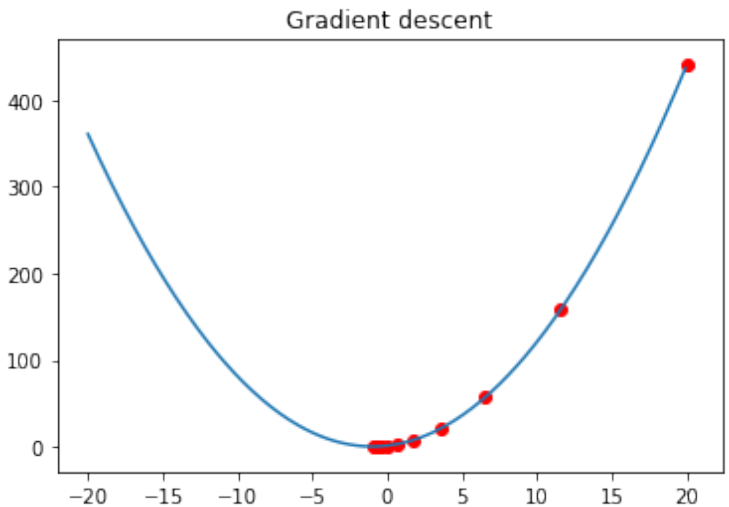
Training neural network

Gradient descent

- Hill climbing algorithm that follows the gradient of the function to be optimized



see notebook Linear - Nonlinear separation

Gradient descent

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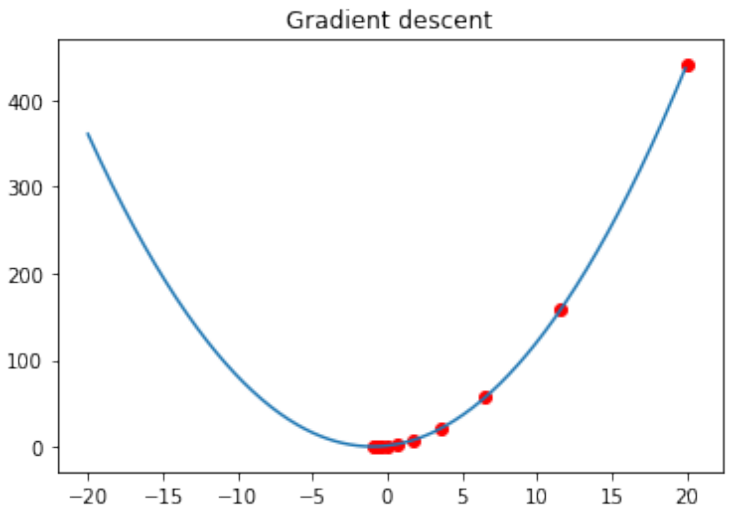


$\mathbf{w} \leftarrow$ any point in the parameter space

loop until convergence **do**

  for each w_i **in w do**

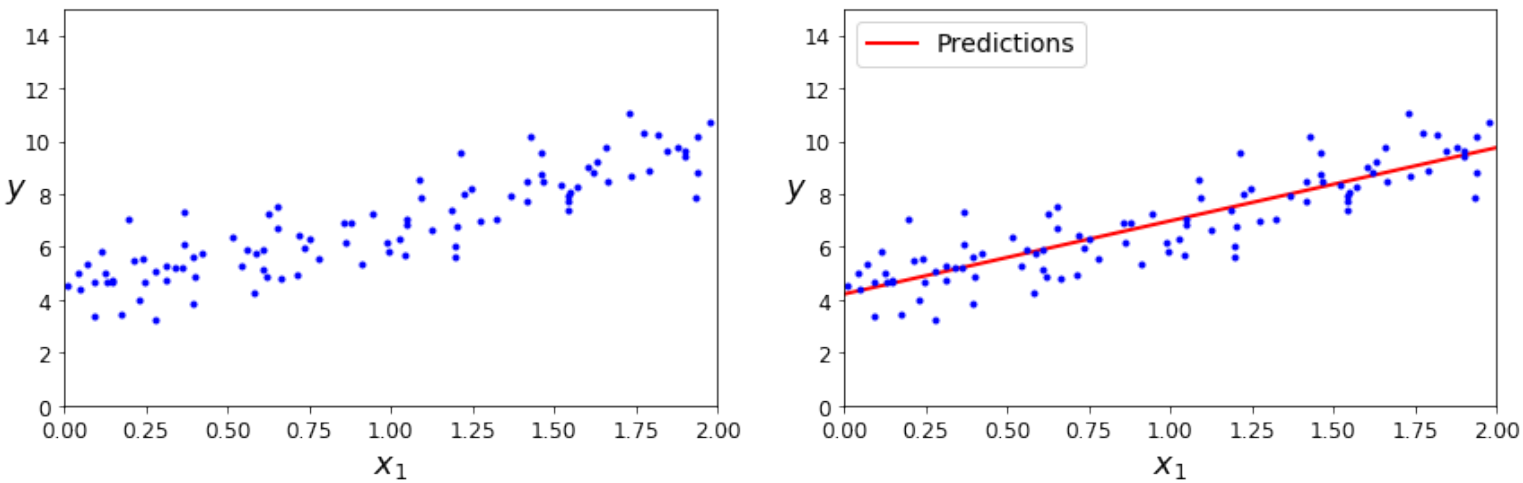
$$w_i \leftarrow w_i - \alpha \frac{\partial}{\partial w_i} Loss(\mathbf{w})$$

see notebook Linear - Nonlinear separation

Linear regression - with gradient descent

- Univariate linear function with input x and output y:
  $y = w_1 * x + w_0$
- Univariate Linear regression - finding the h_w that best fits n point x, y

$$h_w(x) = w_1 x + w0$$



How? with gradient descent in order to minimize the loss
see detailed analysis - it will also be discussed during IS lab

