Homework Set 4

Application of Differentiation

1. Compute the following limit by using L'Hospital's rule.

4.1 $\lim_{x\to 0}\frac{8^x-2^x}{4x}$ 4.2 $\lim_{x\to\infty}\frac{(\ln x)^2}{x}$ 4.3 $\lim_{x\to\infty}\left(e^x+3x\right)^{\frac{2}{x}}$

4.4 $\lim_{x\to 1}\left(\frac{2x}{x^2-1}-\frac{1}{x-1}\right)$ 4.5 $\lim_{x\to\frac{1}{2}}(2x-1)\tan\pi x$ 4.6 $\lim_{x\to 0}\frac{\tan 3x}{2x^2+5x}$

4.1) $\lim_{x\to 0}\frac{8^x-2^x}{4x}$ IF $=\frac{8^0-2^0}{4(0)}=\frac{0}{0}$

$=\lim_{x\to 0}\frac{\frac{d}{dx}(8^x-2^x)}{\frac{d}{dx}4x}$

$=\lim_{x\to 0}\frac{\frac{d}{dx}2^{3x}-2^x}{4}$

$=\lim_{x\to 0}\frac{\frac{3}{\ln 2}2^{3x}-\frac{1}{\ln 2}2}{4}$

$=\lim_{x\to 0}\frac{3\cdot 2^{3x}-2^x}{4\ln 2}$

$=\frac{3\cdot 2^0-2^0}{4\ln 2}$

$=\frac{3-1}{4\ln 2}$

$=\frac{1}{2\ln 2}$

4.2) $\lim_{x\to\infty}\frac{(\ln x)^2}{x}$ IF $=\frac{\infty}{\infty}$

$=\lim_{x\to\infty}\frac{\frac{d}{dx}(\ln x)^2}{\frac{d}{dx}(x)}$

$=\lim_{x\to\infty}\frac{\frac{2}{x}\ln x}{1}$

$=\frac{2}{\infty}\cdot\infty$

$=2\lim_{x\to\infty}\frac{\frac{d}{dx}(\ln x)}{\frac{d}{dx}(x)}$

$=2\cdot\lim_{x\to\infty}\frac{\frac{1}{x}}{1}$

$=2\cdot\frac{1}{\infty}$

$=0$

4.3) $\lim_{x\to\infty} (e^x+3x)^{\frac{2}{x}}$ IF $= \infty^0$

$= \lim_{x\to\infty} e^{\ln(e^x+3x)^{\frac{2}{x}}}$

$= \lim_{x\to\infty} e^{\frac{2\ln(e^x+3x)}{x}}$

$= e^2 \cdot \lim_{x\to\infty} \left(\frac{\ln(e^x+3x)}{x}\right)$

$= e^2 \cdot \lim_{x\to\infty} \left(\frac{e^x+3}{e^x+3x}\right)$

$= e^2 \cdot \lim_{x\to\infty} \left(\frac{e^x}{e^x+3}\right)$

$= e^2 \cdot \lim_{x\to\infty} \left(\frac{e^x}{e^x}\right)$

$= e^{2\cdot 1}$

$= e^2$

4.4) $\lim_{x\to 1} \left(\frac{2x}{x^2-1} - \frac{1}{x-1}\right)$ IF $= \infty - \infty$

$= \lim_{x\to 1} \left(\frac{2x}{x^2-1} - \frac{x+1}{x^2-1}\right)$

$= \lim_{x\to 1} \left(\frac{x-1}{x^2-1}\right)$

$= \lim_{x\to 1} \left(\frac{1}{2x}\right)$

$= \frac{1}{2.1}$

$= \frac{1}{2}$

4.5) $\lim\limits_{x \to \frac{1}{2}} (2x-1) \tan \pi x \quad IF: 0 \cdot \infty$

$$= \lim_{x \to \frac{1}{2}} (2x-1) \frac{\sin \pi x}{\cos \pi x} = \frac{0}{0}$$

$$= \lim_{x \to \frac{1}{2}} \frac{\pi (2x-1) \cos \pi x + 2 \sin \pi x}{-\pi \sin \pi x}$$

$$= \frac{\pi (2 \cdot \frac{1}{2} - 1) \cos \frac{\pi}{2} + 2 \sin \frac{\pi}{2}}{-\pi \sin \frac{\pi}{2}}$$

$$= \frac{0+2}{-\pi}$$

$$= \frac{-2}{\pi}$$

4.6) $\lim\limits_{x \to 0} \frac{\tan 3x}{2x^2 + 5x} \quad IF: \frac{0}{0}$

$$= \lim_{x \to 0} \frac{3 \sec^2 3x}{4x + 5}$$

$$= \frac{3 \sec^2 3 \cdot 0}{4 \cdot 0 + 5}$$

$$= \frac{3 \cdot 1}{0 + 5}$$

$$= \frac{3}{5}$$

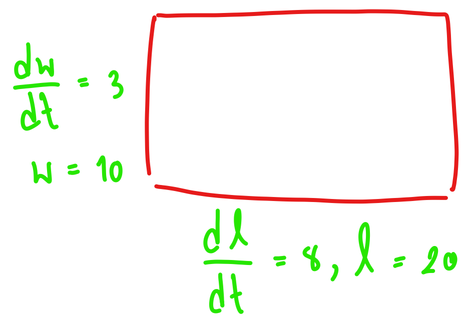
2. Car A is traveling west at and car B is traveling north at 50 mi/h. and car B is traveling north at 60 mi/h. Both are headed for the intersection of the two roads. At what rate are the cars approaching each other when car A is 0.3 mi and car B is 0.4 mi from the intersection?

