Instruction Graph Statics

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1 Validity

p `valid` means that the Program p is a valid program.

$$\frac{(\mathbf{V}(s,\ c) :: vs,\ U)\ \texttt{defined} \qquad (\mathbf{V}(s,\ c) :: vs,\ \emptyset,\ s,\ U)\ \texttt{connected}}{\mathbf{P}(\mathbf{V}(s,\ c),\ vs)\ \texttt{valid}}$$

2 Defined

We let $U \subseteq \mathbb{Z}$ be a subset of the integers.

$(vs,\ U)$ `defined` means that the Vertices vs define exactly the set U of vertex indices.

$$\frac{}{(nil,\ \{\ \})\ \texttt{defined}} \qquad \frac{(vs,\ U)\ \texttt{defined} \qquad n \notin U}{(\mathbf{V}(n,\ c) :: vs,\ U \cup \{n\})\ \texttt{defined}}$$

3 Connected

We let $U \subseteq \mathbb{Z}$ be a subset of the integers.

$(vs,\ U_v,\ n,\ U)$ `connected` means that the vertex represented by n is connected to each vertex represented by an index in U of the vertices in vs, where U_v is the set of vertex indices of vertices already visited.

$$\frac{n \in U_v}{(vs,\ U_v,\ n,\ \emptyset)\ \texttt{connected}} \qquad \frac{\mathbf{V(}n,\ \mathbf{end)} \in vs \qquad n \notin U_v}{(vs,\ U_v,\ n,\ \{n\})\ \texttt{connected}}$$

$$\frac{\begin{array}{c}\mathbf{V(}n,\ \mathbf{do}\ a\ \mathbf{then}\ n'\mathbf{)} \in vs \\ (vs,\ U_v \cup \{n\},\ n',\ U)\ \texttt{connected} \qquad n \notin U_v\end{array}}{(vs,\ U_v,\ n,\ U \cup \{n\})\ \texttt{connected}}$$

$$\frac{\begin{array}{c}\mathbf{V(}n,\ \mathbf{do}\ a\ \mathbf{until}\ cnd\ \mathbf{then}\ n'\mathbf{)} \in vs \\ (vs,\ U_v \cup \{n\},\ n',\ U)\ \texttt{connected} \qquad n \notin U_v\end{array}}{(vs,\ U_v,\ n,\ U \cup \{n\})\ \texttt{connected}}$$

$$\frac{\begin{array}{c}\mathbf{V(}n,\ \mathbf{if}\ cnd\ \mathbf{then}\ n'\ \mathbf{else}\ n''\mathbf{)} \in vs \qquad (vs,\ U_v \cup \{n\},\ n',\ U)\ \texttt{connected} \\ (vs,\ U_v \cup U \cup \{n\},\ n'',\ U')\ \texttt{connected} \qquad n \notin U_v\end{array}}{(vs,\ U_v,\ n,\ U' \cup \{n\})\ \texttt{connected}}$$

$$\frac{\mathbf{V(}n,\ \mathbf{goto}\ n'\mathbf{)} \in vs \qquad (vs,\ U_v \cup \{n\},\ n',\ U)\ \texttt{connected} \qquad n \notin U_v}{(vs,\ U_v,\ n,\ U \cup \{n\})\ \texttt{connected}}$$