Math explanation

$$Loss(h_{\mathbf{w}}) = \sum_{j=1}^{N} L_2(y_j, h_{\mathbf{w}}(x_j)) = \sum_{j=1}^{N} (y_j - h_{\mathbf{w}}(x_j))^2 = \sum_{j=1}^{N} (y_j - (w_1 x_j + w_0))^2$$

For one example:

$$\begin{aligned}
\frac{\partial}{\partial w_i} Loss(\mathbf{w}) &= \frac{\partial}{\partial w_i} (y - h_{\mathbf{w}}(x))^2 \\
&= 2(y - h_{\mathbf{w}}(x)) \times \frac{\partial}{\partial w_i} (y - h_{\mathbf{w}}(x)) \\
&= 2(y - h_{\mathbf{w}}(x)) \times \frac{\partial}{\partial w_i} (y - (w_1 x + w_0))
\end{aligned}$$

$$\frac{\partial}{\partial w_0} Loss(\mathbf{w}) = -2(y - h_{\mathbf{w}}(x)); \qquad \frac{\partial}{\partial w_1} Loss(\mathbf{w}) = -2(y - h_{\mathbf{w}}(x)) \times x$$

Gradient descent rule:

$$w_0 \leftarrow w_0 + \alpha\,(y - h_{\mathbf{w}}(x)); \quad w_1 \leftarrow w_1 + \alpha\,(y - h_{\mathbf{w}}(x)) \times x$$

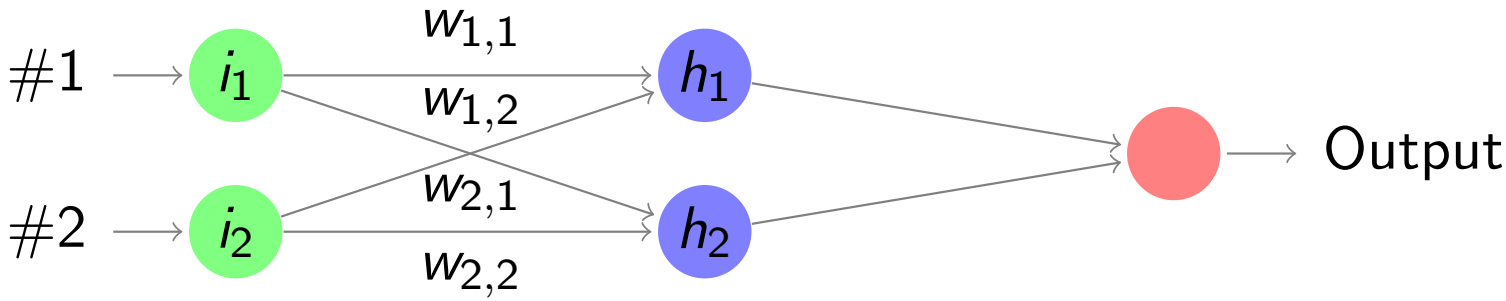
Batch gradient descent

$$w_0 \leftarrow w_0 + \alpha \sum_j (y_j - h_{\mathbf{w}}(x_j)); \quad w_1 \leftarrow w_1 + \alpha \sum_j (y_j - h_{\mathbf{w}}(x_j)) \times x_j$$

Backpropagation

see slide 1 - what is a neural network?

