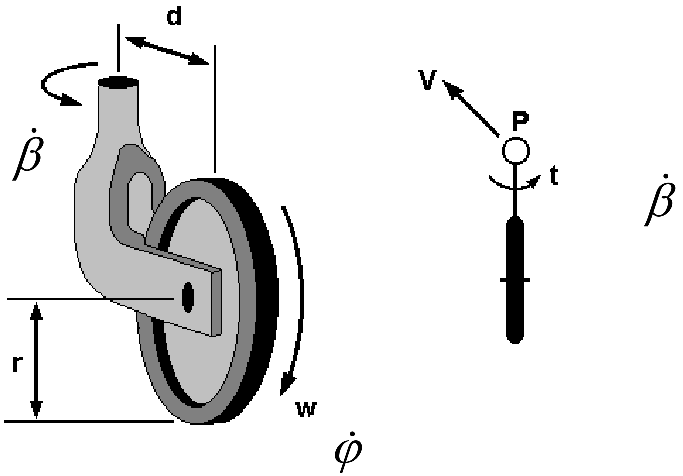
or

$$j(\beta)R(\theta)\dot{\xi}_I - r\dot{\phi} = 0$$

$$c(\beta)R(\theta)\dot{\xi}_I = 0$$

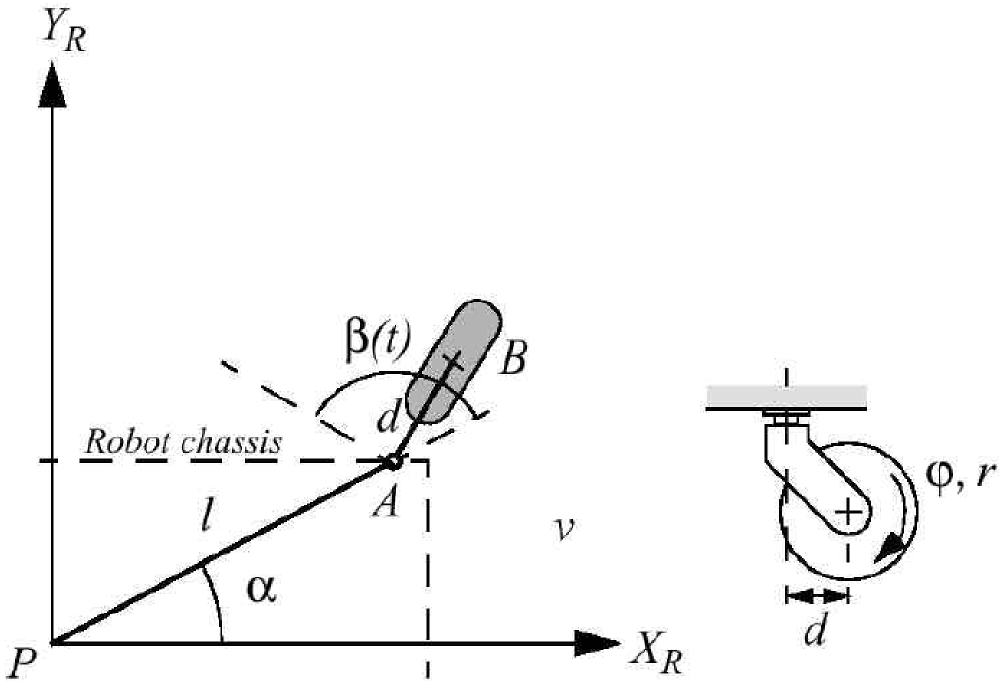


Castor wheel



Castor wheels are *able to steer around a vertical axis*.

However, unlike the steered standard wheel, the vertical *axis of rotation* in a castor wheel *does not pass through the ground contact point*.



The wheel *contact point is now at position B*, which is connected by a rigid rod *AB* of fixed length *d* to point *A*.

