1) $f(x_1, x_2) = 4x_1^2 + 3x_2^2 - 5x_1x_2 - 8x_1 + 10$

$X_1 = [1, 1]$

$$\nabla f = \begin{bmatrix} 8x_1 - 5x_2 - 8 \\ 6x_2 - 5x_1 \end{bmatrix}$$

$$d_k = -\nabla f(X_1) = -\begin{bmatrix} \cancel{8} - 5 - \cancel{8} \\ 6 - 5 \end{bmatrix} = \begin{bmatrix} +5 \\ -1 \end{bmatrix}$$

$$\lambda_1 = \underset{\lambda}{\operatorname{argmin}}\, f\Big(\underbrace{\begin{bmatrix} 1 \\ 1 \end{bmatrix}}_{X_1} + \lambda \cdot \begin{bmatrix} 5 \\ -1 \end{bmatrix}\Big) = \underset{\lambda}{\operatorname{argmin}}\, f\Big(\begin{bmatrix} 1+5\lambda \\ 1-\lambda \end{bmatrix}\Big) =$$

$$= \underset{\lambda}{\operatorname{argmin}} \Big[4(1+5\lambda)^2 + 3(1-\lambda)^2 - 5\overbrace{(1+5\lambda)(1-\lambda)}^{1+4\lambda-5\lambda^2} - 8(1+5\lambda) + 10\Big] =$$

$$= \underset{\lambda}{\operatorname{argmin}} \Big[4 + \cancel{40\lambda} + 100\lambda^2 + 3 - 6\lambda + 3\lambda^2 - 5 - 20\lambda + 25\lambda^2 - 8 - \cancel{40\lambda} + 10\Big] =$$

$$= \underset{\lambda}{\operatorname{argmin}} \Big[128\lambda^2 - 26\lambda + 4\Big]$$

$$\Rightarrow 2 \cdot 128 \cdot \lambda - 26 = 0 \Rightarrow \lambda = \frac{13}{128} \simeq 0{,}1038 \simeq 0{,}1$$

$$\Rightarrow X_2 = X_1 - 0{,}1 \cdot \nabla f(X_1) = \begin{bmatrix} 1 \\ 1 \end{bmatrix} - \frac{1}{10}\begin{bmatrix} 5 \\ -1 \end{bmatrix} = \begin{bmatrix} 1/2 \\ 11/10 \end{bmatrix}$$

$$d_2 = -\nabla f(X_2) = -\begin{bmatrix} \overset{4}{\cancel{8}} \cdot \frac{1}{2} - \frac{\cancel{5} \cdot 11}{\cancel{10}_2} - 8 \\ \overset{3}{\cancel{6}} \cdot \frac{1}{2} - \frac{\cancel{5} \cdot 11}{\cancel{10}_2} \end{bmatrix} = -\begin{bmatrix} -4 - \frac{11}{2} \\ 3 - \frac{11}{2} \end{bmatrix} = \begin{bmatrix} +9{,}5 \\ +2{,}5 \end{bmatrix}$$

$$\Rightarrow \lambda_2 = \underset{\lambda}{\operatorname{argmin}}\, f\Big(\begin{bmatrix} 1/2 \\ 11/10 \end{bmatrix} + \lambda \begin{bmatrix} 9{,}5 \\ 2{,}5 \end{bmatrix}\Big)$$

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$$= \underset{\lambda}{\arg\min}\, f\left(\begin{bmatrix} 0,5+9,5\lambda \\ 5,5+2,5\lambda \end{bmatrix}\right) =$$

$$= \underset{\lambda}{\arg\min}\Big(4(0,5+9,5\lambda)^2 + 3(5,5+2,5\lambda)^2 - 5(1/2+9,5\lambda)(5,5+2,5\lambda)$$

$$-8(0,5+9,5\lambda)+10\Big) = \underset{\lambda}{\arg\min}\Big[1+38\lambda+19^2\lambda^2+3\cdot 5,5^2$$

$$+6\cdot 5,5\cdot 2,5\cdot\lambda + 3\cdot 2,5^2\lambda^2 - 5\left(\tfrac{11}{20}+\tfrac{5}{4}\lambda+\tfrac{19}{2}\cdot\tfrac{11}{2}\lambda+\tfrac{19}{2}\lambda\cdot\tfrac{5}{2}\lambda\right)$$

$$-4-76\lambda+10\Big)\Big] \Rightarrow \lambda = 0,4272$$

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$$\Rightarrow x_3 = x_2 - \lambda\cdot\nabla f(x_2) = \begin{bmatrix} 1/2 \\ 11/2 \end{bmatrix} - 0,42\begin{bmatrix} 9,5 \\ 2,5 \end{bmatrix} \simeq \begin{bmatrix} -3,5 \\ 4,375 \end{bmatrix}$$

2)

$$f(x_1, x_2) = 4x_1^2 + 3x_2^2 - 5x_1x_2 + 8x_1 + 10$$

$$X = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

$$\nabla f = \begin{bmatrix} \frac{\partial f}{\partial x_1} \\ \frac{\partial f}{\partial x_2} \end{bmatrix} = \begin{bmatrix} 8x_1 - 5x_2 - 8 \\ 6x_2 - 5x_1 \end{bmatrix} \quad \nabla f(X) = \begin{bmatrix} -5 \\ 1 \end{bmatrix}$$

$$\nabla^2 f = H = \begin{bmatrix} \frac{\partial^2 f}{\partial x_1^2} & \frac{\partial^2 f}{\partial x_1 \partial x_2} \\ \frac{\partial^2 f}{\partial x_2 \partial x_1} & \frac{\partial^2 f}{\partial x_2^2} \end{bmatrix} = \begin{bmatrix} 8 & -5 \\ -5 & 6 \end{bmatrix}$$

$$H^{-1} = \frac{1}{\det H} \begin{bmatrix} 6 & 5 \\ 5 & 8 \end{bmatrix} = \frac{1}{23} \begin{bmatrix} 6 & 5 \\ 5 & 8 \end{bmatrix}$$

$$X_1 = X - H^{-1} \cdot \nabla f(X) = \begin{bmatrix} 1 \\ 1 \end{bmatrix} - \frac{1}{23} \begin{bmatrix} 6 & 5 \\ 5 & 8 \end{bmatrix} \begin{bmatrix} -5 \\ 1 \end{bmatrix} =$$

$$= \begin{bmatrix} 1 + \frac{25}{23} \\ 1 + \frac{17}{23} \end{bmatrix} = \begin{bmatrix} \frac{48}{23} \\ \frac{40}{23} \end{bmatrix}$$

$$\nabla f(X_1) = \begin{bmatrix} 8 \cdot \frac{48}{23} - \frac{40}{23} \cdot 5 - \overset{23}{8} \\ \frac{40}{23} \cdot 6 - 5\frac{48}{23} \end{bmatrix} = \begin{bmatrix} \frac{\cancel{8 \cdot 48} - \cancel{200} - \cancel{8 \cdot 23}}{23} \\ \frac{\cancel{8 \cdot 5 \cdot 6} - \cancel{5 \cdot 8 \cdot 6}}{23} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$\Rightarrow X_1$ este minimul căutat