Neural networks



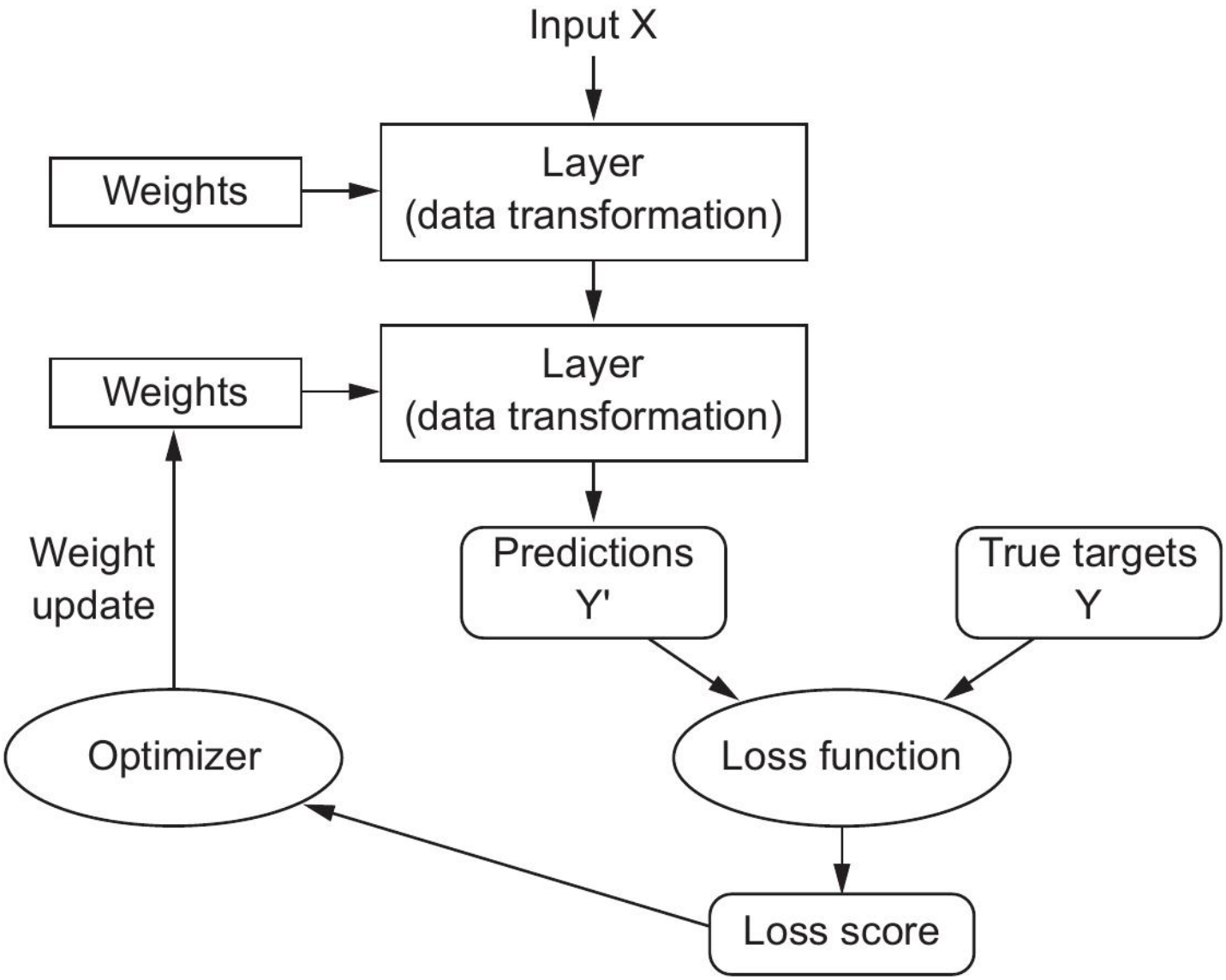
$$a_j = g(in_j)$$

$$a_j = g(\sum_i (w_{i,j} * a_i))$$

- a_i output of neuron i,
- $a_0 = 1$ bias
- $W_{i,j}$ weight between neuron i and neuron j
- g activation function

example: $a_{h_1} = g(w_{1,1} * a_i + w_{2,1} * a_2 + w_{0,1})$

Layered transformation



Loss

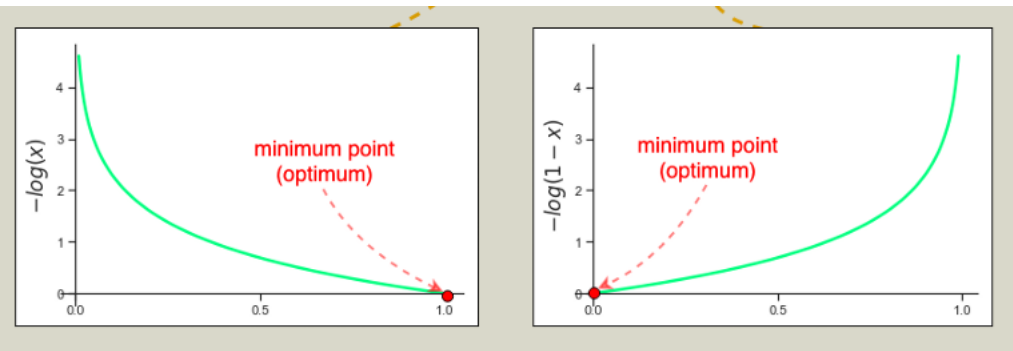
- Mean square error MSE

$$MSE = \frac{1}{N} \sum_{(x,y) \in D} (y - prediction(x))^2$$

- Log Loss

$$LogLoss = \sum_{(x,y) \in D} -ylog(y') - (1-y)log(1-y')$$

  - $(x, y) \in D$ - labeled dataset; y - label for sample x, x is a vector of features. For binary classification y is 0 or 1; y' - $prediction(x)$



Backpropagation - based on Gradient Descent

Back-propagation

- Output layer

$$w_{j,i} = w_{j,i} + \alpha * a_j * \Delta_i$$

, unde $\Delta_i = Error_i * g'(in_i)$

- Hidden layer (k node - j node - i nodes)
  Propagated error:

$$\Delta_j = g'(in_j) \sum_i w_{j,i} \Delta_i$$

  - hidden node j is responsible for some fraction of the error Δ_i in each of the output nodes to which it connects; stronger connection (higher weight) means higher contribution to error

$$w_{k,j} = w_{k,j} + \alpha * a_k * \Delta_j$$

Backpropagation

- compute the Δ values for the output units using the observed error (loss)
- starting with output layer, repeat fpr each layer in the network, until earliest hiddent layer is reached:
  - propagate the *Delta* value back to the previous layer
  - update the weights between two layers

Math demo (Chain rule (nice explanations AIMA, notes)

Neural Network Playground

https://playground.tensorflow.org/

What can go wrong

- Vanishing gradient
- Exploding gradient
- Overfitting/underfitting
  - Adjust size of the network
  - Use regularization: reduce complexity

$$minimize(Loss(Data|Model) + complexity(Model))$$

    - L2 penalizes $weight^2$, L1 penalizes $||weight||$.
    - the derivative of L2 is 2 * weight, The derivative of L1 is k (a constant, whose value is independent of weight). L2 does not normally drive weights to zero, while L1 can $\rightarrow$ L1 eliminates irrelevant features
  - Dropout - Dropout, is useful for neural networks. It works by randomly "dropping out" unit activations in a network for a single gradient step. The more you drop out, the stronger the regularization:
  - Early stopping
  - Adjust learning rate

