Appendix B: Instruction Graph Statics

1 Validity

p `valid` means that the Program p is a valid program.

$$\frac{(\mathbf{V}(s,\ c) :: vs,\ U)\ \texttt{defined} \qquad (\mathbf{V}(s,\ c) :: vs,\ \emptyset,\ s,\ U)\ \texttt{connected}}{\mathbf{P}(\mathbf{V}(s,\ c),\ vs)\ \texttt{valid}}\,(\mathrm{P}_{\texttt{valid}})$$

2 Defined

We let $U \subseteq \mathbb{Z}$ be a subset of the integers.

$(vs,\ U)$ `defined` means that the Vertices vs define exactly the set U of vertex indices.

$$\frac{}{(nil,\ \{\ \})\ \texttt{defined}}\,(\texttt{nil}_{\texttt{defined}})$$

$$\frac{(vs,\ U)\ \texttt{defined} \qquad n \notin U}{(\mathbf{V}(n,\ c) :: vs,\ U \cup \{n\})\ \texttt{defined}}\,(\texttt{cons}_{\texttt{defined}})$$

3 Connected

We let $U \subseteq \mathbb{Z}$ be a subset of the integers.

$(vs,\ U_v,\ n,\ U)$ `connected` means that there exists a path from the vertex represented by n to each vertex represented by an index in U without going through any vertex in U_v. By "represented" we mean that vs contains a vertex for that index.

$$\frac{(vs,\ U)\ \texttt{defined} \qquad U_v \subseteq U \qquad n \in U_v}{(vs,\ U_v,\ n,\ \emptyset)\ \texttt{connected}}(\texttt{visited}_{\texttt{connected}})$$

$$\frac{(vs,\ U)\ \texttt{defined} \qquad U_v \subseteq U \qquad \mathbf{V}(n,\ \mathbf{end}) \in vs \qquad n \notin U_v}{(vs,\ U_v,\ n,\ \{n\})\ \texttt{connected}}(\texttt{end}_{\texttt{connected}})$$

$$\frac{\begin{array}{c}\mathbf{V}(n,\ \mathbf{do}\ a\ \mathbf{then}\ n') \in vs \\ (vs,\ U_v \cup \{n\},\ n',\ U)\ \texttt{connected} \qquad n \notin U_v\end{array}}{(vs,\ U_v,\ n,\ U \cup \{n\})\ \texttt{connected}}(\texttt{doonce}_{\texttt{connected}})$$

$$\frac{\begin{array}{c}\mathbf{V}(n,\ \mathbf{do}\ a\ \mathbf{until}\ cnd\ \mathbf{then}\ n') \in vs \\ (vs,\ U_v \cup \{n\},\ n',\ U)\ \texttt{connected} \qquad n \notin U_v\end{array}}{(vs,\ U_v,\ n,\ U \cup \{n\})\ \texttt{connected}}(\texttt{dountil}_{\texttt{connected}})$$

$$\frac{\begin{array}{c}\mathbf{V}(n,\ \mathbf{if}\ cnd\ \mathbf{then}\ n'\ \mathbf{else}\ n'') \in vs \qquad (vs,\ U_v \cup \{n\},\ n',\ U)\ \texttt{connected} \\ (vs,\ U_v \cup U \cup \{n\},\ n'',\ U')\ \texttt{connected} \qquad n \notin U_v\end{array}}{(vs,\ U_v,\ n,\ U \cup U' \cup \{n\})\ \texttt{connected}}(\texttt{ifthen}_{\texttt{connected}})$$

$$\frac{\mathbf{V}(n,\ \mathbf{goto}\ n') \in vs \qquad (vs,\ U_v \cup \{n\},\ n',\ U)\ \texttt{connected} \qquad n \notin U_v}{(vs,\ U_v,\ n,\ U \cup \{n\})\ \texttt{connected}}(\texttt{goto}_{\texttt{connected}})$$