
Case Study Cyber Physical Production System Using AM

INTERNAL GEAR PUMP

Optimal Gear Solutions

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Abstract

Gear pump utilizes the meshing of gears to pump fluids through displacement. There are two main classifications: external gear pumps with two external spur gears, and internal gear pumps with an external and an internal spur gear (where the teeth face inwards). This paper emphasizes primarily on internal gear pumps and their characteristics. An internal gear pump transfers fluid by continuously encompassing a constant volume between interconnecting gears, mechanically moving it to produce a steady pulse-free movement equivalent to the rotational velocity of its gears. It has just two moving parts and comprises of an Internal Gear, Pinion (driving gear) and the Crescent Filler. Based on their geometric theory focusing on flowrate and meshing characteristics, a systematic study has been presented in this report. Information gathered from this research will be used for parametric modeling of the internal gear pumps in IceSL with the help of the Lua script. Moreover, an educational video will be compiled to elaborate on their functional characteristics.

Zahnradpumpen nutzen das Prinzip der Verdrängung um Flüssigkeiten zu fördern. Es gibt zwei Hauptklassifizierungen: Außenzahnradpumpen mit zwei Außenstirnrädern und Innenzahnradpumpen mit einem Außen- und einem Innenstirnrad (bei denen die Zähne nach innen zeigen). In diesem Beitrag geht es hauptsächlich um Innenzahnradpumpen und ihre Eigenschaften. Eine Innenzahnradpumpe fördert Flüssigkeit, indem sie kontinuierlich ein konstantes Volumen zwischen miteinander verbundenen Zahnrädern umschließt und mechanisch so bewegt, dass eine gleichmäßige, impulsfreie Bewegung entsteht, die der Rotationsgeschwindigkeit der Zahnräder entspricht. Es hat nur zwei bewegliche Teile und besteht aus einem Innenzahnrad, einem Ritzel (Antriebszahnrad) und dem Krümmerfüller. Basierend auf ihrer geometrischen Theorie, die sich auf den Durchfluss und die Eingriffseigenschaften konzentriert, wurde in diesem Bericht eine systematische Studie vorgestellt. Die aus dieser Untersuchung gewonnenen Informationen werden für die parametrische Modellierung der Innenzahnradpumpen in IceSL mit Hilfe des Lua-Skripts verwendet. Darüber hinaus wird ein Lehrvideo erstellt, in dem die Funktionseigenschaften der Pumpen erläutert werden.

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1 Introduction

Gear pump is a positive displacement (PD) pump and utilizes the meshing of gears to pump fluids through displacement. There are two main types of gear pumps: external and internal. An external gear pump consists of two external spur gears, whereas an internal gear pump has an external and an internal spur gear whose teeth face inwards towards the centre of the gear. When compared to external gear pumps, internal gear pumps have lesser flowrate pulsation allowing them to function quietly. Hence, they are widely used in applications where environmental noise is a huge concern.

An internal gear pump produces a steady pulse-free movement equivalent to the rotational velocity of its gears. They are bi-directional consisting of only two moving parts. As shown in the figure 1.1, an internal gear pump has three significant components, namely, the Internal Gear, Pinion (driving gear) and a Crescent Filler separating them. This crescent-shaped portion attached to the pump body divides the shape, splits the liquid, and serves as an adhesive between the suction and discharge ports. The operating cycle of an internal gear pump can be primarily divided into three phases: loading, transition, and delivery.

In the course of pump action, the driving and driven gears are split, causing the suction capacity defined by them to expand, resulting in vacuum. Fluid can be ingested from the suction port due to the vacuum. In the meantime, at the outlet side, the two gears mesh with one another, reducing the output volume so that the fluid can be squeezed out of the discharge vent. The crescent here divides the suction and discharge volumes and defines the distribution zone. This mechanism is repeated in a timely manner as the gears rotate. In these types of configurations, there is a line of contact at various locations between the gear couples and the fixed crescent, so pressure cannot readily migrate to the neighbouring cells. This has the benefit of generating more pressure than a standard internal gear pump with no crescent.