Vanishing gradient

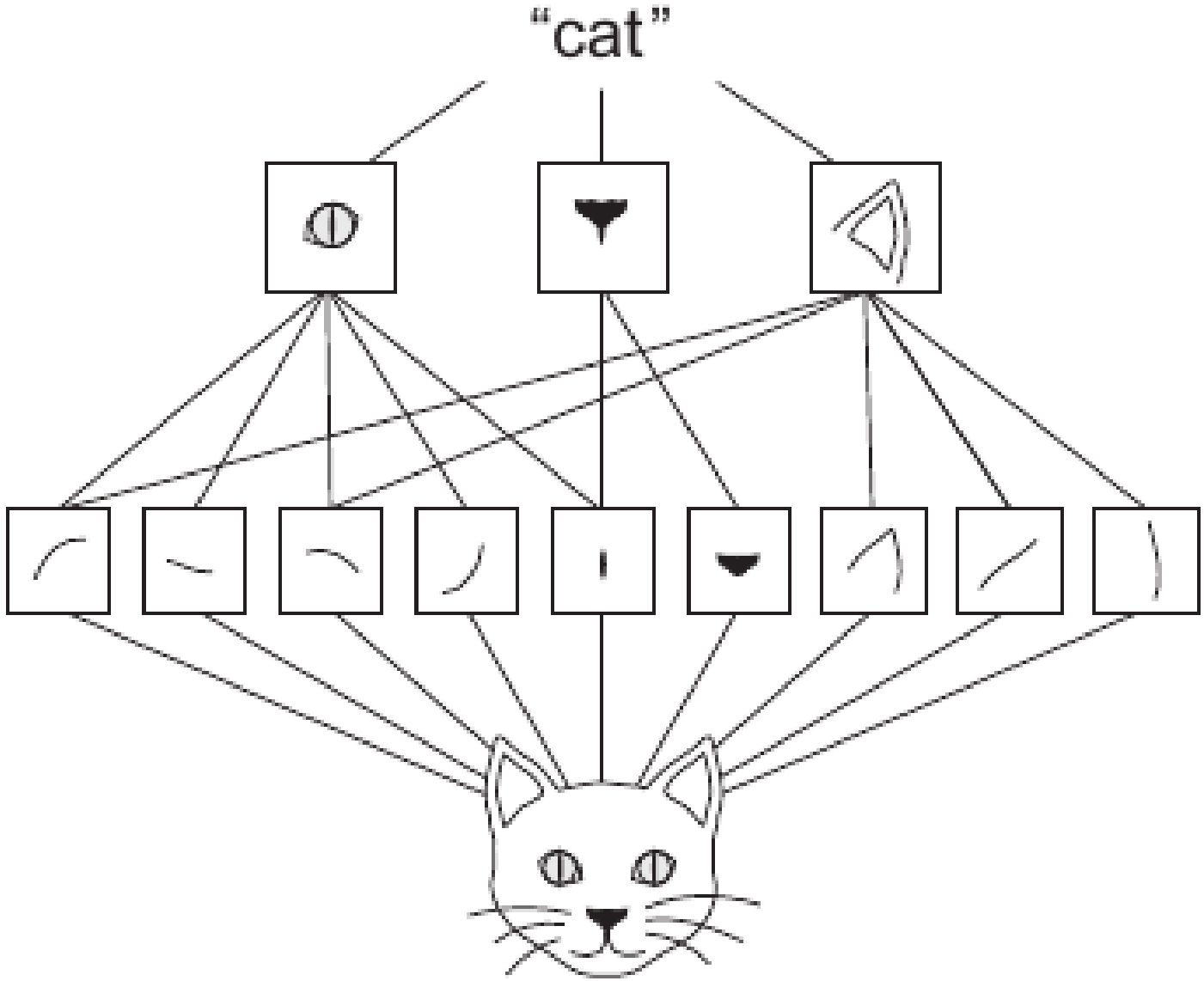
- The gradients for the lower layers (closer to the input) can become very small. In deep networks, computing these gradients can involve taking the product of many small terms.
- When the gradients vanish toward 0 for the lower layers, these layers train very slowly, or not at all.
- The ReLU activation function can help prevent vanishing gradients.

see Google Machine Learning Crash Course

Exploding gradient

- If the weights in a network are very large, then the gradients for the lower layers involve products of many large terms. In this case you can have exploding gradients: gradients that get too large to converge.
- Batch normalization can help prevent exploding gradients, as can lowering the learning rate.

