Calculation of lepton fluxes in the Earth's atmosphere

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Abstract

When the primary cosmic rays enter Earth's atmosphere and interact whit the air nuclei, they produce several amounts of the secondary particles. Some of these particles are not absorbed in the atmosphere, such as muons and neutrinos. This way, from the spectra of each secondary particle, we can obtain information about the primary cosmic rays and their physical interactions through the atmosphere. In the present work, we solve the coupled Cascade equations numerically with MCEq code. The presented approach is very flexible and allows the use of different hadronic interaction models, realistic parametrizations of the primary cosmic-ray flux and the Earth's atmosphere, and a detailed treatment of particle interactions and decays. Finally, we study in detail the seasonal variation of atmospheric muons at very high energy.

Coupled cascade equations

The equations describe the evolution of the differential flux, defined as the differential of the particle flux ϕ w.r.t. energy per unit area, unit solid angle, and time

$$\Phi = \frac{d\phi}{dE} = \frac{dN}{dE\,dA\,d\Omega\,dt} \tag{1}$$

The coupled cascade equations

$$\frac{d\phi_h(E,X)}{dX} = -\frac{\phi_h(E,X)}{\lambda_{int,h}(E)} - \frac{\phi_h(E,X)}{\lambda_{dec,h}(E,X)}$$
$$-\frac{\partial}{\partial E}(\mu(E)\phi_h(E,X)) + \sum_l \int_E^\infty dE_l \frac{dN_{l(E_l)\to(E_h)}}{dE}\frac{\phi_l(E_l,X)}{\lambda_{int,l}(E_l)}$$
$$+\sum_l \int_E^\infty dE_l \frac{dN^{dec}_{l(E_l)\to(E_h)}}{dE}\frac{\phi_l(E_l,X)}{\lambda_{dec,l}(E_l,X)} \tag{2}$$

The energy dependence of the interaction lengths and decay lengths

$$\lambda_{int,h}(E) = \frac{m_{air}}{\sigma^{inel}_{h-air}(E)} \qquad \lambda_{dec,h}(E,X) = \frac{c\tau_h E \rho_{air}(X)}{m_h} \tag{3}$$

Calculation Method

The coupled cascade equations can be solved with Monte Carlo simulations. However, this method becomes inefficient at energies above several hundreds GeV. Therefore, the MCEq software was used in this work to solve the cascade equations, which is fast enough calculation method for systematic studies of atmospheric lepton uncertainties.

Matrix cascade equations

The open-source software Matrix Cascade Equations (MCEq)[2] solves the system of discrete coupled cascade equations:

$$\frac{d\Phi^h_{E_i}}{dX} = -\frac{\Phi^h_{E_i}}{\lambda_{int,E_i}} - \frac{\Phi^h_{E_i}}{\lambda_{dec,E_i}(X)}$$
$$+\sum_{E_k\geq E_i}\sum_l \frac{c_{l(E_k)\to h(E_i)}}{\lambda^l_{int,E_k}}\Phi^l_{E_k} + \sum_{E_k\geq E_i}\sum_l \frac{d_{l(E_k)\to h(E_i)}}{\lambda^l_{dec,E_k}(X)}\Phi^l_{E_k} \tag{4}$$

On the other hand, an efficient numerical computing scheme can be found by rewriting the cascade equations into matrix form

$$\frac{\mathbf{d\phi}}{\mathbf{dX}} = -\nabla_{\mathbf{E}}(\mathrm{diag}(\mu)\phi) + (\mathbf{-1}+\mathbf{C})\mathbf{\Lambda_{int}}\phi + \frac{\mathbf{1}}{\rho(\mathbf{X})}(\mathbf{-1}+\mathbf{D})\mathbf{\Lambda_{dec}}\phi \tag{5}$$

The matrices $\mathbf{C}$ and $\mathbf{D}$ contain the coefficients c and d arranged in a way so as to represent the coupling sums (source terms) from equation (4) above. ∇_μ is the first derivative operator,μ the mean energy loss or the stopping power of muons in dry air.

References

[1] A. Fedynitch. The hadronic interaction model sibyll 2.3c and inclusive lepton fluxes, 2018. URL arXiv:1806.04140v1.

[2] A. Fedynitch and R. Engel. Mceq - efficient computationof particle particle cascades in the atmosphere. 2018. URL https://github.com/afedynitch/MCEq.

[3] D. S. P. D. Serap Tilav, Thomas K. Gaisser. Seasonal variation of atmospheric muons in icecube. 2019. URL arXiv:1909.02036.