For the castor wheel, the rolling constraint is identical to the standard wheel because the offset axis plays no role during motion that is *aligned with the wheel plane*:

$$\left[sin(\alpha + \beta) \quad -cos(\alpha + \beta) \quad -l\cos\beta \right] R(\theta)\dot{\xi}_I - r\dot{\varphi} = 0$$

or

$$j(\beta)R(\theta)\dot{\xi}_I - r\dot{\varphi} = 0$$

The castor geometry does, however, have significant impact on the sliding constraint.

$$\left[cos(\alpha + \beta) \quad sin(\alpha + \beta) \quad l\sin\beta \right] R(\theta)\dot{\xi}_I + d\dot{\beta} = 0$$

or

$$c(\beta)R(\theta)\dot{\xi}_I + d\dot{\beta} = 0$$

It can be surmised from the above equations that:

Given any robot chassis motion ξ_I, there exists some value for spin speed $\dot{\varphi}$ and steering speed $\dot{\beta}$ such that the constraints are met.

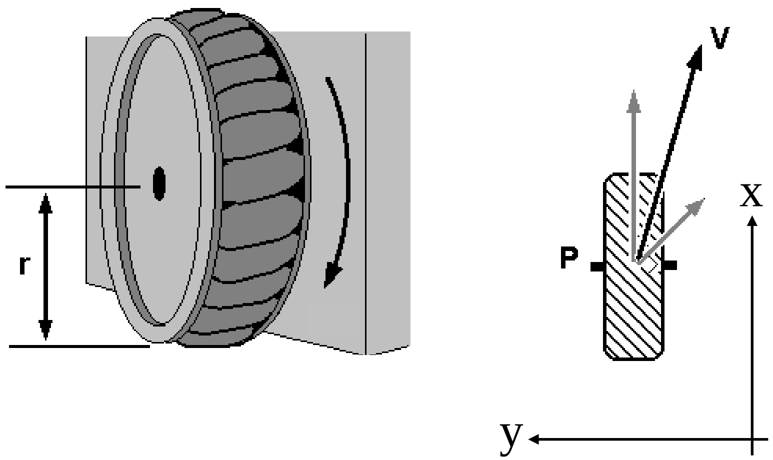
Therefore, a robot with only castor wheels can move with any velocity in the space of possible robot motions. We term such systems ***omnidirectional***.

Thus, although the kinematics of castor wheels is somewhat complex, such wheels do not impose any real constraints on the kinematics of robot chassis.

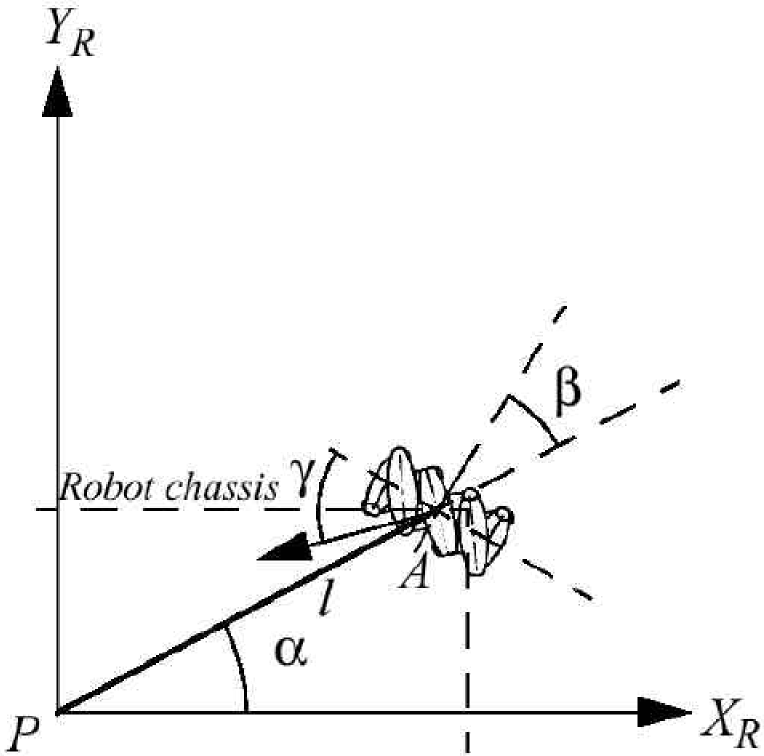
A real-world example of such a system is the five-castor wheel office chair.

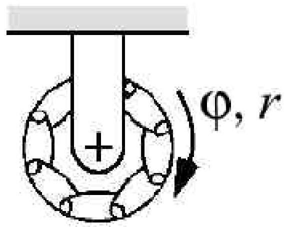
Swedish wheel

Swedish wheels are able to move *omnidirectionally* like the castor wheel. This is achieved by *adding a degree of freedom to the fixed standard wheel*.