CNN - Hierachies of patterns



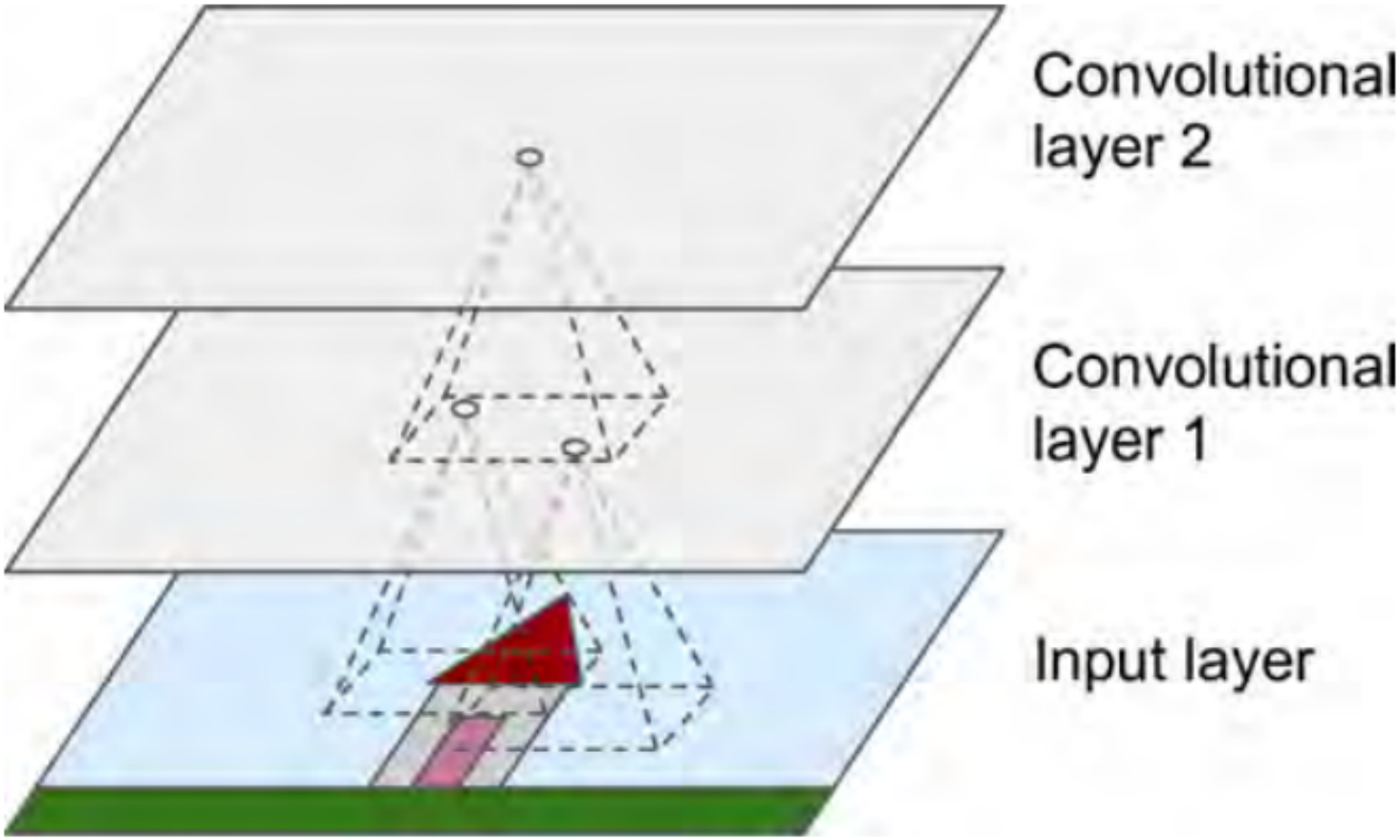
1

[1] Chollet. Deep Learning with python

CNN

- Receptive fields
  - First convolutional layer: neurons are not connected to every single pixel in the input image, but only pixels in thier receptive fields
  - Second convolutional layer: each neuron is connected only to neurons located within a small rectangle
- the networks concentrates on
  - low-level features in the first hidden layer, then
  - assemble them into higher-level features in the next hidden layer,
  - and so on.

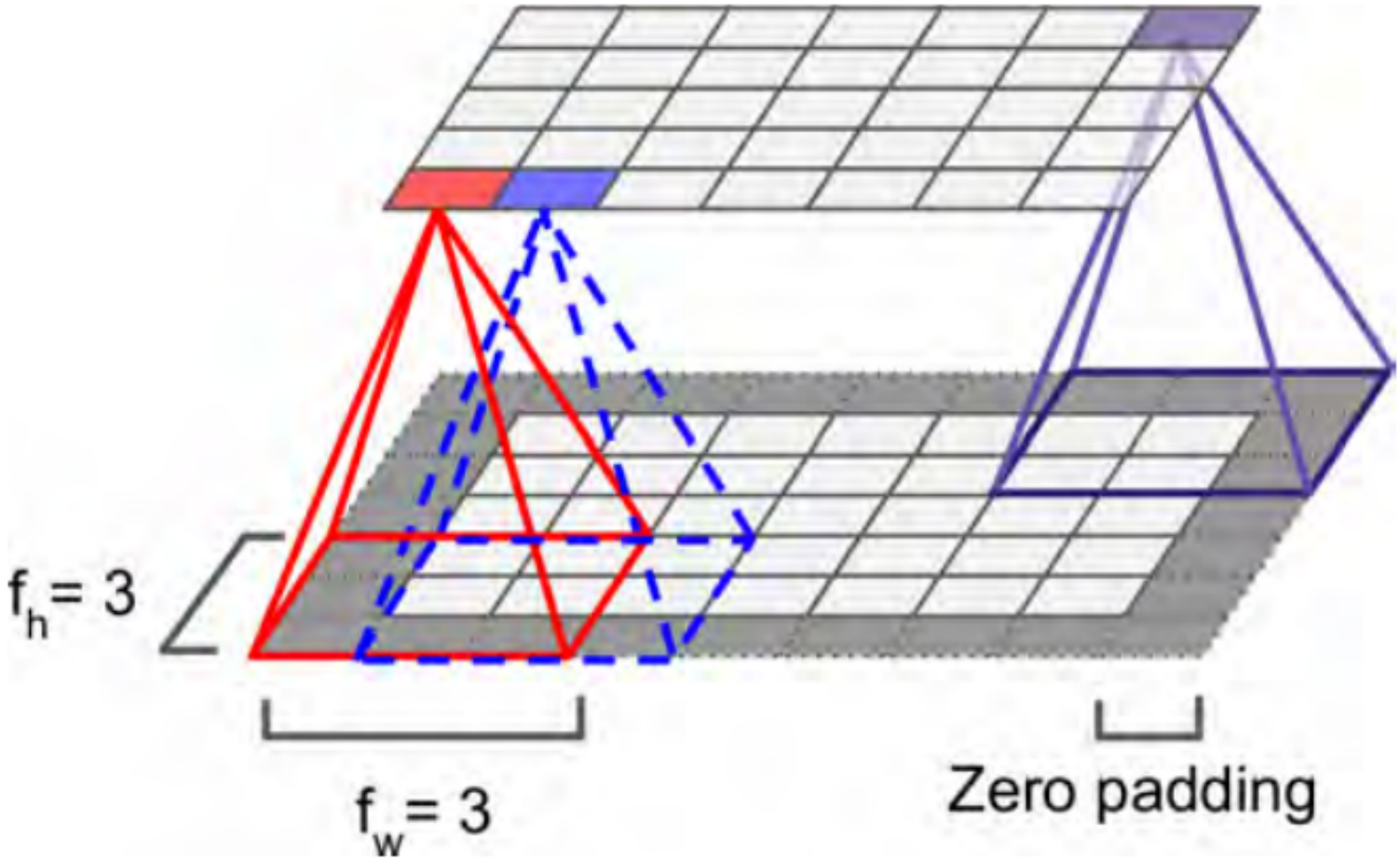
CNN layers with rectangular local receptive fields



