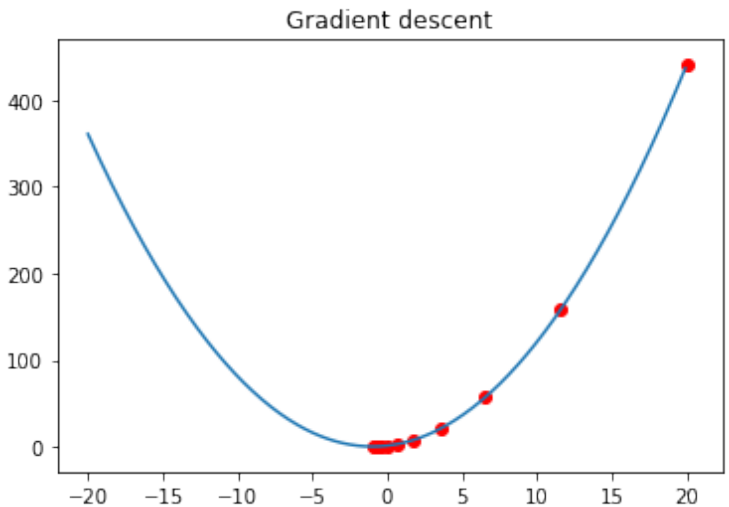
Training neural network

Gradient descent

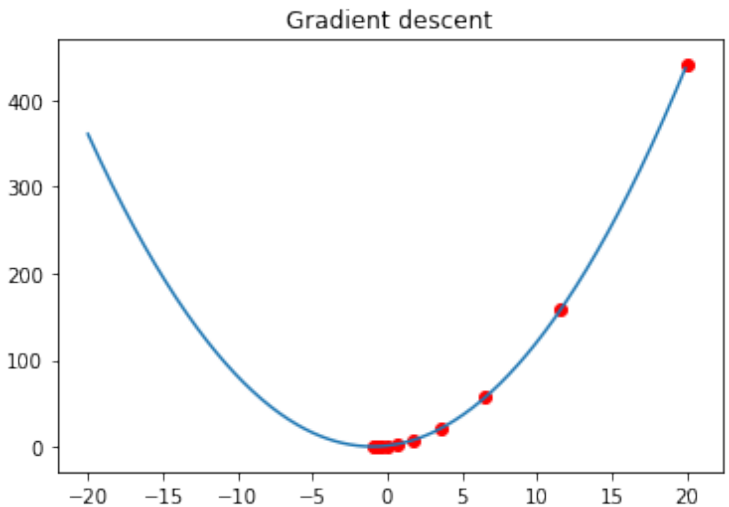
- Hill climbing algorithm that follows the gradient of the function to be optimized



see notebook Linear - Nonlinear separation

Gradient descent

- Hill climbing algorithm that follows the gradient of the function to be optimized



$\mathbf{w} \leftarrow$ any point in the parameter space

loop until convergence **do**

  for each w_i **in w do**

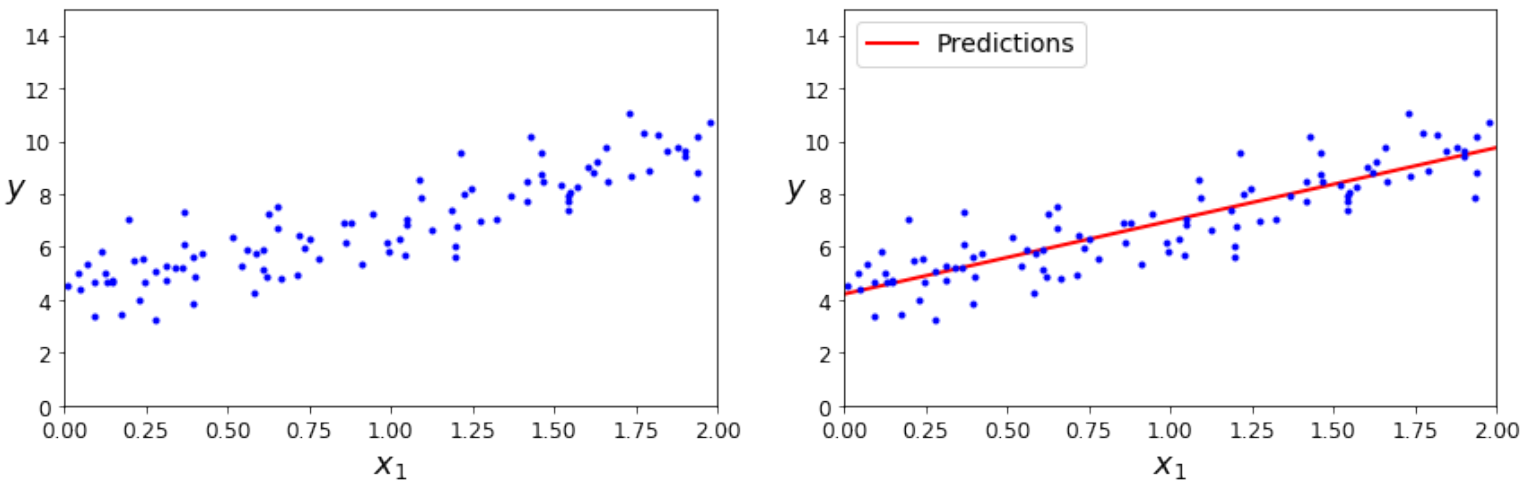
$$w_i \leftarrow w_i - \alpha \frac{\partial}{\partial w_i} Loss(\mathbf{w})$$