Swedish wheels consist of ***a fixed standard wheel with rollers*** *attached to axes that are antiparallel to the main axis of the fixed wheel component*.





The pose of a Swedish wheel is similar to fixed standard wheel, with the addition of an angle, γ, which represents the angle between the main wheel plane and the axis of rotation of the small rollers.

The rolling constraint is

$$\left[sin(\alpha+\beta+\gamma) \quad -cos(\alpha+\beta+\gamma) \quad -l\,cos(\beta+\gamma) \right] R(\theta)\dot{\xi}_I$$
$$-r\dot{\varphi}\cos\gamma = 0$$

$$j\ R(\theta)\dot{\xi}_I - r\cos\gamma\ \dot{\varphi} = 0$$

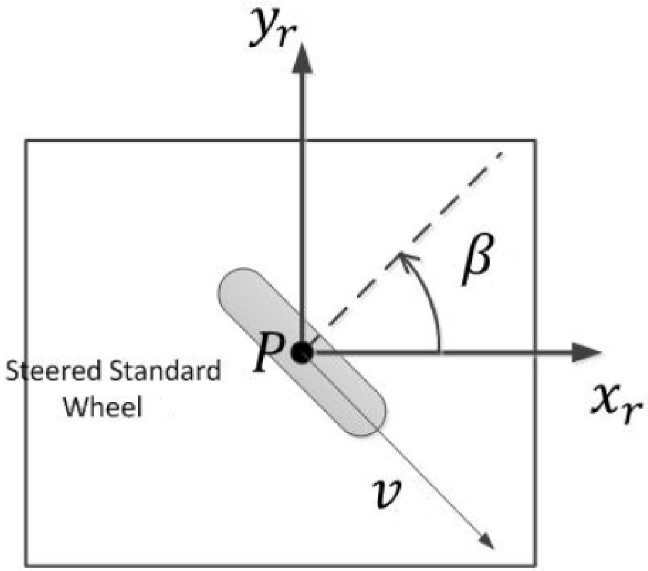
Orthogonal to this direction the *motion is not constrained* because of the free rotation $\dot{\varphi}_{sw}$ of the small rollers. The sliding constraint is

$$\left[cos(\alpha+\beta+\gamma) \quad sin(\alpha+\beta+\gamma) \quad l\,sin(\beta+\gamma) \right] R(\theta)\dot{\xi}_I$$
$$-r\dot{\varphi}\sin\gamma - r_{sw}\dot{\varphi}_{sw} = 0$$

where r_{sw} is the radius of the small rollers.

Example 3: Derive the rolling and sliding constraints for the single wheel robot.

Note that the wheel is in a position such that α =0, l = 0.



The rolling and sliding constraints are:

$$\begin{bmatrix}\sin(\beta) & -\cos(\beta) & 0\end{bmatrix} R(\theta)\dot{\xi}_I - r\dot{\phi} = 0$$

$$\begin{bmatrix}\cos(\beta) & \sin(\beta) & 0\end{bmatrix} R(\theta)\dot{\xi}_I = 0$$

where β is a variable,

3.1.4 Robot kinematic constraints

We now consider a general mobile robot with N wheels. We use the following subscripts to identify quantities relative to these 4 classes of wheels:

- f for fixed standard wheel,
- s for steerable standard wheel,
- c for castor wheels, and
- sw for Swedish wheels.

For example, the numbers of wheels of each type are denoted N_f , N_s,, N_c,, N_{sw}, φ_{f}, φ_{s}, φ_{c}, φ_{sw} denote the rotation angles of the wheels, and β_{s}, β_{c} denote the steering angles of the wheels .

Combining the wheel constraints imposes the overall constraints for the vehicle.