2

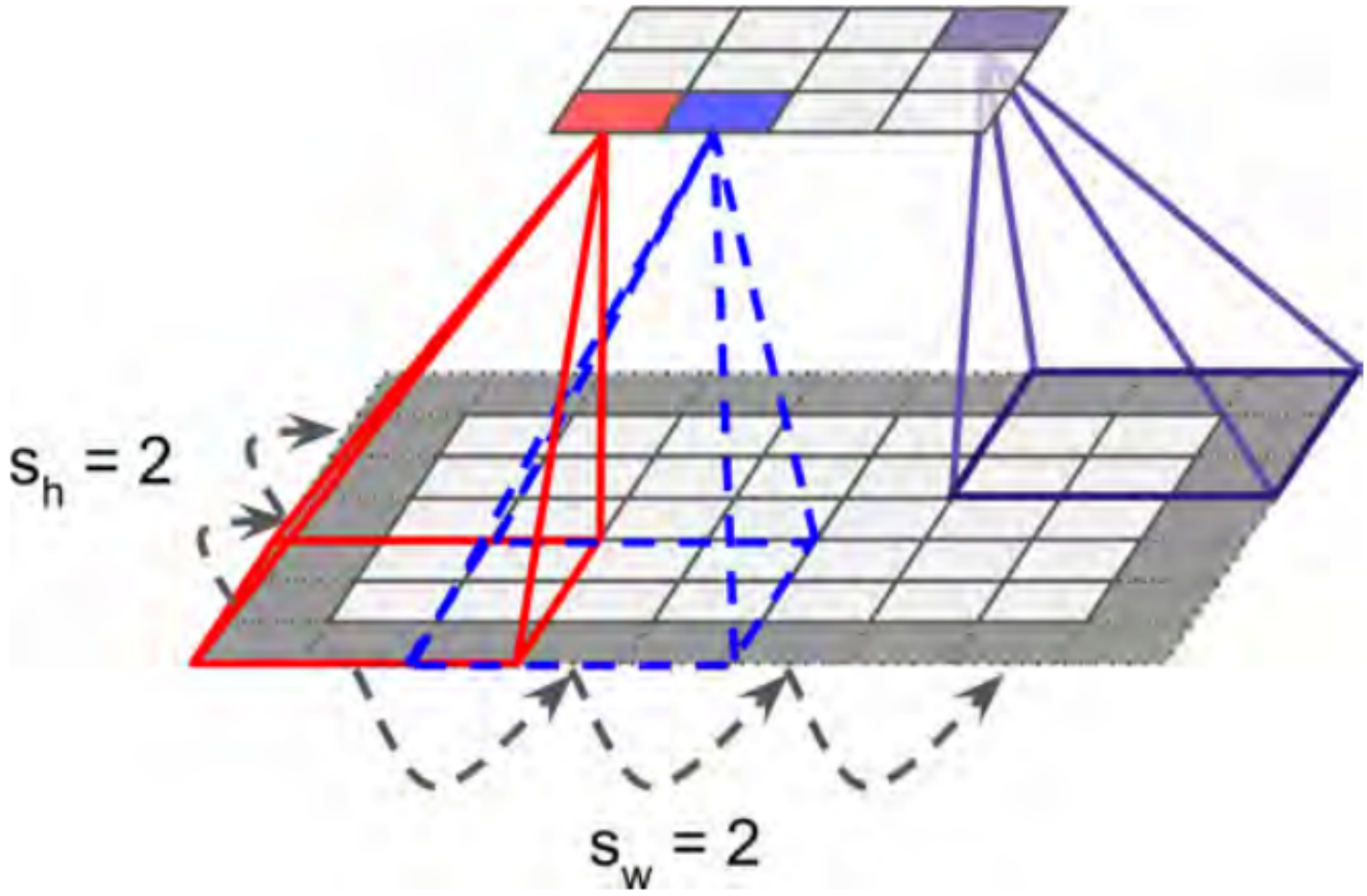
[2] Pictures taken from Geron. Hands-on Machine Learning

Connections between layers (with zero padding)



- neuron i, j - connected to neurons in previous layer in rows i to $i + f_h - 1$, columns j to $j + f_w - 1$.
- the layer has the same height and width as the previous layer (with zero padding)

