NANYANG TECHNOLOGICAL UNIVERSITY

SEMESTER 1 EXAMINATION 2022-2023

EE6222 – MACHINE VISION

November / December 2022 Time Allowed: 3 hours

INSTRUCTIONS

1. This paper contains 5 questions and comprises 4 pages.
2. Answer all 5 questions.
3. All questions carry equal marks.
4. This is a closed-book examination.
5. Unless specifically stated, all symbols have their usual meanings.

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1. A digital image $f(x,y)$ of size 7×7 is given in Figure 1.

0	3	0	0	0	0	0
3	9	3	3	3	9	3
0	3	0	0	0	0	0
0	3	0	0	0	0	0
0	9	0	0	3	9	3
0	3	0	0	3	9	3
0	3	0	0	3	3	3

Figure 1

(a) An LSI filter of size 1×3 with mask [1/3 1/3 1/3] is applied to the image $f(x,y)$. Give the output image $g_1(x,y)$ of size 7×7 with the assumption that $f(x,y) = 0$ outside the image in Figure 1.

(7 Marks)

(b) A median filter of size 1×3 is applied to the image $f(x,y)$. Give the output image $g_2(x,y)$ of size 7×7 with the assumption that $f(x,y) = 0$ outside the image in Figure 1.

(7 Marks)

(c) Suppose that the image $f(x,y)$ contains information of a horizontal line, a vertical line and a square. These structures are corrupted by noise. Compare the two filtered images $g_1(x,y)$ and $g_2(x,y)$.

(6 Marks)

2. A binary image whose black pixels constitute set A is shown in Figure 2. A structure element (a filled circle) whose black pixels constitute set B is shown in Figure 3. Assume that the origin of B is at its center. Note that the structure element B is larger than the smallest circle and the width of the strip in A, and is smaller than the two larger circles in A. The boundary of A is shown in Figure 4.

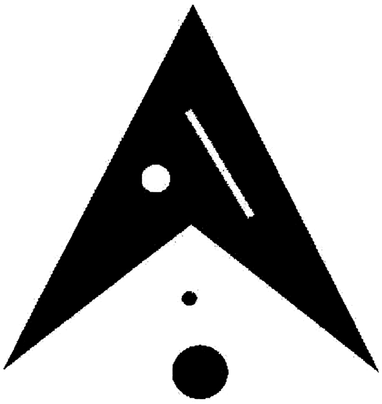

Figure 2

Figure 3

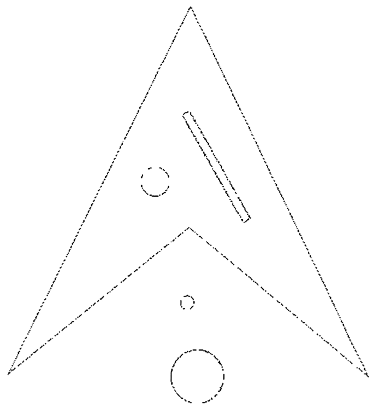

Figure 4

(a) Sketch the boundary of A using dotted line (or pencil) and sketch the boundary of the dilation of A by B, $A \oplus B$, using solid line (or pen) in the same plot.

(5 Marks)

(b) Sketch the boundary of A using dotted line (or pencil) and sketch the boundary of the erosion of A by B, $A \ominus B$, using solid line (or pen) in the same plot.

(5 Marks)

(c) Sketch the boundary of A using dotted line (or pencil) and sketch the boundary of the closing of A by B, $A \bullet B$, using solid line (or pen) in the same plot.

(5 Marks)

(d) Sketch the boundary of A using dotted line (or pencil) and sketch the boundary of the opening of A by B, $A \circ B$, using solid line (or pen) in the same plot.

(5 Marks)

3. Let $p_{\omega_i}(\omega_i)$ be the prior probability of class ω_i and $p_{\omega_i}(\omega_i|\boldsymbol{x})$ be the posterior probability of class ω_i. Let $p_x(\boldsymbol{x}|\omega_i)$ be the class-conditional probability density function and $p_x(\boldsymbol{x})$ be the probability density function over all classes. For symbolic simplicity, the subscripts of p_{ω_i} and p_x are omitted in the following, where it is clear from the context, which probability density function is referred to.

(a) If you know the value of the data $\boldsymbol{x}$ and make a decision that $\boldsymbol{x}$ belongs to a class ω_i, find the probability of the wrong decision $p(e_i|\boldsymbol{x})$ and hence derive the decision rule that minimizes $p(e_i|\boldsymbol{x})$, using one of the above quantities.

(5 Marks)

Note: Question No. 3 continues on page 3.