Results

Muon fluxes at very high energy

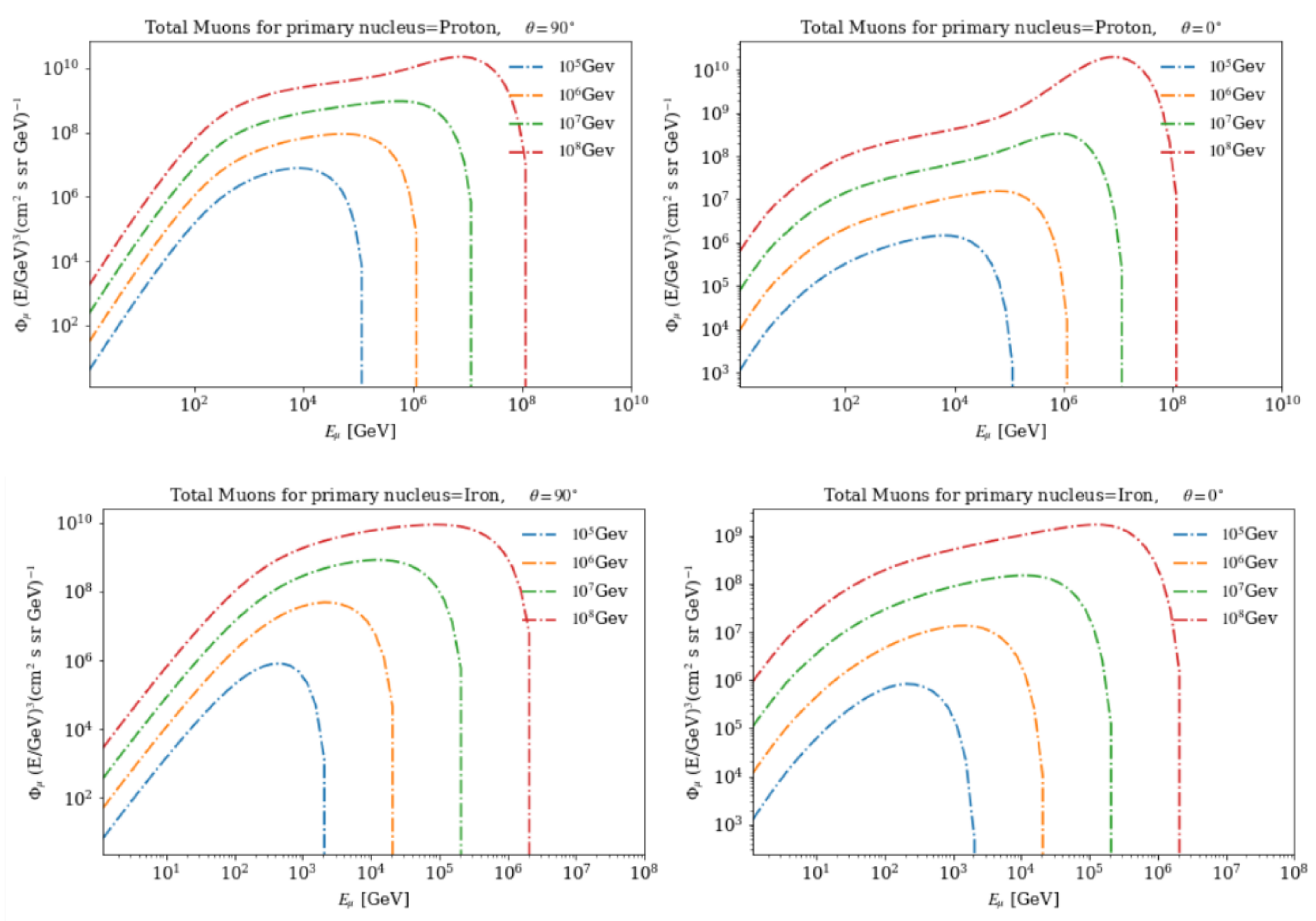

Figure 1: Muon flux for two different angles. We used the Proton and Iron as a primary nuclei. The primary model is H3a and the interaction model SIBYLL-2.3 RC1.