see notebook Linear - Nonlinear separation

Linear regression - with gradient descent

- Univariate linear function with input x and output y:
  $y = w_1 * x + w_0$
- Univariate Linear regression - finding the h_w that best fits n point x, y

$$h_w(x) = w_1 x + w0$$



How? with gradient descent in order to minimize the loss
see detailed analysis - it will also be discussed during IS lab

Math explanation

$$Loss(h_{\mathbf{w}}) = \sum_{j=1}^{N} L_2(y_j, h_{\mathbf{w}}(x_j)) = \sum_{j=1}^{N} (y_j - h_{\mathbf{w}}(x_j))^2 = \sum_{j=1}^{N} (y_j - (w_1 x_j + w_0))^2$$

For one example:

$$\begin{aligned}
\frac{\partial}{\partial w_i} Loss(\mathbf{w}) &= \frac{\partial}{\partial w_i} (y - h_{\mathbf{w}}(x))^2 \\
&= 2(y - h_{\mathbf{w}}(x)) \times \frac{\partial}{\partial w_i} (y - h_{\mathbf{w}}(x)) \\
&= 2(y - h_{\mathbf{w}}(x)) \times \frac{\partial}{\partial w_i} (y - (w_1 x + w_0))
\end{aligned}$$

$$\frac{\partial}{\partial w_0} Loss(\mathbf{w}) = -2(y - h_{\mathbf{w}}(x)); \qquad \frac{\partial}{\partial w_1} Loss(\mathbf{w}) = -2(y - h_{\mathbf{w}}(x)) \times x$$

Gradient descent rule:

$$w_0 \leftarrow w_0 + \alpha\,(y - h_{\mathbf{w}}(x)); \quad w_1 \leftarrow w_1 + \alpha\,(y - h_{\mathbf{w}}(x)) \times x$$

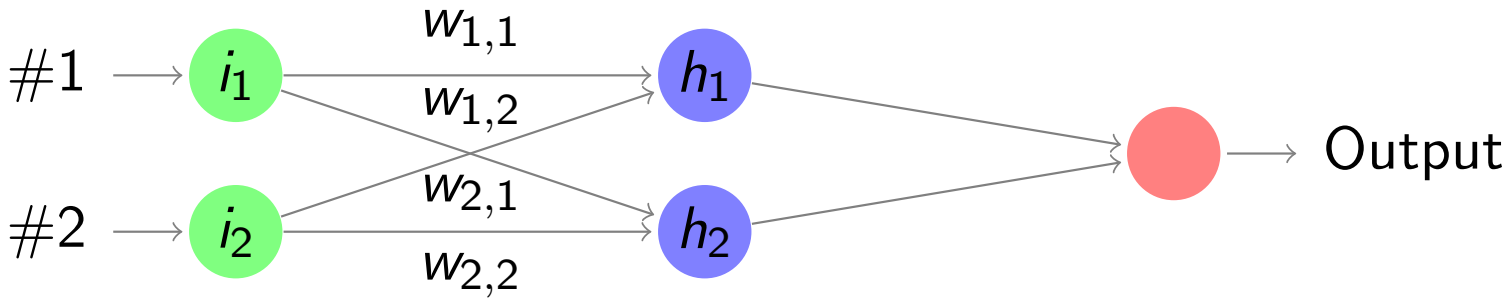
Batch gradient descent

$$w_0 \leftarrow w_0 + \alpha \sum_j (y_j - h_{\mathbf{w}}(x_j)); \quad w_1 \leftarrow w_1 + \alpha \sum_j (y_j - h_{\mathbf{w}}(x_j)) \times x_j$$

Backpropagation

see slide 1 - what is a neural network?

