

# EE7401 Homework 2

### Instructions:

- Please write your final solutions in the boxes shown below, then scan (if needed) and upload via NTULearn/Assignments/Homework2 by 23:59, 2 November 2023.
- Please do not copy from others, or let others copy your solution.
- Penalties may apply for late submission.
- Homework 2 carries 10% overall marks.

**Q1) Dilemma in lion city:** Two students living in *Singapore*, Dil Singh and Emma Lyon, can't decide whether to take 179 or 179A to JP (Jurong Point). They know the following facts.

- 179A takes 5 minutes less than 179 to reach JP.
- 179A arrival is a Poisson arrival with arrival rate  $\lambda$ . In other words,  $\mathbf{x}(t) = \mathbf{n}(0, t)$  is a Poisson process, where  $\mathbf{n}(0, t)$  is the number of arrivals of 179A between time 0 and  $t$  minute.  $\mathbf{n}(0, t)$  is  $k$  with probability  $P\{\mathbf{n}(0, t) = k\} = e^{-\lambda t} (\lambda t)^k / k!$  where  $t$  is in minutes.

**1.1)** Dil, Emma are waiting at the bus stop, and a 179 arrives at time  $t = 0$  minute. Dil prefers to take the 179. Emma, however, prefers to wait for the 179A.

(1.1.a) Find  $\hat{t}$ , the time of arrival of the 179A, such that 179A (Emma's preference) reaches JP at the same time as the 179 (Dil's preference). □

$\hat{t} =$  (2 marks)

(1.1.b) If the 179A arrives after  $\hat{t}$ , then Emma's preference reaches JP late. Find the probability  $P_{Emma}^{late}$  that Emma's preference reaches late, that is, the probability that no 179A arrives within time  $\hat{t}$ , or  $P\{\mathbf{x}(\hat{t}) = 0\}$ , in terms of  $\lambda$ .

$P_{Emma}^{plate} =$   (2 marks)

(1.1.c) Find the probability  $P_{Dil}^{late}$  that Dil's preference reaches late, in terms of  $\lambda$ .

$P_{Dil}^{late}$  =  

(3 marks)

(1.1.d) Find a condition on the arrival rate  $\lambda$  such that Emma's preference is better, that is,  $p_{Emma}^{late} < p_{Dil}^{late}$ .

(3 marks)

**1.2)** Dil, Emma decides to meet at the bus stop and take a bus to JP. Dil comes to the bus stop at time  $t = 0$  and waits for Emma.  $p$  179A buses arrive between 0 and 10 minutes. Emma finally comes to the bus stop at time  $t = 10$  and finds a visibly angry Dil waiting. Dil argues that the probability (say,  $P_1$ ) of at least one 179A arriving in the next 5 minutes, given that he has already seen  $p$  179A arriving in the past 10 minutes, is less than the usual probability of at least one 179A arriving in the next 5 minutes (say,  $P_2$ ). Emma feels otherwise. Dil and Emma are comparing  $P_1 =$  the conditional probability of greater than  $p$  arrivals in 0 to 15 minutes given there were  $p$  arrivals in 0 to 10 minutes  $= P\{\mathbf{x}(15) > p \mid \mathbf{x}(10) = p\}$ , with  $P_2 =$  the probability of at least 1 arrival in 10 to 15 minutes  $= P\{\mathbf{x}(15) - \mathbf{x}(10) \geq 1\}$ .

(1.2.a) Compare  $P\{\mathbf{x}(15) > p \mid \mathbf{x}(10) = p\}$  and  $P\{\mathbf{x}(15) - \mathbf{x}(10) \geq 1\}$  to see if Dil is correct ( $P_1 < P_2$ ), or if Emma is correct ( $P_1$  not less than  $P_2$ ). Show your steps.

(10 marks)

(1.2.b) Evaluate  $P_2 = P\{\mathbf{x}(15) - \mathbf{x}(10) \geq 1\}$  for  $\lambda = 0.2$  arrival/minute.

$$P\{\mathbf{x}(15) - \mathbf{x}(10) \geq 1\} = \boxed{\phantom{0.0000}}$$

(5 marks)

(5 marks)

(2.d) Unfortunately Mr. Vestor never took EE7401 or any similar course, and failed to appreciate orthogonality. However, he understands what correlated means (recall that you already explained correlation to him in part 1.a). To convince him, show that for a given  $t_1$ , the random variables  $\mathbf{x}(t_1)$  and  $\mathbf{x}'(t_1)$  are uncorrelated.

(5 marks)