Gearing theory is often used by investigators to address issues in designing and manufacturing of internal gear pumps. While internal gear pairs have been long researched, further attempts are made to gain a better comprehension of their flowrate performances and meshing characteristics. The work of Song, Zhou and Zhao [SZZ13] demonstrated the models of the internal gear pairs with conjugated straight-line profile of the pinion to identify the issues of overcutting, interferences

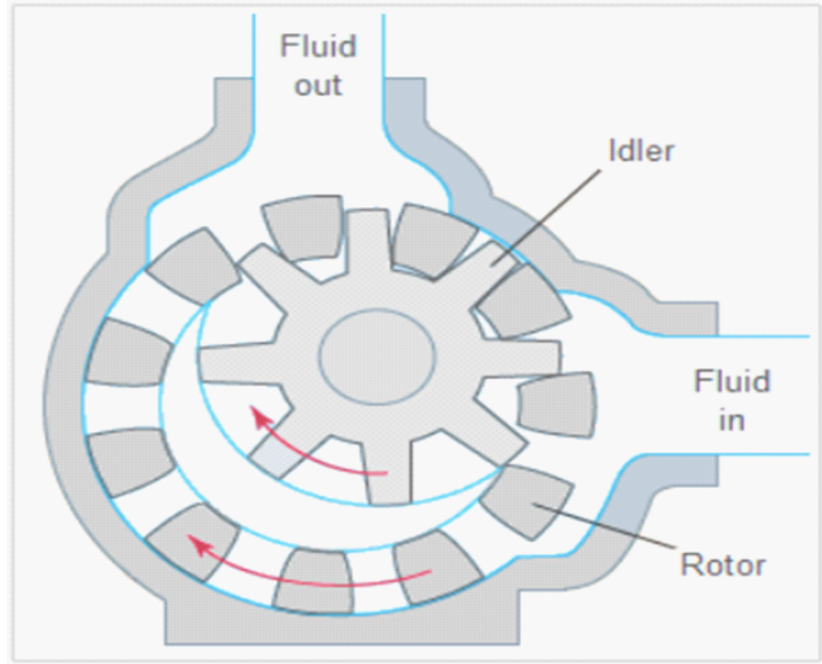


Figure 1.1: Internal gear pumps [Jen12]

and the performances of fluid power gear machine. Zhou and Song [ZS13] have presented the conceptual flowrate characteristics of conjugated pumps which resulted in a better performance than the conventional pumps. To design it, a proper shifting coefficient, a larger tooth number of the pinion, a smaller tooth number of the internal gear, a larger pressure angle and a larger fillet radius, under the condition that no tooth interference occurs. For conjugate gear pairs, Yang [Yan07] investigated on their meshing and tooth contact analysis. Additionally, he has proposed that the driven gear of a set of internal gear pairs can be estimated based on the double envelope notion.

Based on the theory of gearing, Colbourne [Col12] [Col75] researched on the shape of internal gear pumps with specific reference to the curvature of the gear teeth and the outcome of the tooth shapes on the flow rate. Moreover, approaches were recommended for decreasing the differences in flow rate, which can create pressure pulses, and for enhancing the minimum tooth radii of curvature. With respect to the involute internal gear pumps, Ichikawa [Ich59] estimated the ideal flowrate characteristics from a kinematic view, emphasizing on fluctuations of trapped volume. Results demonstrated that internal gear pumps have less pulsation factor and variation of the trapped volume compared to the external gear pumps.

Since several researchers [CXHL18], [LF04], [ZDXY17] have shown that gear pairs

directly influence the efficiency of internal gear pumps, so sufficient knowledge must be established of the properties of gear pairs in order to design highly efficient internal gear pumps. The internal gear pairs can enhance the efficiency of gear pumps in terms of noise level and service life, and can form the basis for the design process of fluid power gear machines, including gear pumps and gear motors.

