Metoda secțiunii de aur

1) $[a_0, b_0] = [0,2]$ $\varepsilon = 0,3$

$f : [0,2] \to \mathbb{R}$, $f(x) = x^4 - 14x^3 + 60x^2 - 70x$

$\alpha = 0,618$

$\lambda_1 = a_0 + (1-\alpha)(b_0 - a_0) = 0 + 0,382 \cdot 2 = 0,7640$

$\mu_1 = a_0 + \alpha(b_0 - a_0) = 0 + 0,618 \cdot 2 = 1,236$

$|b_0 - a_0| > \varepsilon$

$f(\lambda_1) = f(0,764) = (0,764)^4 - 14 \cdot (0,764)^3 + 60 \cdot (0,764)^2 - 70 \cdot 0,764 = -24,36$

$f(\mu_1) = f(1,236) = (1,236)^4 - 14 \cdot (1,236)^3 + 60 \cdot (1,236)^2 - 70 \cdot 1,236 = -18,95$

$f(\mu_1) > f(\lambda_1)$

$\Rightarrow [a_1, b_1] = [a_0, \mu_1] = [0, 1,236]$

$\lambda_2 = a_1 + (1-\alpha)(b_1 - a_1) = 0 + 0,382 \cdot 1,236 = 0,472$

$\mu_2 = \lambda_1 = 0,764$

$|b_1 - a_1| > \varepsilon$

$f(\mu_2) = f(\lambda_1) = -24,36$

$f(\lambda_2) = f(0,472) = -21,09$ $f(\lambda_2) > f(\mu_2)$

$\Rightarrow [a_2, b_2] = [\lambda_2, b_1] = [0,472, 1,236]$

$\lambda_3 = \mu_2 = 0,764$

$\mu_3 = a_2 + \alpha(b_2 - a_2) = 0,472 \cdot 0,618 \cdot (1,236 - 0,472) = 0,944$

$|b_2 - a_2| > \varepsilon$

$f(\lambda_3) = f(\mu_2) = -24,36$

$f(\mu_3) = f(0,944) = -23,59$ $\Rightarrow f(\mu_3) > f(\lambda_3)$

$\Rightarrow [a_3, b_3] = [a_2, \mu_2] = [0,472, 0,944]$

$\lambda_4 = a_3 + (1-\alpha)(b_3 - a_3) = 0,6525$

$\mu_4 = \lambda_3 = 0,764$

$|b_3 - a_3| > \varepsilon$

$f(\lambda_4) = f(0,65\overline{25}) = -23,83$

$f(\mu_4) = f(0,764) = -24,35 \Rightarrow f(\lambda_4) > f(\mu_4)$

$\Rightarrow [a_4, b_4] = [\lambda_4, b_3] = [0,6525, 0,944]$

$\lambda_5 = \mu_4 = 0,764$

$\mu_5 = a_4 + \alpha(b_4 - a_4) = 0,8328$ } Algoritmul le calculează, dar nu mai sunt utilizate

Ne oprim $|b_4 - a_4| < \varepsilon$

$\Rightarrow x_{min} = \frac{a_4 + b_4}{2} = \frac{0,6525 + 0,944}{2} = 0,78$

Metoda lui Fibonacci

$f(x) = x^4 - 14x^3 + 60x^2 - 70x$

$\varepsilon = 0,3 \quad [a_1, b_1] = [0,2] \Rightarrow F_n > \frac{b-a}{\varepsilon} = 6,66$

$\Rightarrow F_n > 6,66$ Alegem $F_n = 8 \Rightarrow n = 5$ $F_0\ F_1\ F_2\ F_3\ F_4\ F_5$
$1, 1, 2, 3, 5, 8$

$\lambda_1 = a_1 + \frac{F_3}{F_5}(b_1 - a_1) = 0,75$

$\mu_1 = a_1 + \frac{F_4}{F_5}(b_1 - a_1) = 1,25$

$f(\mu_1) = f(\frac{5}{4}) = -18,6$

$f(\lambda_1) = f(\frac{3}{4}) = -24,3$ $\Rightarrow f(\lambda_1) < f(\mu_1) \Rightarrow [a_2, b_2] = [a_1, \mu_1] = [0, 1,25]$

$\lambda_2 = a_2 + \frac{F_2}{F_4}(b_2 - a_2) = \frac{2}{5} \cdot \frac{5}{4} = \frac{1}{2}$

$\mu_2 = \lambda_1 = 0,75$

$f(\mu_2) = f(0,75) = -24,3$

$f(\lambda_2) = f(0,5) = -21,68$

②

$\Rightarrow f(\lambda_2) > f(\mu_2) \Rightarrow [a_3, b_3] = [\lambda_2, \mu_2] = [0{,}5; 1{,}25]$

$\Rightarrow \lambda_3 = \mu_2 = 0{,}75$

$\mu_3 = a_3 + \frac{F_2}{F_3}(b_3 - a_3) = \frac{1}{2} + \frac{2}{3}\left(\frac{5}{4} - \frac{2}{4}\right) = \frac{1}{2} + \frac{2}{3} \cdot \frac{3}{4} = 1$

$f(\lambda_3) = f(0{,}75) = -24{,}3$

$f(\mu_3) = f(1) = 1^4 - 14 \cdot 1^3 + 60 \cdot 1^2 - 70 \cdot 1 = 61 - 84 = -23$

$\Rightarrow f(\mu_3) > f(\lambda_3) \Rightarrow [a_4, b_4] = [a_3, \mu_3] = [0{,}5; 1]$

$\Rightarrow \lambda_4 = a_4 + \frac{F_0}{F_2}(b_4 - a_4) = \frac{1}{2} + \frac{1}{2} \cdot \frac{1}{2} = \frac{3}{4}$

$\mu_4 = \lambda_3 = 0{,}75$

$f(\lambda_4) = f(\mu_4) \Rightarrow [a_5, b_5] = [a_4, \mu_4] = [0{,}5; 0{,}75]$

$\lambda_5 = \ldots$ } Nu mai avem nevoie deoarece $|b_5 - a_5| < 2\varepsilon$

$\mu_5 = \ldots$ }

$\Rightarrow x_{min} = \frac{a_5 + b_5}{2} = \frac{0{,}5 + 0{,}75}{2} = 0{,}625$