(b) Suppose none of the above quantities is available but we can use the available training data to estimate $p(\omega_i)$, $p(\boldsymbol{x}|\omega_i)$ and $p(\boldsymbol{x})$, derive the decision rule that minimizes the probability of the wrong decision so that you can make a decision based on the estimated $p(\omega_i)$ and $p(\boldsymbol{x}|\omega_i)$.

(5 Marks)

(c) Estimate $p(\omega_i)$, $p(\boldsymbol{x}|\omega_i)$ and $p(\boldsymbol{x})$ so that the decision rule that minimizes the probability of the wrong decision leads to the k-nearest neighbor classifier.

(10 Marks)

4. Given a $m \times n$ data matrix $\boldsymbol{X}$ that contains n centralized training samples, i.e., the sample mean vector is zero.

(a) Compute the covariance matrix $\boldsymbol{\Sigma}$ of the training data $\boldsymbol{X}$ and give out the definition of the eigenvalues λ_i and eigenvectors $\phi_i$of the covariance matrix $\boldsymbol{\Sigma}$. Then show that the variance $\sigma_i^2$of the projected training data on the unit-length eigenvector ϕ_i equals the eigenvalue λ_i.

(8 Marks)

(b) Prove that two eigenvectors ϕ_k and $\phi_j$corresponding to two different eigenvalues λ_k and λ_j, $\lambda_k \neq \lambda_j$, are orthogonal.

(6 Marks)

(c) Derive the covariance σ_{kj}^2 between the projected training data on the eigenvector ϕ_k and the projected training data on the eigenvector ϕ_j, where $\lambda_k \neq \lambda_j$.

(6 Marks)

5. (a) Given four collinear points A, B, C, D, which are projected on the image plane as points a, b, c, d, the definition of the cross-ratio of $[A, B, C, D]$ is $\frac{\|AC\|}{\|BC\|} / \frac{\|AD\|}{\|BD\|}$. Prove that the cross-ratio is projective invariant.

(4 Marks)

(b) Suppose point $\boldsymbol{P}$ is projected onto two image planes at points $\boldsymbol{p}$ and $\boldsymbol{p}'$ which are shown in Figure 5 (on page 4). Suppose the intrinsic matrix of the two cameras are both $\boldsymbol{K}$ while the extrinsic matrix of Camera 2 with respect to Camera 1 is defined as $[\boldsymbol{R}, \boldsymbol{t}]$. Derive the epipolar constraint with the essential matrix and the fundamental matrix and explain the mathematical relationship between the fundamental matrix and the essential matrix. Describe the constraints of leveraging the epipolar constraint with the essential matrix.

(6 Marks)

Note: Question No. 5 continues on page 4.

(c) Given the baseline of an ideal stereo camera as B, the focal length as f and the disparity as d, describe how to obtain the depth of an object Z with the disparity equation. If there is a disparity error of Δd, derive the estimated depth error ΔZ in terms of the disparity error Δd and the estimated depth Z.
Note: the actual disparity is assumed to be unknown, only the disparity error is known. The estimated depth error should be in its simplest form via Taylor series approximation.

(4 Marks)

(d) The brightness constancy is a key assumption in estimating optical flow. Given at time t a pixel at (x, y) with brightness I, which moves to $(x + u, y + v)$ at time $t + 1$, derive the brightness constancy constraint equation. Describe limitations of the brightness constancy constraint and state other key assumptions required for estimating optical flow. Briefly discuss the mathematical conditions where the estimation of optical flow may succeed.

(6 Marks)

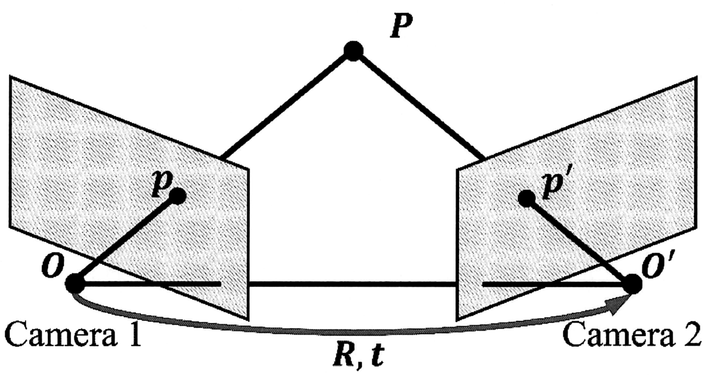

Figure 5

END OF PAPER

EE6222 MACHINE VISION

Please read the following instructions carefully:

1. **Please do not turn over the question paper until you are told to do so. Disciplinary action may be taken against you if you do so.**

2. You are not allowed to leave the examination hall unless accompanied by an invigilator. You may raise your hand if you need to communicate with the invigilator.

3. Please write your Matriculation Number on the front of the answer book.

4. Please indicate clearly in the answer book (at the appropriate place) if you are continuing the answer to a question elsewhere in the book.