This paper emphasizes primarily on the geometric characteristics of internal gear pumps. With a mathematical approach, the geometric models of the gear pairs based on their meshing and flowrate characteristics are investigated. It is expected that this study should be helpful in the designing and parametrical modelling of the internal gear pairs in IceSL with the help of Lua script. Moreover, to compile an educational video elaborating the functional characteristics of the internal gear pumps.

2 Geometrical and Mathematical Study

Functional description of the internal gear pump is done considering its significant components of internal gear pair which is used along with a separator to create difference in pressure at various contact and operating points to pump the fluid in the required directions.

Depict in the figure 2.1 is the circumference of the gear divided by the pressure regions made by the meshing gears and crescent fillers. This chapter summarizes the technical aspects that are needed for a reader to understand about the mathematical characteristics of the parts along with the required technical representation and conceptual discussion involved in the effective operation of Internal Gear Pump [SCZ16].

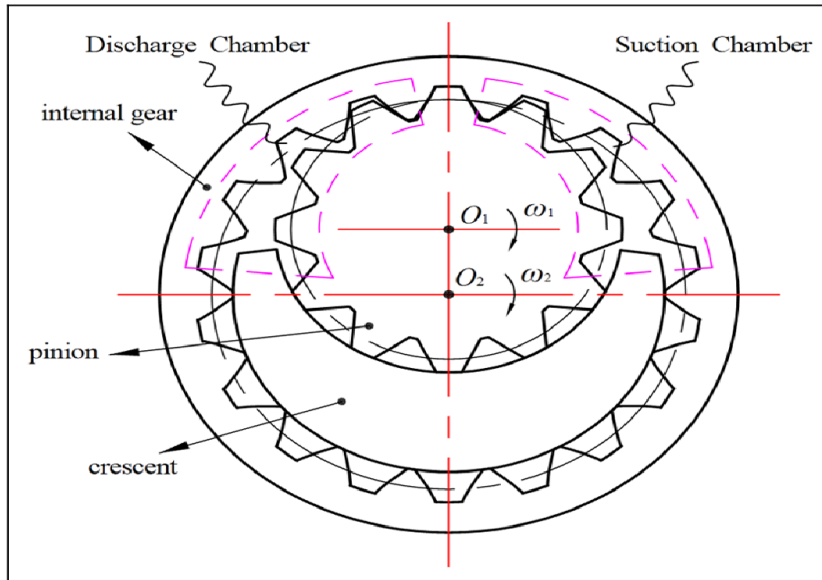


Figure 2.1: Internal gear pumps cross section [SCZ16]

2.1 Internal Gear

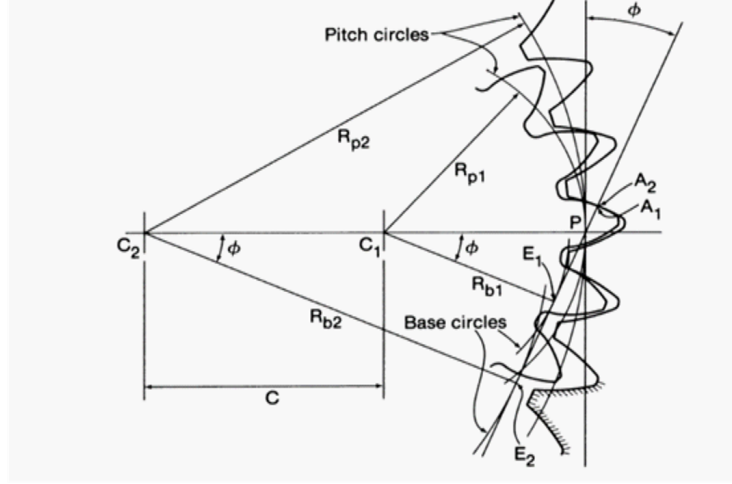


Figure 2.2: Internal Gear Pair Meshing Nomenclature [Col12]

The Internal Gear is a very essential part of the Internal Gear Pump as the name suggests. The two interlocking gears in the internal gear pump are of different sizes with one rotating inside the other. The internal gear whose teeth are concave is the largest gear in this assembly. The engagement occurs between the concave profile of the internal gear and the convex profile of the external gear keeping the rotation in the same direction.