Neural networks



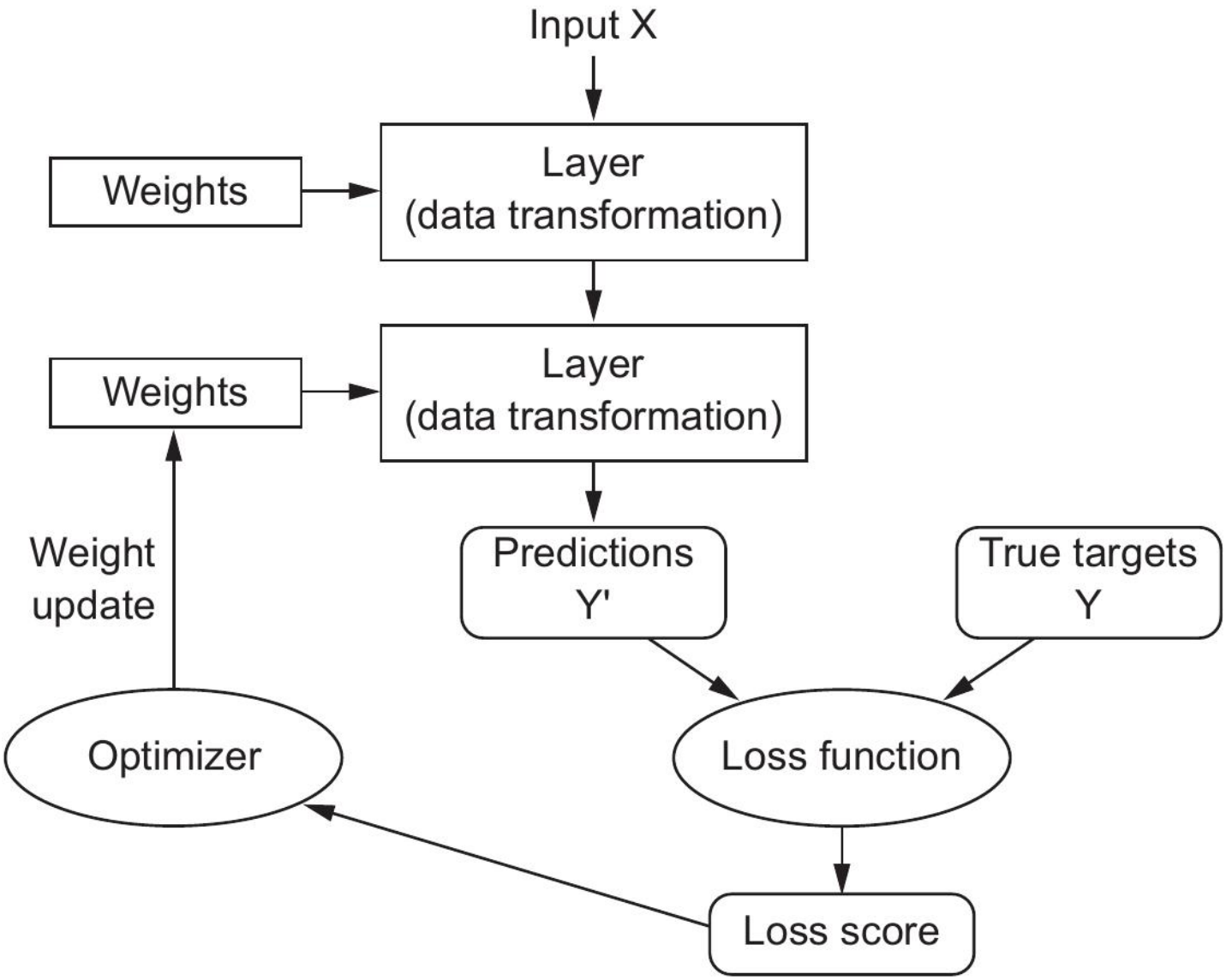
$$a_j = g(in_j)$$

$$a_j = g(\sum_i (w_{i,j} * a_i))$$

- a_i output of neuron i,
- $a_0 = 1$ bias
- $W_{i,j}$ weight between neuron i and neuron j
- g activation function

example: $a_{h_1} = g(w_{1,1} * a_i + w_{2,1} * a_2 + w_{0,1})$

Layered transformation



Loss

- Mean square error MSE

$$MSE = \frac{1}{N} \sum_{(x,y) \in D} (y - prediction(x))^2$$

- Log Loss

$$LogLoss = \sum_{(x,y) \in D} -ylog(y') - (1 - y)log(1 - y')$$

  - $(x, y) \in D$ - labeled dataset
  - y - label for sample x, x is a vector of features. For binary classification y is 0 or 1
  - y' - $prediction(x)$

Backpropagation - based on Gradient Descent

Back-propagation

- Output layer

$$w_{j,i} = w_{j,i} + \alpha * a_j * \Delta_i$$

, unde $\Delta_i = Error_i * g'(in_i)$

- Hidden layer (k node - j node - i nodes)
  Propagated error:

$$\Delta_j = g'(in_j) \sum_i w_{j,i} \Delta_i$$

  - hidden node j is responsible for some fraction of the error Δ_i in each of the output nodes to which it connects; stronger connection (higher weight) means higher contribution to error

$$w_{k,j} = w_{k,j} + \alpha * a_k * \Delta_j$$

Backpropagation

- compute the Δ values for the output units using the observed error (loss)
- starting with output layer, repeat fpr each layer in the network, until earliest hiddent layer is reached:
  - propagate the *Delta* value back to the previous layer
  - update the weights between two layers

Math demo (Chain rule (nice explanations AIMA, notes)

Neural Network Playground

https://playground.tensorflow.org/

What can go wrong

- Vanishing gradient
- Exploding gradient
- Overfitting/underfitting
- Adjust size of the network
- Use regularization: L1, L2
- Dropout
- Early stopping
- Adjust learning rate