EE7401 Probability and Random Processes

RA 2 Solutions

Note that there can be many possible solutions. This is just one possible approach.

1.

$$\begin{aligned}\mathbb{P}(X > 0) &= 1 - \mathbb{P}(X \leq 0) \\ &= 1 - F_X(0) \\ &= 1 - \left(\frac{1}{3} + \frac{2}{3}\right) \\ &= 0.\end{aligned}$$

$$\begin{aligned}\mathbb{P}(|X| \geq 1) &= \mathbb{P}(X \leq -1) + \mathbb{P}(X \geq 1) \\ &= F_X(-1) + (1 - F_X(1)) \\ &= \frac{1}{3}.\end{aligned}$$

2.

$$\begin{aligned}\int \frac{p_{Y|X}(z \mid x)}{p_{X|Y}(x \mid z)} \,\mathrm{d}z &= \int \frac{p_{X,Y}(x,z)}{p_X(x)} \frac{p_Y(z)}{p_{X,Y}(x,z)} \,\mathrm{d}z \\ &= \int \frac{p_Y(z)}{p_X(x)} \,\mathrm{d}z \\ &= \frac{1}{p_X(x)}.\end{aligned}$$

We then have

$$p_{X,Y}(x,y) = p_{Y|X}(y \mid x) p_X(x) = \frac{p_{Y|X}(y \mid x)}{\int \frac{p_{Y|X}(z|x)}{p_{X|Y}(x|z)} \,\mathrm{d}z}.$$

3. Since

$$\mathbb{E}[\theta] = 0.4 \cdot 3 + 0.6 \cdot 8 = 6$$

and $\mathbb{E}[Y_1 \mid \theta] = \mathbb{E}[Y_1] = 10$ because $Y_1 \perp\!\!\!\perp \theta$, we have

$$\begin{aligned}
\mathbb{E}[X] &= \mathbb{E}[\mathbb{E}[X \mid \theta]] \\
&= \mathbb{E}[\mathbb{E}[Y_1 + Y_2 + \cdots + Y_\theta \mid \theta]] \\
&= \mathbb{E}[\theta \mathbb{E}[Y_1 \mid \theta]] \\
&= \mathbb{E}[10\theta] \\
&= 10\mathbb{E}[\theta] \\
&= 60.
\end{aligned}$$