

# Instruction Graph Statics

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## 1 Validity

$p$  **valid** means that the Program  $p$  is a valid program.

$$\frac{(\mathbf{V}(s, c) :: vs, U) \text{ defined} \quad (\mathbf{V}(s, c) :: vs, \emptyset, s, U) \text{ connected}}{\mathbf{P}(\mathbf{V}(s, c), vs) \text{ valid}} (\mathbf{P}_{\text{valid}})$$

## 2 Defined

We let  $U \subseteq \mathbb{Z}$  be a subset of the integers.

$(vs, U)$  **defined** means that the Vertices  $vs$  define exactly the set  $U$  of vertex indices.

$$\frac{}{(\text{nil}, \{ \}) \text{ defined}} (\text{nil}_{\text{defined}})$$
$$\frac{(vs, U) \text{ defined} \quad n \notin U}{(\mathbf{V}(n, c) :: vs, U \cup \{n\}) \text{ defined}} (\text{cons}_{\text{defined}})$$

## 3 Connected

We let  $U \subseteq \mathbb{Z}$  be a subset of the integers.

$(vs, U_v, n, U)$  **connected** means that there exists a path from the vertex represented by  $n$  to each vertex represented by an index in  $U$  without going through any vertex in  $U_v$ . By “represented” we mean that  $vs$  contains

a vertex for that index.

$$\begin{array}{c}
\frac{(vs, U) \text{ defined} \quad U_v \subseteq U \quad n \in U_v}{(vs, U_v, n, \emptyset) \text{ connected}} (\text{visited}_{\text{connected}}) \\
\\
\frac{(vs, U) \text{ defined} \quad U_v \subseteq U \quad \mathbf{V}(n, \text{end}) \in vs \quad n \notin U_v}{(vs, U_v, n, \{n\}) \text{ connected}} (\text{end}_{\text{connected}}) \\
\\
\frac{\mathbf{V}(n, \text{do } a \text{ then } n') \in vs \quad (vs, U_v \cup \{n\}, n', U) \text{ connected} \quad n \notin U_v}{(vs, U_v, n, U \cup \{n\}) \text{ connected}} (\text{doonce}_{\text{connected}}) \\
\\
\frac{\mathbf{V}(n, \text{do } a \text{ until } \text{cnd} \text{ then } n') \in vs \quad (vs, U_v \cup \{n\}, n', U) \text{ connected} \quad n \notin U_v}{(vs, U_v, n, U \cup \{n\}) \text{ connected}} (\text{dountil}_{\text{connected}}) \\
\\
\frac{\mathbf{V}(n, \text{if } \text{cnd} \text{ then } n' \text{ else } n'') \in vs \quad (vs, U_v \cup \{n\}, n', U) \text{ connected} \quad (vs, U_v \cup U \cup \{n\}, n'', U') \text{ connected} \quad n \notin U_v}{(vs, U_v, n, U \cup U' \cup \{n\}) \text{ connected}} (\text{ifthen}_{\text{connected}}) \\
\\
\frac{\mathbf{V}(n, \text{goto } n') \in vs \quad (vs, U_v \cup \{n\}, n', U) \text{ connected} \quad n \notin U_v}{(vs, U_v, n, U \cup \{n\}) \text{ connected}} (\text{goto}_{\text{connected}})
\end{array}$$