

# Appendix C: Instruction Graph Dynamics

## 1 Configuration

To describe a state midway through execution, we define

**Configuration**  $cfg ::= (n, vs, I, O)$  configurations

where  $n \in \mathbb{Z}$ , the integers,  $vs \in \mathbf{Vertices}$ ,  $I$  is a *bool* list, representing the input used to satisfy a **Condition**  $cond$ , and  $O$  is an **Action** list, representing the ordered (but reversed) list of actions that are executed.

## 2 Terminated

$(n, vs, I, O)$  **terminated** means that the state with vertex represented by  $n$  in vertices  $vs$  with remaining input  $I$  and current output  $O$  is in a finished state for the program execution context.

$$\frac{V(n, \mathbf{end}) \in vs}{(n, vs, I, O) \mathbf{terminated}} (\mathbf{cfg}_{\mathbf{terminated}})$$

## 3 Waiting

$(n, vs, I, O)$  **waiting** means that the state with vertex represented by  $n$  in vertices  $vs$  with remaining input  $I$  and current output  $O$  cannot proceed, as it requires more input to continue.

$$\frac{\mathbf{V}(n, \text{do } a \text{ until } cnd \text{ then } n') \in vs}{(n, vs, [], O) \text{ waiting}} (\text{dountil}_{\text{waiting}})$$

$$\frac{\mathbf{V}(n, \text{if } cnd \text{ then } n' \text{ else } n'') \in vs}{(n, vs, [], O) \text{ waiting}} (\text{ifthen}_{\text{waiting}})$$

## 4 Steps

$(n, vs, I, O) \mapsto (n', vs, I', O')$  means that the state with vertex represented by  $n$  in vertices  $vs$  with remaining input  $I$  and current output  $O$  continues to the state with vertex represented by  $n'$  in vertices  $vs$  with remaining input  $I'$  and current output  $O'$ .

$$\frac{\mathbf{V}(n, \text{do } a \text{ then } n') \in vs}{(n, vs, I, O) \mapsto (n', vs, I, a :: O)} (\text{doonce}_{\text{steps}})$$

$$\frac{\mathbf{V}(n, \text{do } a \text{ until } cnd \text{ then } n') \in vs}{(n, vs, true :: I, O) \mapsto (n', vs, I, a :: O)} (\text{dountil}_{\text{steps}}^1)$$

$$\frac{\mathbf{V}(n, \text{do } a \text{ until } cnd \text{ then } n') \in vs}{(n, vs, false :: I, O) \mapsto (n, vs, I, a :: O)} (\text{dountil}_{\text{steps}}^2)$$

$$\frac{\mathbf{V}(n, \text{if } cnd \text{ then } n' \text{ else } n'') \in vs}{(n, vs, true :: I, O) \mapsto (n', vs, I, O)} (\text{ifthen}_{\text{steps}}^1)$$

$$\frac{\mathbf{V}(n, \text{if } cnd \text{ then } n' \text{ else } n'') \in vs}{(n, vs, false :: I, O) \mapsto (n'', vs, I, O)} (\text{ifthen}_{\text{steps}}^2)$$

$$\frac{\mathbf{V}(n, \text{goto } n') \in vs}{(n, vs, I, O) \mapsto (n', vs, I, O)} (\text{goto}_{\text{steps}})$$