Instruction Graph Dynamics

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1 Configuration

To describe a state midway through execution, we define

$$\texttt{Configuration}\quad cfg\quad ::=\quad (n,\ vs,\ I,\ O)\qquad \text{configurations}$$

where $n \in \mathbb{Z}$, the integers, $vs \in \texttt{Vertices}$, I is a *bool* list, representing the input used to satisfy a `Condition` cnd, and O is an `Action` list, representing the ordered (but reversed) list of actions that are executed.

2 Terminated

$(n,\ vs,\ I,\ O)$ `terminated` means that the state with vertex represented by n in vertices vs with remaining input I and current output O is in a finished state for the program execution context.

$$\frac{\mathbf{V}(n,\ \mathbf{end}) \in vs}{(n,\ vs,\ I,\ O)\ \texttt{terminated}}$$

3 Waiting

$(n,\ vs,\ I,\ O)$ `waiting` means that the state with vertex represented by n in vertices vs with remaining input I and current output O cannot proceed, as it requires more input to continue.

$$\frac{\mathbf{V}(n,\ \mathbf{do}\ a\ \mathbf{until}\ cnd\ \mathbf{then}\ n') \in vs}{(n,\ vs,\ [\,],\ O)\ \texttt{waiting}}$$

$$\frac{\mathbf{V}(n,\ \mathbf{if}\ cnd\ \mathbf{then}\ n'\ \mathbf{else}\ n'') \in vs}{(n,\ vs,\ [\,],\ O)\ \texttt{waiting}}$$

4 Steps

$(n,\ vs,\ I,\ O) \longmapsto (n',\ vs,\ I',\ O')$ means that the state with vertex represented by n in vertices vs with remaining input I and current output O continues to the state with vertex represented by n' in vertices vs with remaining input I' and current output O'.

$$\frac{\mathbf{V}(n,\ \mathbf{do}\ a\ \mathbf{then}\ n') \in vs}{(n,\ vs,\ I,\ O) \longmapsto (n',\ vs,\ I,\ a :: O)}$$

$$\frac{\mathbf{V}(n,\ \mathbf{do}\ a\ \mathbf{until}\ cnd\ \mathbf{then}\ n') \in vs}{(n,\ vs,\ \mathit{true} :: I,\ O) \longmapsto (n',\ vs,\ I,\ a :: O)}$$

$$\frac{\mathbf{V}(n,\ \mathbf{do}\ a\ \mathbf{until}\ cnd\ \mathbf{then}\ n') \in vs}{(n,\ vs,\ \mathit{false} :: I,\ O) \longmapsto (n,\ vs,\ I,\ a :: O)}$$

$$\frac{\mathbf{V}(n,\ \mathbf{if}\ cnd\ \mathbf{then}\ n'\ \mathbf{else}\ n'') \in vs}{(n,\ vs,\ \mathit{true} :: I,\ O) \longmapsto (n',\ vs,\ I,\ O)}$$

$$\frac{\mathbf{V}(n,\ \mathbf{if}\ cnd\ \mathbf{then}\ n'\ \mathbf{else}\ n'') \in vs}{(n,\ vs,\ \mathit{false} :: I,\ O) \longmapsto (n'',\ vs,\ I,\ O)}$$

$$\frac{\mathbf{V}(n,\ \mathbf{goto}\ n') \in vs}{(n,\ vs,\ I,\ O) \longmapsto (n',\ vs,\ I,\ O)}$$