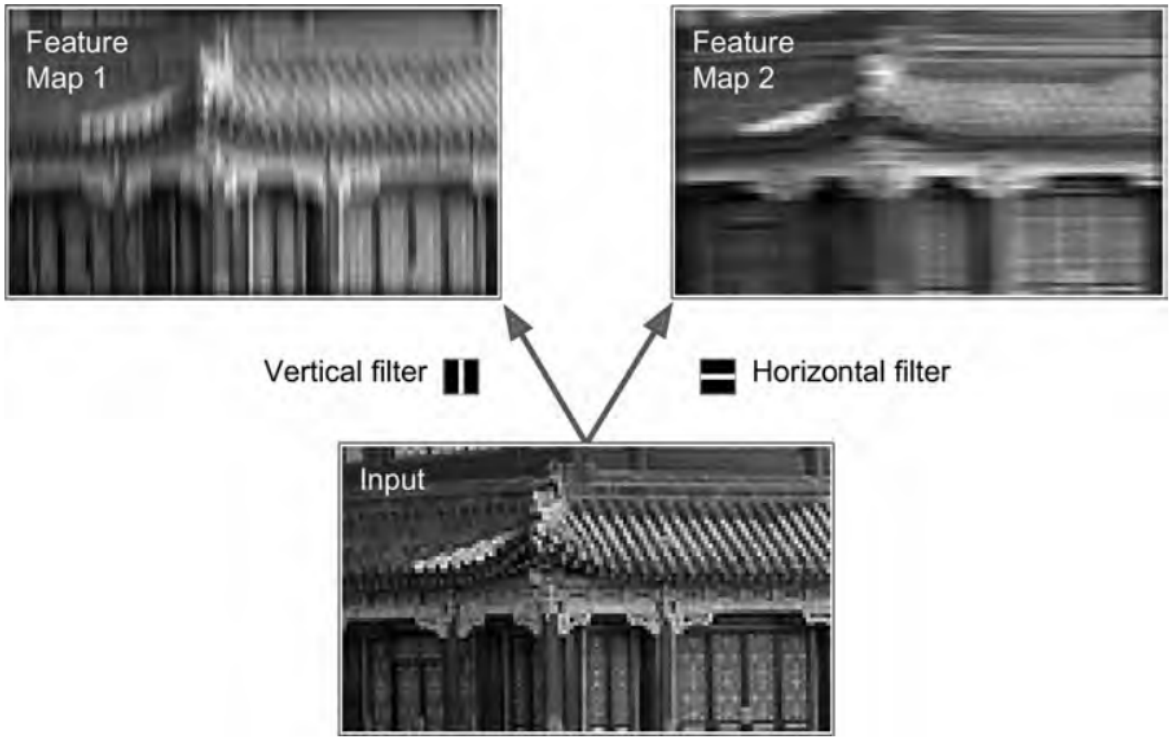
Reduce dimensionality using a stride



Input layer : $5x7$. Filter: $3x3$. Stride: 2.Result: $3x4$

Filters

- a neuron's weight can be seen as a small image the size of the receptive field
- a layer full of neurons using the same filter gives a $=$ highlights the areas in an image that are most similar to the filter



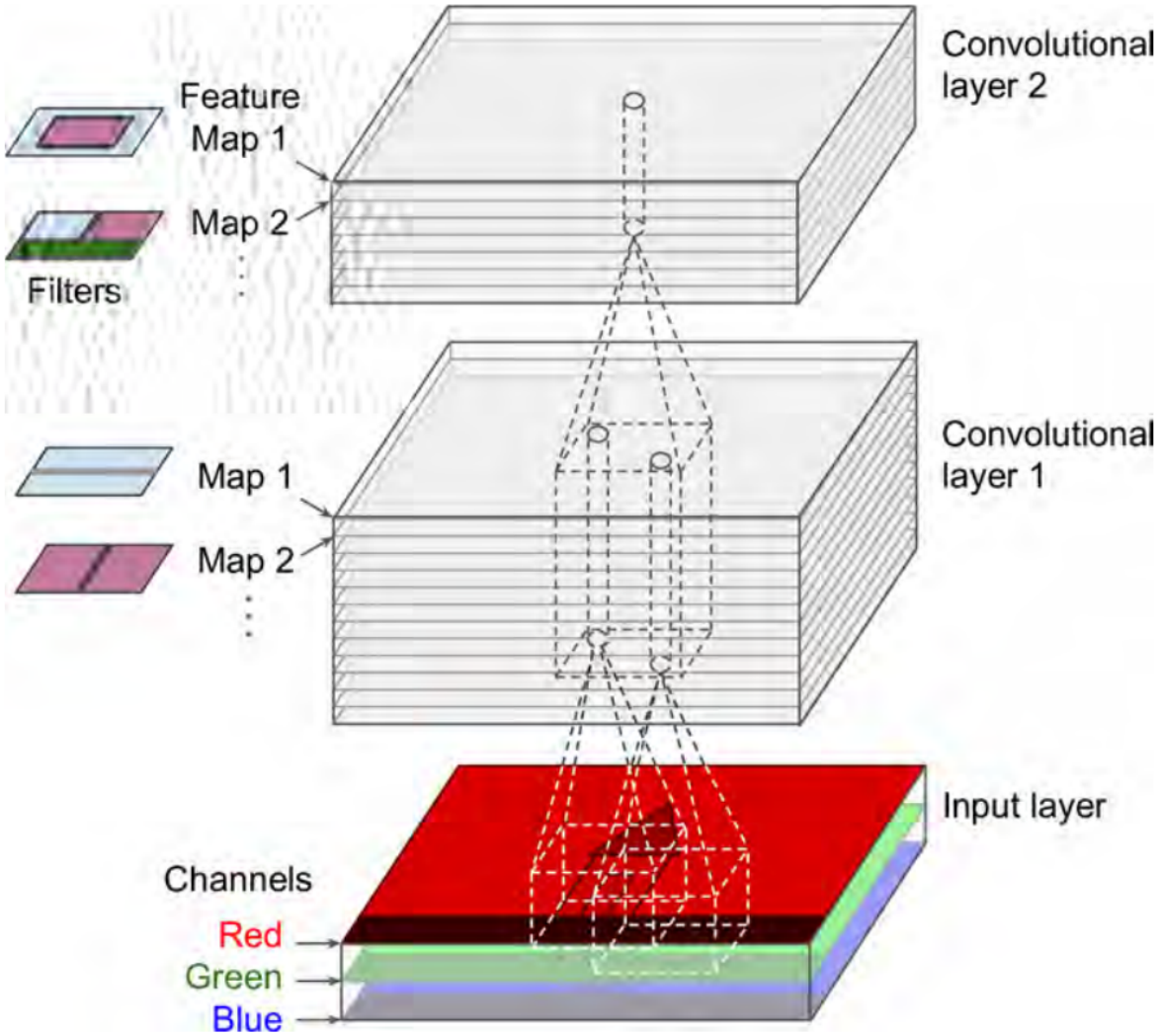
Time for notebook

Stacking multiple feature maps

Obs: All the previous representations were simplifications

- a convolutional layer is composed of several feature maps of equal sizes
- within one feature map, all neurons share the same parameters
- different features maps may have different parameters
- a neuron's receptive field includes a limited number of neurons but across all the features maps of the previous layer

