EE7401 Probability and Random Processes

RA 1

1. Monty Hall game show. A gold bar is placed in one of three boxes A, B or C. A contestant is asked to choose one of the boxes. The game host Monty Hall opens one of the unselected empty boxes. The contestant has a choice either to switch her selection to the third box or not.

(a) What is the sample space?

(b) Assume the box containing the gold bar is random, the contestant has no prior information and makes her first choice randomly and independently of the gold bar placement, and Monty Hall's choice of the empty box is random among the alternatives. Specify the probability measure for this random experiment and use it to compute the probability of winning the gold bar if the contestant decides to switch.

(c) (Optional) What would be your answer to (b) if now there are n boxes in total and Monty Hall reveals p boxes?

2. Let A be the event that a patient develops long COVID symptoms and B be the event that the patient is unvaccinated. A study has found that if a patient is unvaccinated, she is more likely to develop long COVID symptoms, i.e., $\mathbb{P}(A \mid B) \geq \mathbb{P}(A)$. Given a *vaccinated* patient, what can you say about the conditional probability of her developing long COVID symptoms? Justify your answer rigorously.