Matrix Cascade Equation (MCEq)

The cascade equations for particle h can be written for one discrete energy bin E_i:

$$\frac{d\Phi^h_{E_i}}{dX} = -\frac{\Phi^h_{E_i}}{\lambda_{int,E_i}} - \frac{\Phi^h_{E_i}}{\lambda_{dec,E_i}(X)} + \sum_{E_k \geq E_i} \sum_{l} \frac{c_{l(E_k)\to h(E_i)}}{\lambda^l_{int,E_k}} \Phi^l_{E_k} +$$

$$+ \sum_{E_k \geq E_i} \sum_{l} \frac{d_{l(E_k)\to h(E_i)}}{\lambda^l_{dec,E_k}} \Phi^l_{E_k}$$

Where

$$\lambda^h_{int,E_i} = m_{air}/\sigma^{inel}_{p-air}(E_i) \qquad \lambda^h_{dec,E_i}(X) = c\tau_h E_i \rho_{air}(X)/m_h$$

- The coupled cascade equations are solved numerically by formulating them as a matrix equation.

$$\frac{\mathrm{d}}{\mathrm{d}X}\boldsymbol{\Phi} = \left[(-\mathbf{1} + \boldsymbol{C})\boldsymbol{\Lambda}_{int} + \frac{1}{\rho(X)}(-\mathbf{1} + \boldsymbol{D})\boldsymbol{\Lambda}_{dec}\right]\boldsymbol{\Phi}.$$

- We have been rewritten these equations into a matrix form to make use of linear algebra algorithms.
- The power of the MCEq is illustrated by calculating lepton flux predictions for a number of different scenarios. In this case we are interested only in the muon spectra.

Atmospheric muon flux

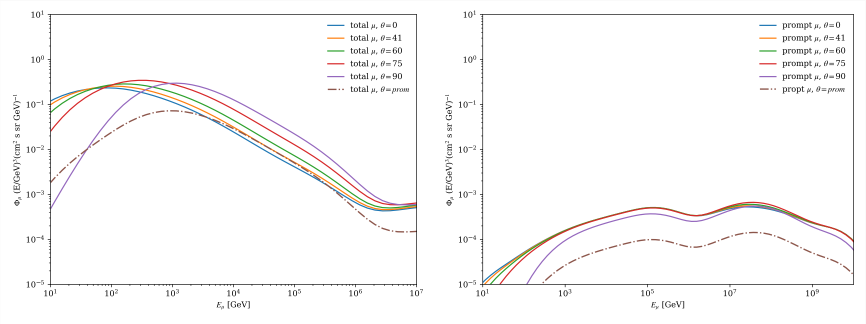

Figure 1: Muon flux for different angles with the GST4 primary flux model and ('CORSIKA', ('PLSouthPole', 'August')) density model.

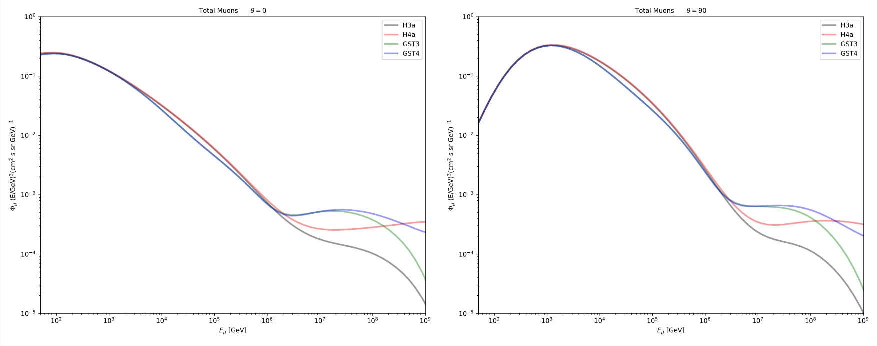

Figure 2: Primary model dependence of the atmospheric prompt and total muon flux.

Number of muons for each primary flux model

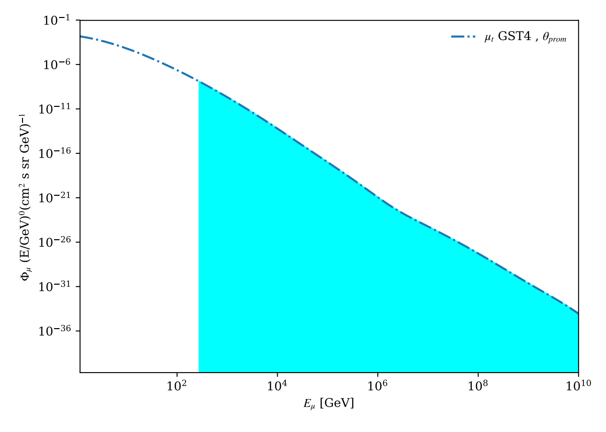

Figure 3: Number of particles with $E_0 > 300$GeV. The primary flux model is GST4, the density model is ('CORSIKA', ('PLSouthPole', 'August') and the angle is the average of 10 values between 0° and 90°.