The *rolling constraints* of all wheels can now be collected in the following general expressions in matrix form:

$$J_1(\beta_s, \beta_c)R(\theta)\dot{\xi}_I + J_2\dot{\varphi} = 0$$

The sliding constraints of all standard wheels can be expressed into a single expression

$$C(\beta_s, \beta_c)R(\theta)\dot{\xi}_I + D\dot{\beta} = 0$$

For a vehicle with only standard wheels (fixed or steered), the above equation reduces to:

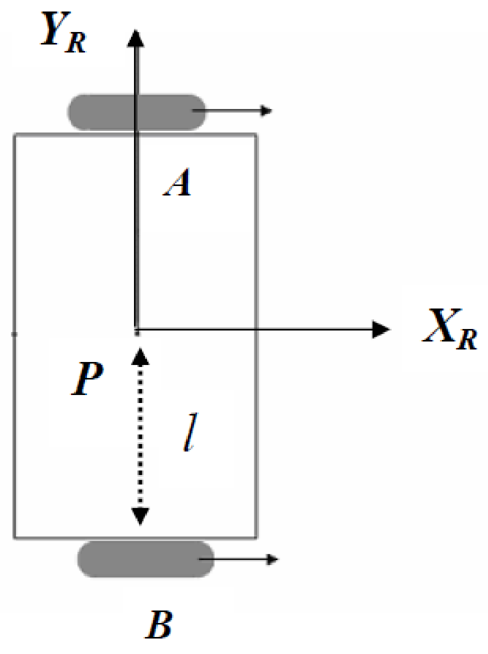
$$C(\beta_s)R(\theta)\dot{\xi}_I = 0$$

Example 4:

Consider the following robot with two standard fixed wheels as in example 2.

Note that wheel 1 is in a position such that $\alpha = \frac{\pi}{2}$, $\beta = 0$ and

wheel 2 is in a position such that $\alpha = -\frac{\pi}{2}$, $\beta = \pi$.



Note the value of β of the wheel 2 is necessary to ensure that positive spin causes motion in the $+X_R$ direction.

The rolling constraint for standard fixed wheels is given as:

$$\left[sin(\alpha + \beta) \quad -cos(\alpha + \beta) \quad -l\cos\beta \right] R(\theta)\dot{\xi}_I - r\dot{\varphi} = 0$$

Hence, we have

wheel A: $\left[1 \quad 0 \quad -l \right] R(\theta)\dot{\xi}_I - r\dot{\varphi}_{fA} = 0$

wheel B: $\left[1 \quad 0 \quad l \right] R(\theta)\dot{\xi}_I - r\dot{\varphi}_{fB} = 0$

Combining the above equations yield

$$\underbrace{\begin{bmatrix} 1 & 0 & -l \\ 1 & 0 & l \end{bmatrix}}_{J_1} R(\theta)\dot{\xi}_I - \underbrace{\begin{bmatrix} r & 0 \\ 0 & r \end{bmatrix}}_{J_2} \begin{bmatrix} \dot{\varphi}_{fA} \\ \dot{\varphi}_{fB} \end{bmatrix} = 0$$

The *sliding constraint* is given as:

$$\begin{bmatrix} cos(\alpha+\beta) & sin(\alpha+\beta) & l\sin\beta \end{bmatrix} R(\theta)\dot{\xi}_I = 0$$

Hence, we have

$$\underbrace{\begin{bmatrix} 0 & 1 & 0 \\ 0 & 1 & 0 \end{bmatrix}}_{C} R(\theta)\dot{\xi}_I = 0$$

Fusing these two equations yields the following expression:

$$\begin{bmatrix} J_1 \\ C_1 \end{bmatrix} R(\theta)\dot{\xi}_I = \begin{bmatrix} J_2\dot{\varphi} \\ 0 \end{bmatrix} = 0$$

where $\dot{\varphi} = \begin{bmatrix} \dot{\varphi}_{fA} \\ \dot{\varphi}_{fB} \end{bmatrix}$.

Hence

$$\begin{bmatrix} 1 & 0 & -l \\ 1 & 0 & l \\ 0 & 1 & 0 \end{bmatrix} R(\theta)\dot{\xi}_I = \begin{bmatrix} r\dot{\varphi}_{fA} \\ r\dot{\varphi}_{fB} \\ 0 \end{bmatrix}$$

Inverting the above equation yields the kinematic equation of the mobile robot:

$$\dot{\xi}_I = R^T(\theta) \begin{bmatrix} 1 & 0 & -l \\ 1 & 0 & l \\ 0 & 1 & 0 \end{bmatrix}^{-1} \begin{bmatrix} r\dot{\varphi}_{fA} \\ r\dot{\varphi}_{fB} \\ 0 \end{bmatrix}$$