3. The length of a rectangle is increasing at a rate of 8 cm/s and its width is increasing at a rate of 3 cm/s. When the length is 20 cm and the width is 10 cm, how fast is the area of the rectangle increasing?

Area = width × length

$$A = wl$$

$$\frac{dA}{dt} = w\frac{dl}{dt} + l\frac{dw}{dt}$$



$$\frac{dA}{dt} = 10(8) + 20(3)$$

$$= 80 + 60$$

$$\frac{dA}{dt} = 140 \text{ cm}^2/\text{s}$$

4. Find the critical points of the following

4.1 $y = x^3 - 2x^2$ 4.2 $y = \frac{x-1}{x^2}$

4.1) $y' = 3x^2 - 4x$

Find critical points $3x^2 - 4x = 0$

$x\ 3x - 4 = 0$

Critical points $x = 0, \frac{4}{3}$

4.2) $y' = \frac{x^2(1) - (x-1)(2x)}{(x^2)^2}$

$y' = \frac{-x^2 + 2x}{x^4}$

$= \frac{-x+2}{x^3}$

Find critical points $\frac{-x+2}{x^3} = 0$

$-x + 2 = 0$

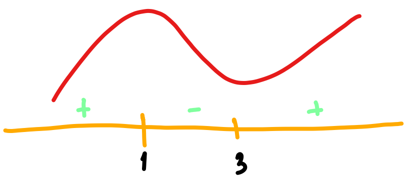
critical $x = 2$ and $x \neq 0$

5. Locate the intervals of x where each function is increasing and where it is decreasing and find the local minimum and maximum values of $f(x) = x^3 - 6x^2 + 9x - 5$.

$f(x) = 3x^2 - 12x + 9 = 0$

$= x^3 - 4x + 3 = 0$

$= (x-3)(x-1) = 0$



increasing $(-\infty, 1] \cup [3, \infty)$

decreasing $[1, 3]$

$f(x) = x^3 - 6x^2 - 9x - 5$

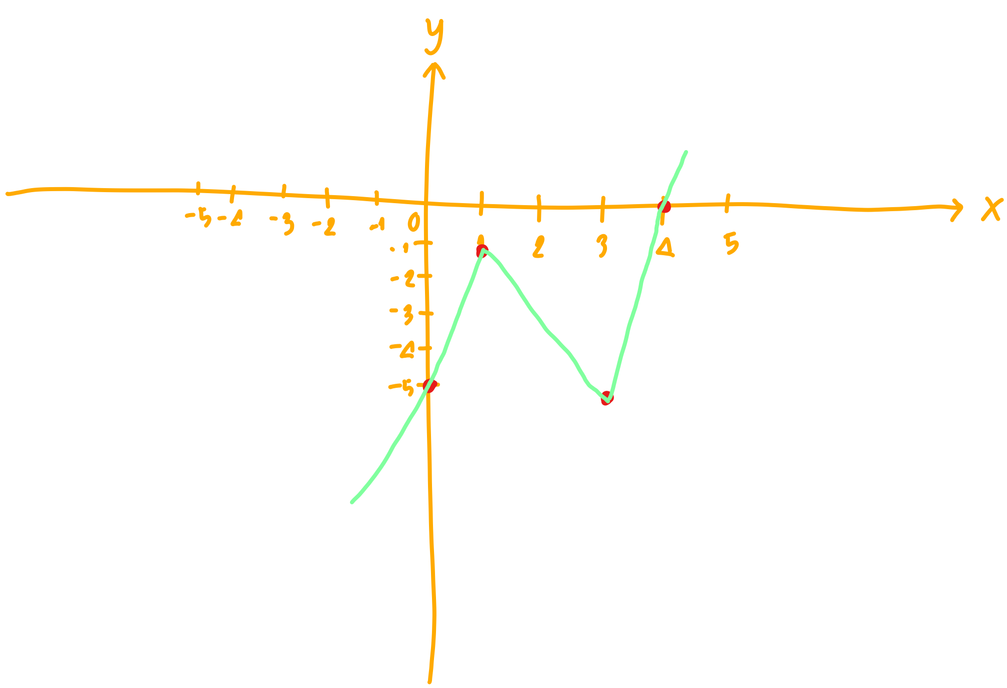
$f(1) = 1^3 - 6(1)^2 + 9(1) - 5$

$= 1 - 6 + 9 - 5$

$= -1$ local maximun

$f(x) = 3^3 - 6(3)^2 + 9(3) - 5$

$= -5$ local minimun



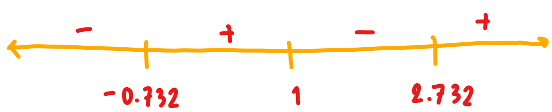
6. Analyze and sketch a graph of $y = x^4 - 4x^3 + 8x - 2$.

$y' = 4x^3 - 12x^2 + 8$

critical points $4x^3 - 12x^2 + 8 = 0$

$x^3 - 3x^2 + 2 = 0$

using calculator x $-0.732, 2.732, 1$



find max/min

$f(-0.732) \approx -6$

$f(1) \approx 3$

$f(2.732) \approx 6$