According to the law of gearing for the gear couple to be conjugate, for a gear profile chosen arbitrarily, makes it probable to find the profile of the other gear fit and its required installation parameters. For a determined pinion surface, the internal gear is regarded as an outer conjugate envelope. Hence the internal gear is considered as a generated surface for the generating pinion profile [Col12].

The basic concept and parametrical representation of the Internal gear is made in the following [Col12].

$$N_1\omega_1 = N_2\omega_2 \quad (2.1)$$

where N_1 and N_2 are the number of teeth for the pinion and the internal gear and ω_1 and ω_2 are their angular velocity.

And another equation can also be stated with the help of angular velocities and the Radii of the pitch circles of Pinion and the Internal Gear respectively can be written as follows:

$$R_{p1}\omega_1 = R_{p2}\omega_2 \quad (2.2)$$

where R_{p1} and R_{p2} are the pitch circle radius of the pinion and the internal gear.

Also, it can be seen from the Figure2.2 that

$$R_{p1} - R_{p2} = C \quad (2.3)$$

Where C is the center distance which is stated as the difference between the pitch circles.

Thus the pitch point meshing relation of the Pinion and the Internal Gear can be derived as below:

$$R_{p1} = \frac{N_1 C}{N_2 - N_1} \quad (2.4)$$

$$R_{p2} = \frac{N_2 C}{N_2 - N_1} \quad (2.5)$$

The above equations can help the reader to determine the Number of Teeth, Pitch Circle and the Centre Distance of the Pinion and the Internal Gear respectively. The Working Pressure Angle is as well related and required for the meshing of the Pinion and the Internal Gear and that can also be derived from the above equations.

$$\alpha = \cos^{-1}\left(\frac{R_{p2} - R_{p1}}{C}\right) \quad (2.6)$$

The other significant parameters important for defining the internal gear and the meshing pair are listed in the table below 2.1 [Den19].

Item	Symbol	Formula
Module	m	
Pressure angle	α	
Number of Teeth	Z	
Working Pressure Angle	α_w	$\cos^{-1}\left(\frac{\cos\alpha}{\frac{Z_2-Z_1}{2}+1}\right)$
Center Distance	a	$\left(\frac{Z_2-Z_1}{2} + y\right)m$
Center Distance Modification Coefficient	y	$\frac{Z_2-Z_1}{2} \left(\frac{\cos\alpha}{\cos\alpha_w} - 1\right)$
Profile Shift Coefficient Difference	$x_2 - x_1$	$\left(\frac{(Z_2-Z_1)(\text{inv}\alpha_w - \text{inv}\alpha)}{2\tan\alpha}\right)m$

Table 2.1: Internal Gear Parameter with Profile Shift Calculation [Den19]

The complete calculation required for the internal gear and the corresponding meshing gear is presented in the book by The author J. R. Colbourne [Col12] and also discussed in the article [Den19]. The study here [Den19] also discusses various interference occurring with the internal gear meshing, one such is the the

involute interference occurred due to the narrow distance between addendum and dedendum of the internal gear mesh pair.

To mathematically illustrate the pinion and the internal gear the coordinate structure is defined with x, y coordinates and center O . Thus $S_t(O_t, x_t, y_t)$ and $S_1(O_1, x_1, y_1)$ are the coordinates that represented and rigidly connects the tooth profile to the gear. Using the law of gearing for the profile and along with the application of the coordinate transform the pinion and the internal gear profile can be algebraically represented as [ZS13] [SZZ13]:

$$R_1^p(\phi_1, x_t) = M_1(\phi_1)R_t^c(x_t) \quad (2.7)$$

$$R_2^i(\phi_1, \phi_2, x_t) = M_{21}(\phi_1, \phi_2)R_1^p(\phi_1, x_t) \quad (2.8)$$

Where, R_1^p and R_2^i represents the profile of pinion and the internal gear and M_ϕ is the coordinate transformation matrix.