Convolution layers with multiple feature maps and image with three channels



cont. Feature maps

A neuron located

- in row i, column j of the feature map k is connected
- to the outputs of the neurons in the previous layer located in rows ixs_w to $ixs_w + f_w - 1$, columns jxs_h to $jxs_h + f_h - 1$ across all feature maps

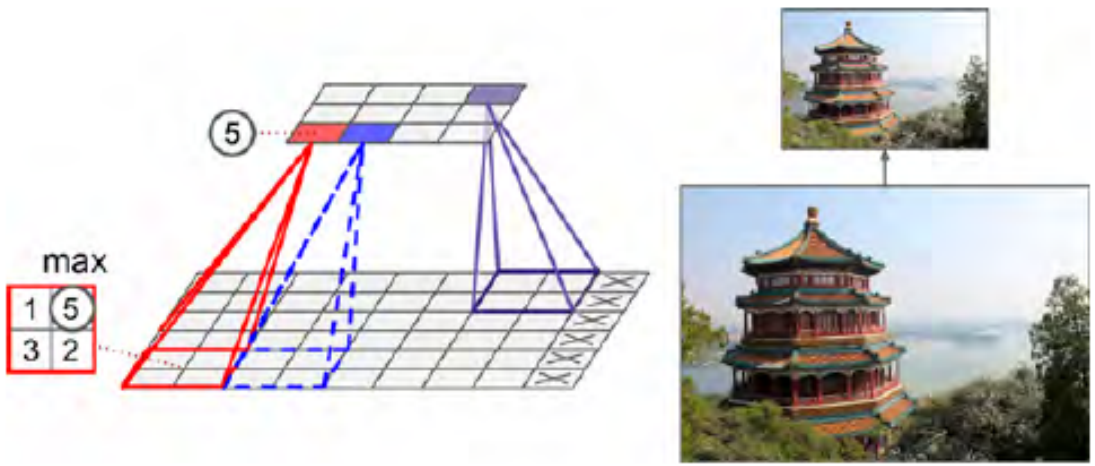
All neurons located in teh same row and column but in different feature maps are connected to the outputs of the exact same neurons in the previous layer

Time for notebook

- Fashion mnist with only dense layers (fully connected)
- Fashion mnist with convolutional layers
  - CNN
  - number of parameters (model.summary)
  - improvement over fully connected

Pooling layer: max, mean

- Subsample the input image - reduce number of parameteres - reduce overfitting
- Required info: size, stride, padding type
- Has no weights
- Worls on every input channel independently: output depth is the same as input depth



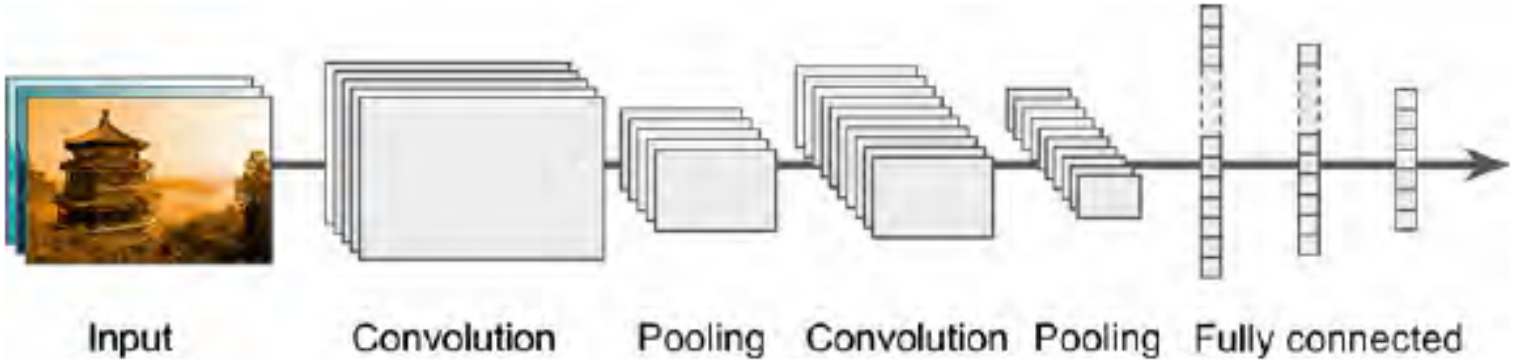
max pooling, 2x2, stride 2, no padding

Why not simply use fully connected layers

- Image 100x100. First dense layer 1000 neurons (Obs: it restricts the amount of information transmitted to the next layer) $\rightarrow 10^7$ parameters for only one layer!!!
- the learnt patterns are translation invariants: a certain pattern can be recognized anywhere
- cnn can learn hierarchies of patterns

Time for notebook

Typical CNN architecture



Pooling effect on FMNIST
Transfer knowledge (ImageNet) - fmnist
Data augmentation - dogs/cats
Covid radiology