EE7401 Probability and Random Processes

RA 3

1. The per bit error rate over a binary communication channel is 10^{-8}. No other statistics are known about the channel or data.

(a) What is the expected number of erroneous bits in a block of 1000 bits? Hint: Let X_i be 1 if the i-th bit is erroneous. You are given that $\mathbb{P}(X_i = 1) = 10^{-8}$.

(b) Use the Markov inequality to find an upper bound on the probability that a block of 1000 bits has 10 or more erroneous bits.

2. (Estimation vs. Detection) Let

$$X = \begin{cases} 1 & \text{with probability } \frac{1}{2}, \\ -1 & \text{with probability } \frac{1}{2}, \end{cases}$$

and the noise $Z \sim \mathrm{Unif}\,(-2, 2)$ be independent random variables. Their sum $Y = X + Z$ is observed.

(a) Find the conditional pmf $p_{X|Y}(x \mid y)$. Find the MMSE of X given Y and its MSE.

(b) Suppose we use a decoder to decide whether $X = 1$ or -1. Using the pmf $p_{X|Y}(x \mid y)$ found in part (a), find the MAP decoder and its probability of error. Compare the MAP decoder's MSE to the minimum MSE.