Before carrying forward to the topics of meshing gears it is also significant to understand the involute tooth profile geometry and calculation because of its simple tool geometry with favourable meshing characteristics. The complete description of the involute profile modelling and calculation is presented in the article [sci18].

2.2 Internal Gear Couple Meshing

As already the name states the gears pairs and their meshing takes the direct effect on the trapped fluid and flowrate pulsation directly affecting the performance of the gear pump operations.

The derivation of the line of action creates the base for defining the mesh characteristics between different contact points among the gear couple and the chambers which causes the change in the cavity volumes between the suction, transitional, discharge and trapped chamber [SCZ16].

The author J. R. Colbourne [Col12] establishes the law of gearing could also be applied for the couple of gears which states that the contact points of the tooth passes through a fixed set of points. Also discussed in the study is the conjugate characteristics of the gear profiles. The pair of profiles that satisfy the law of gearing are said to be conjugate, and for such conjugate pair makes it possible to derive the other surface. Here the derived pinion which becomes the generating surface and the internal gear becomes the generated surface [Col12].

The conjugated gear profiles can be represented as follows [SCZ16] [SZZ13]:

$$R_1^p = R_1^p(x_1, s, z_1) \quad (2.9)$$

$$R_2^i = R_2^i(x_2, s, z_1, z_2) \quad (2.10)$$

Here, x_1, x_2 are the x-coordinates, s is the tooth thickness and z_1, z_2 are the number of tooth of the pinion profile R_1^p and internal gear profile R_2^i .

The book Gear Geometry and Applied Theory by Litvin, Faydor L., Fuentes, Alfonso [LF04] also discusses the line of action as the locus of the contact points between the tooth surfaces of the gear couple. Hence the line of action can be represented by the equation [SCZ16]:

$$R_g^l(\phi_l, x_1, s, \beta, z_1) = M_{gl}(\phi_l) R_l^p(x_1, s, \beta, z_l) \quad (2.11)$$

$$M_{gl}(\phi_l) = \begin{bmatrix} \cos \phi_l & -\sin \phi_l & 0 \\ \sin \phi_l & \cos \phi_l & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad (2.12)$$

Here ϕ_l represents the rotation of the pinion, R_g^l represents the line of action and $M_{gl}(\phi_l)$ represents the matrix of coordinate transform. The combination of the above equations with the internal gear model, the line of action can be obtained.

While discussing the meshing of the internal involute gear couples, besides the meshing of the involute profile the fillet also is participated in the meshing of the gear couple. Understanding all the contact points along with analysis on various interferences helps to obtain the smooth gear transmission and thus a steady fluid delivery for the pump could be obtained.

Here the meshing process of the conjugated gear couple rotating in clockwise direction can be summarized from the figure2.3. Here seen is the involute teeth profiles of both the pinion and the internal gear. For better analysis and study one tooth pair is considered for the illustration of the meshing process, where seen is the contact points of the couple marked by the hollow circles and the target tooth pair involved in the description are filled with shadow lines [SCZ16].

For the considered single tooth pair shown in the figure the meshing process could be interpreted as follows [ZS13] [SCZ16]:

1. The line of action for the illustrated clockwise rotating gear tooth pair is represented by curve AB, where the contact points moves along the curve AB.
2. At point A the tooth pair begins the meshing process.
3. At point C the target tooth pair meshes along the involute profile of the pinion. With the rotation the contact points shifted along the curve AB.
4. As the rotation forwards the second tooth pair makes its contact point at A and the target pair moves its contact point along AB to D, forming two contact points simultaneously then onwards.