Geometry and Atmosphere

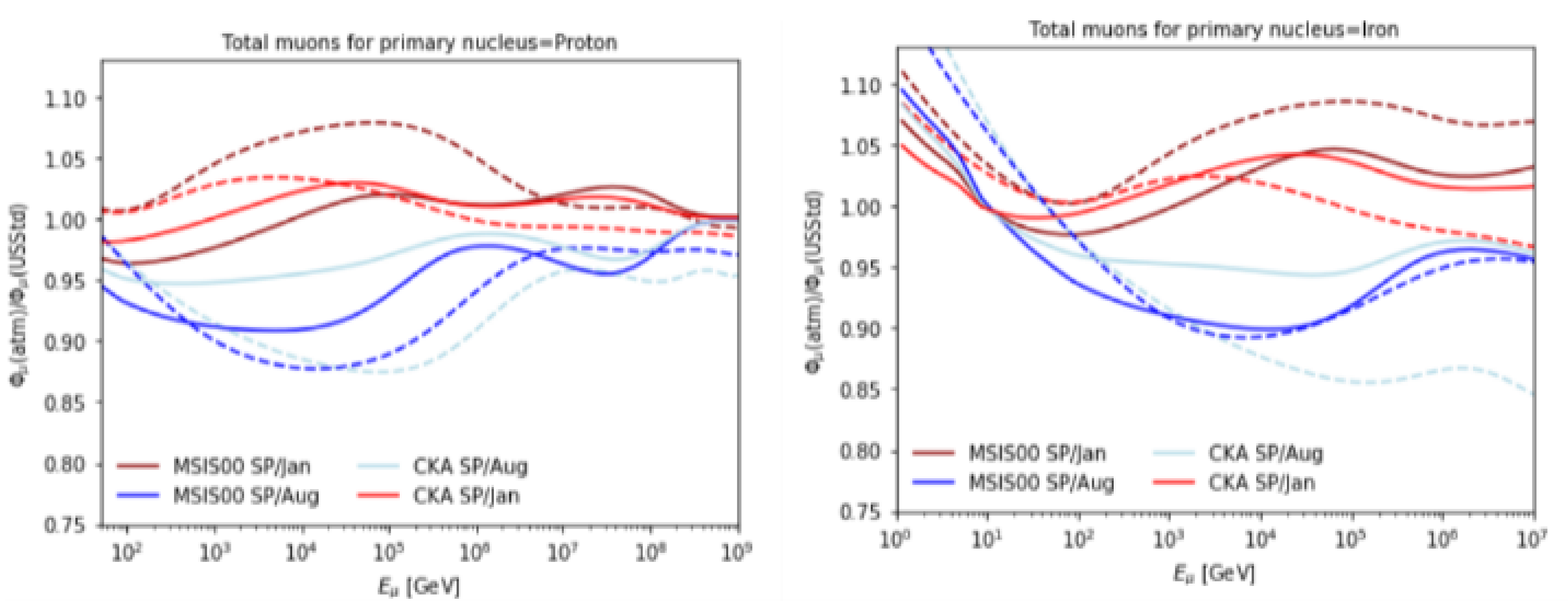

Figure 2: Ratio of the flux calculated with different atmospheric models to the flux with US Standard atmosphere (USStd) for two different primary nucleus. The primary model is H3a and the interaction model SIBYLL-2.3 RC1. A vertical trajectory (θ=0°) is represented by solid and a horizontal ($= 90°$) by dashed lines.

Seasonal variation

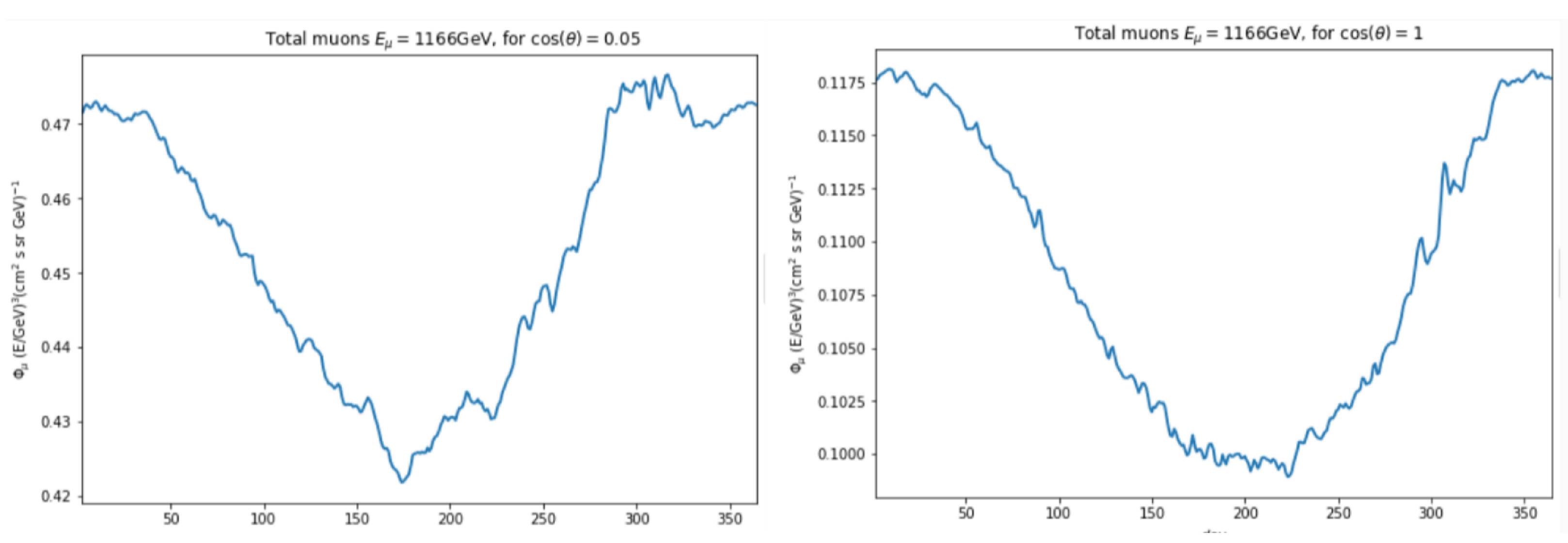

Figure 3: Seasonal variation of atmospheric muons for two different incident angles: $\cos(\theta) = 0.05$ (left) and $\cos(\theta) = 1.00$ (right).

Conclusions

- Muon flux depends strongly on the primary particle that generates the air showers.
- MCEq allows to take into account a detailed configuration of the antartic atmosphere through the AIRS satelite.
- Muon flux is found to be much less during antartic winter than during its summer.