Position Control

The servo control law is

$$v_2 = \ddot{y}_d + k_{2v}\dot{e} + k_{2p}e$$

where $e = y_d - y$. Therefore,

$$\ddot{e} + k_{2v}\dot{e} + k_{2p}e = 0$$

Force Control

The normal force exerted on the surface is given by:

$$f = k_e(x - x_e)$$

where k_e is the surface stiffness. Since $\ddot{f} = k_e\ddot{x}$, the dynamics equation can be written as

$$\frac{1}{k_e}\ddot{f} = v_1$$

Let $e_f = f_d - f$, the force servo controller is:

$$v_1 = \frac{1}{k_e}(\ddot{f}_d + k_{1v}\dot{e}_f + k_{1p}e_f)$$

Thus

$$\ddot{e}_f + k_{1v}\dot{e}_f + k_{1p}e_f = 0$$

The force controller requires f and $\dot{f}$, which can be calculated from the position and velocity:

$$f = k_e(x - x_e)$$

$$\dot{f} = k_e \dot{x}$$

In general, suppose the robot dynamics is given as:

$$M_x \ddot{X} + C_x + g_x + f_e = \tau$$

The hybrid position and force controller takes the form:

$$\tau = \alpha v + \beta$$

$$\alpha = M_x$$

$$\beta = C_x + g_x + f_e$$

Substitute into the robot dynamics, we have

$$M_x \ddot{X} + C_x + g_x + f_e = \alpha v + \beta$$

which gives:

$$\ddot{X} = v$$