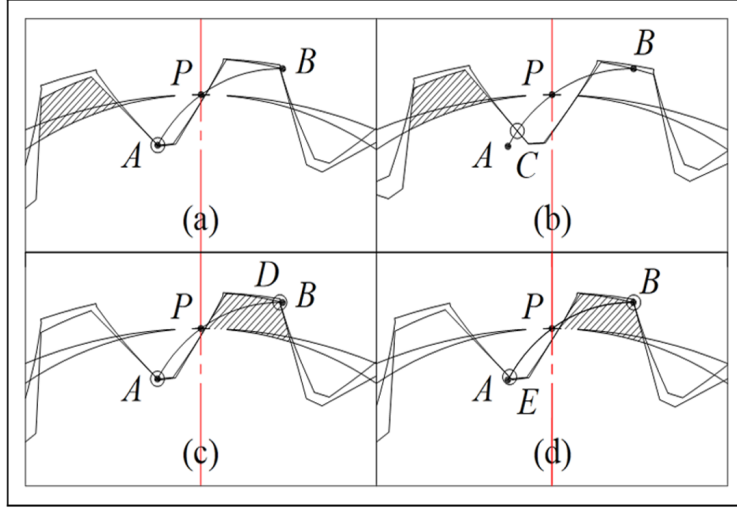


Figure 2.3: Gear couple meshing process. (a) Beginning of the target tooth pair meshing (b) Complete meshing of the target tooth pair (c) Second tooth pair comes into meshing (d) End of the target tooth pair meshing. [SCZ16]

5. At the end meshing of the target tooth pair makes its contact point at B along AB, where at the same time the second tooth pair meshes at point E which is close to the point A indicating the end of meshing of the targeted tooth pair meshing.

The above discussed meshing process considering the single tooth pair creates the foundation for interpretation of the steady delivery of required fluid along with the process of the trapped volume with the assumption that there would be a backlash between the contact points and no relief groove in the pump.

2.3 Flow characteristics based on the gear meshing

The gear pumps as the name states are known for the fluid delivery based on the meshing motion of the gear couple. From the above discussion on the single gear tooth pair depicts the periodic meshing at different contact points. Hence the corresponding fluid displacement within the periodic motion can be interpreted for one such tooth pair motion [ZS13].

The initiation of the meshing of the targeted tooth pair closes the chamber creating the fluid inside the chamber, squeeze into the discharge chamber which

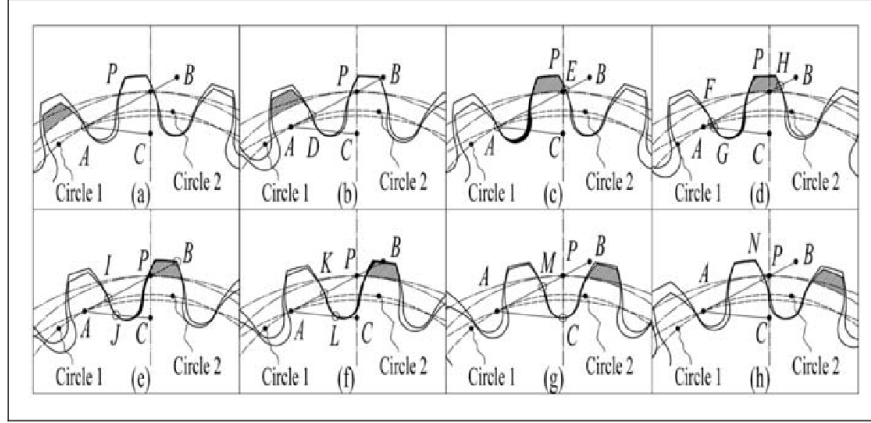


Figure 2.4: Single Tooth Pair Meshing Fluid displacement Illustration [ZS13]

illustrates the beginning process of the fluid delivery. Shown in the figure2.4 (a) to (c) represents the effective fluid delivery process caused by the pressure created by the meshing process of the targeted tooth. The figure2.4 (c) to (g) shows the process of trapped fluid resulting in pressure pulsation and unbalanced force thus creating a opportunity for optimizing the vibration and noise level. The trapped volume enclosed by the involute profiles and two contact points respectively. Thus creating a periodic sections for deciding effective section and discharge chamber [ZS13].

