# Instruction Graph Proofs

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## 1 Progress

If $cfg$ `cfgvalid`, then either

1. $cfg$ `terminated`
2. $cfg$ `waiting`
3. $\exists\ cfg'$ `s.t.` $cfg \longmapsto cfg'$

### 1.1 Proof

We proceed by induction on the judgment $cfg$ `cfgvalid`. There is only one rule that concludes $cfg$ `cfgvalid`:

$$\frac{\mathbf{P}(v,\ vs)\ \texttt{valid} \qquad \mathbf{V}(n,\ c) \in v :: vs}{(n,\ v :: vs,\ I,\ O)\ \texttt{cfgvalid}}$$

So we know $cfg$ is of the form $(n,\ v :: vs,\ I,\ O)$ and $\mathbf{V}(n,\ c) \in vs$. We continue by structural induction on $c$, which is of the sort `Content`.

Case $c$ is **do** $a$ **then** $n'$:

Then by the rule

$$\frac{\mathbf{V}(n,\ \mathbf{do}\ a\ \mathbf{then}\ n') \in vs}{(n,\ vs,\ I,\ O) \longmapsto (n',\ vs,\ I,\ a :: O)}$$

we can conclude $(n,\ v :: vs,\ I,\ O) \longmapsto (n',\ v :: vs,\ I,\ a :: O)$.

Case $c$ is **do** $a$ **until** $cnd$ **then** $n'$:

We use structural induction on I.

Inner case $I$ is $[\,]$:

Then by the rule

$$\frac{\mathbf{V}(n,\ \mathbf{do}\ a\ \mathbf{until}\ cnd\ \mathbf{then}\ n') \in vs}{(n,\ vs,\ [\,],\ O)\ \texttt{waiting}}$$

we can conclude $(n,\ v :: vs,\ I,\ O)\ \texttt{waiting}$.

Inner case $I$ is $true :: I'$:

Then by the rule

$$\frac{\mathbf{V}(n,\ \mathbf{do}\ a\ \mathbf{until}\ cnd\ \mathbf{then}\ n') \in vs}{(n,\ vs,\ true :: I,\ O) \longmapsto (n',\ vs,\ I,\ a :: O)}$$

we can conclude $(n,\ v :: vs,\ I,\ O) \longmapsto (n',\ v :: vs,\ I',\ a :: O)$.

Inner case $I$ is $false :: I'$:

Then by the rule

$$\frac{\mathbf{V}(n,\ \mathbf{do}\ a\ \mathbf{until}\ cnd\ \mathbf{then}\ n') \in vs}{(n,\ vs,\ false :: I,\ O) \longmapsto (n,\ vs,\ I,\ a :: O)}$$

we can conclude $(n,\ v :: vs,\ I,\ O) \longmapsto (n,\ v :: vs,\ I',\ a :: O)$.

Case $c$ is **if** $cnd$ **then** $n'$ **else** $n''$:

We use structural induction on I.

Inner case $I$ is $[\,]$:

Then by the rule

$$\frac{\mathbf{V}(n,\ \textbf{if}\ \mathit{cnd}\ \textbf{then}\ n'\ \textbf{else}\ n'') \in \mathit{vs}}{(n,\ \mathit{vs},\ [\,],\ O)\ \texttt{waiting}}$$

we can conclude $(n,\ v :: \mathit{vs},\ I,\ O)\ \texttt{waiting}$.

Inner case $I$ is $\mathit{true} :: I'$:

Then by the rule

$$\frac{\mathbf{V}(n,\ \textbf{if}\ \mathit{cnd}\ \textbf{then}\ n'\ \textbf{else}\ n'') \in \mathit{vs}}{(n,\ \mathit{vs},\ \mathit{true} :: I,\ O) \longmapsto (n',\ \mathit{vs},\ I,\ O)}$$

we can conclude $(n,\ v :: \mathit{vs},\ I,\ O) \longmapsto (n',\ v :: \mathit{vs},\ I',\ O)$.

Inner case $I$ is $\mathit{false} :: I'$:

Then by the rule

$$\frac{\mathbf{V}(n,\ \textbf{if}\ \mathit{cnd}\ \textbf{then}\ n'\ \textbf{else}\ n'') \in \mathit{vs}}{(n,\ \mathit{vs},\ \mathit{false} :: I,\ O) \longmapsto (n'',\ \mathit{vs},\ I,\ O)}$$

we can conclude $(n,\ v :: \mathit{vs},\ I,\ O) \longmapsto (n'',\ v :: \mathit{vs},\ I',\ O)$.

Case $c$ is $\textbf{goto}\ n'$:

Then by the rule

$$\frac{\mathbf{V}(n,\ \textbf{goto}\ n') \in \mathit{vs}}{(n,\ \mathit{vs},\ I,\ O) \longmapsto (n',\ \mathit{vs},\ I,\ O)}$$

we can conclude $(n,\ v :: \mathit{vs},\ I,\ O) \longmapsto (n',\ v :: \mathit{vs},\ I,\ O)$.

Case $c$ is **end**:

Then by the rule

$$\frac{\mathbf{V}(n,\ \mathbf{end}) \in vs}{(n,\ vs,\ I,\ O)\ \texttt{terminated}}$$

we can conclude $(n,\ v :: vs,\ I,\ O)$ `terminated`.

## 2 Preservation

If $cfg$ `cfgvalid` and $cfg \longmapsto cfg'$ then $cfg'$ `cfgvalid`.