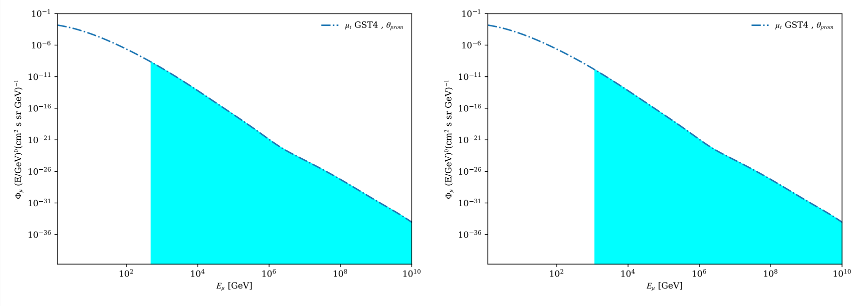

Figure 4: Number of particles with $E_0 > 500\text{GeV}$ (left) and $E_0 > 1\text{TeV}$ (right). The primary flux model is GST4, the density model is ('CORSIKA', ('PLSouthPole', 'August') and the angle is the average of 10 values between 0° and 90°.

Results

'The number of the total muons for E>300GeV is:'

'H3a:'1.5849962693061645e-06

'H4a:'1.6234849002848547e-06,

'GST3:'1.5983203389744605e-06,

'GST4:'1.5965450625765415e-06,

'Ratios for $E > 300$'

'H4a/H3a:'1.0242831050924421

'GST3/H3a:' 1.0084063728895265

'GST4/H4a:' 1.007286322052627

'The number of the total muons for E>500GeV is:'

'H3a:', 4.5091817502512534e-07,

'H4a:', 4.619374592350055e-07,

'GST3:', 4.539689526057299e-07,

'GST4:', 4.5337554835804654e-07

'Ratios for $E > 500$ GeV'
'H4a/H3a:' 1.024437436369173
'GST3/H3a:' 1.006765701073004
'GST4/H4a:' 1.005449710100473

'The number of the total muons for E>1TeV is:'
'H3a:'6.282432774403802e-08,
'H4a:'6.437360496806326e-08,
'GST3:'6.1889268957261e-08,
'GST4:'6.178423275559578e-08

'Ratios for $E > 1$TeV:'
'H4a/H3a:' 1.024660466409404
'GST3/H3a:' 0.9851162945891488
'GST4/H4a:' 0.9834443912765793

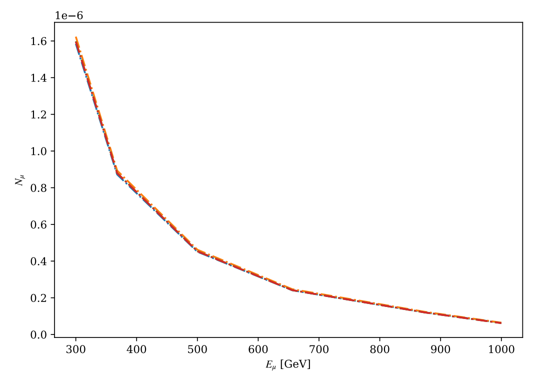

Figure 5: Number of particles in function of the energies thresholds.

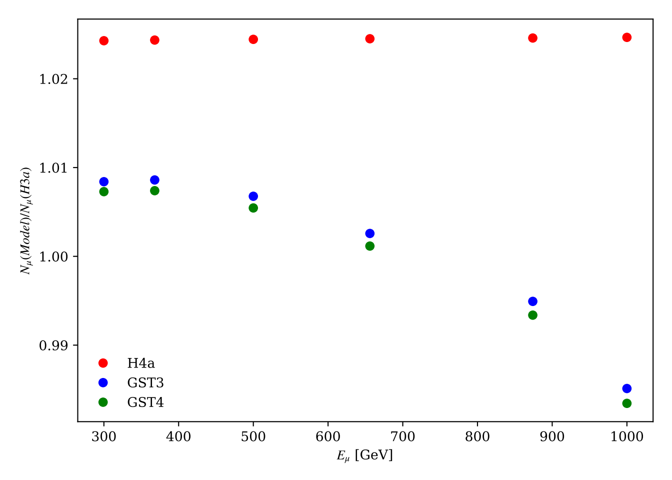

Figure 6: Ratio of number of muons for different primary flux models. The angle is de average of 10 values between 0°- 90°, the density model used is ('CORSIKA', ('PLSouthPole', 'August').

The ratio of the flux Φ