2.3.1 Instantaneous flowrate formula

The volumetric capacity of the pump could be interpreted as displacement of the fluid which is the volume delivered for every rotation. To determine the flowrate the control flow approach is used [Col75]. The control volumes V_1 and V_2 corresponds to the pinion and the internal gear respectively2.5. Firstly, the discharge volume dV is defined as the difference of the inputs and output volumes considering the hypotheses of pumps with rigid parts, incompressible fluid, and no leakage [GMC07] [ZS13].

$$dV = dV_i - dV_o = (dV_{i1} + dV_{i2}) - (dV_{o1} + dV_{o2}) \quad (2.13)$$

where dV_i is the input volume and dV_o is the output volume, hence

$$dV_{i1} = \frac{1}{2}r_{a1}^2 d\phi_1, \quad dV_{o1} = \frac{1}{2}r_{f1}^2 d\phi_1, \quad dV_{i2} = \frac{1}{2}r_{a2}^2 d\phi_2, \quad dV_{o2} = \frac{1}{2}r_{f2}^2 d\phi_2 \quad (2.14)$$

The above could be seen from the figure 2.5 r_{a1} and r_{a2} , is the addendum radius for the pinion and internal gear, r_{f1} and r_{f2} are the centre distance from the

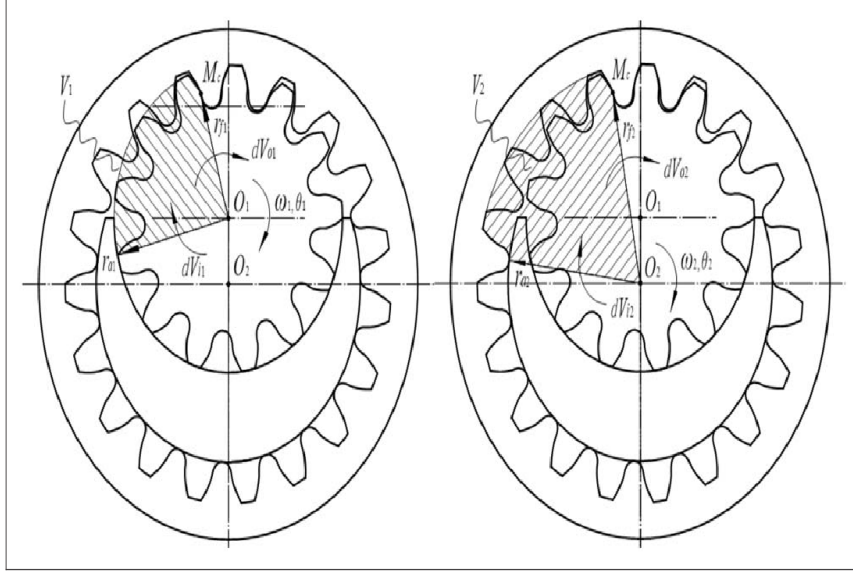


Figure 2.5: Control Volume Illustration [SCZ16]

contact point of the pinion and internal gear, and ϕ_2 is the rotation of the internal gear [SCZ16].

Hence the instantaneous flowrate could be derived from the discharge volume and the fundamental law of gearing [SCZ16]. It can be mathematically represented as:

$$Q(x_1, s, \beta, z_1, z_2) = \frac{dV}{dt} \quad (2.15)$$

Determining Displacement and Pulsation

The displacement is defined as the fluid flow of volume delivered for the every revolution of the pump. Thus single tooth delivering volume can be mathematically represented as [ZS13]:

$$V_Z = \int_t Q dt \quad (2.16)$$

where t denotes the lasting time for the contact points during meshing. The discription of detailed derivation of the displacement is presented by Zhou and Song [ZS13].

Design parameter	Value	Displacement	Flowrate pulsation	Trapped volume	Difference of trapped volumes
x	great		V-shaped		
	low				
z_1	great	↗	↘	↗	↘
	low	↘	↗	↘	↗
z_2	great	insensitive	↗	insensitive	↗
	low		↘		↘
α	great	↗	↘	↗	↘
	low	↘	↗	↘	↗
r_o^*	great	↗	↘	↗	↘
	low	↘	↗	↘	↗

Figure 2.6: Gear parametric influence on flow characteristics of the pump [ZS13]

Though the study discusses the nonlinear line of action for the internal gear pair, thus the study for the one such consideration where flowrate cannot be expressed as including time dependent variable.

Pulsation caused by fluctuations in pressure. The flow pulsation occurs from the change in the meshing points between driving and driven gears periodically. In Consequence the pulsation coefficient can be represented as [ZS13];

$$\gamma = \frac{Q_{max} - Q_{min}}{q\omega_1} \quad (2.17)$$

where q is the displacement of the pump [ZS13].

Determination of the Trapped fluid is also considered significant for the modeling of the gear pump flow rate. The volume of the trapped fluid is the volume isolated between the involute profiles of the pinion and the internal gear. Trapped volume could be represented as using the flow rate relation as:

$$\Delta V = \int_t Q \quad (2.18)$$

The table presented in 2.6 give a quick illustration on how the various gear parameters affect the flow characteristics of the pump. In this illustration shown is the upward arrow which represent increase and downward arrow which notates the decrease [ZS13].

3 Conclusion

Throughout this study, investigations on the geometrical and mathematical characteristics of the internal gear couple influencing the operational performance are effectuated. Furthermore, a detailed examination of the internal gear pair geometry including the internal gear profile, conjugate characteristics, meshing properties and instantaneous flowrate has been conducted. The following findings were identified from this study:

1. Based on the law of gearing, the geometric theory of the internal gear pair profiles has been inspected. Additionally, examined the different overcutting, interference, and overlap analyses for the internal gear pair and its contact characteristics.
2. The mathematical depictions of the internal gear pair profile are derived. Also, formulas of basic geometric dimensions for the meshing and flow characteristics are presented.
3. A line of action creates the base for defining the mesh properties between gear pair and the chambers. This results in the change of cavity volumes between the suction, transitional, discharge, and trapped chambers.
4. Volumetric capacity is the displacement of the fluid delivered for every rotation. Hence, the control-flow approach is utilized to determine the flow rate.
5. The corresponding fluid displacement within the periodic meshing at different contact points for a single gear tooth pair is obtained.

This report is addressed as a primary resource for the subsequent modelling and analysis purpose. Moreover, the study provides insights for future works to explore and delve deeper into the geometric approaches of the internal gear couple along with its mesh and flowrate characteristics.

3.1 Discussion

The underlying section addresses numerous empirical researches on the operational attributes of the internal gear pump, which is interpreted through the benefits, challenges as well as potential workaround solutions.

The internal gears are considered extremely efficient for high-viscosity liquids due to their constant and evenly non-pulsating discharge of liquids regardless of pressure conditions. Moreover, being bi-directional with only two moving parts, it requires a low Net Positive Suction Head (NPSH) and has a single adjustable end clearance. As it provides application customization with its flexible design, they are comparatively easy to maintain. However, internal gear pumps typically need moderate speeds and have pressure limitations in addition to the overhung load on shaft bearing. Although these challenges can be tackled by utilizing close-coupled and seal-less designs, it is not a long term solutions.

Hence, other workaround can be obtained by swapping the traditional involute gear shape with the Truninger teeth shape (for further this study [Gmb] could be referred) which enhances the sealing surface and delivers better performance. A general drawback with the internal gear pumps is the insufficient suction or low flow rate of the fluids which is mostly caused by air leakage in the suction pipe or a narrower diameter of the suction inlet. This can be addressed by shortening and widening the suction line diameter, as well as extending the suction pipe. Furthermore, by employing an internal gear pump with axial and radial play compensation. Internal leakage is another typical problem with internal gear pumps due to the interference between the meshing surfaces. Hence, incorporating Additive Manufacturing introduces the prospect of enhancing flowrate characteristics and optimizing contact points and thereby improving the meshing processes.

Bibliography

- [Col75] COLBOURNE, JR: Gear shape and theoretical flow rate in internal gear pumps. In: *Transactions of the Canadian Society for Mechanical Engineering* 3 (1975), Nr. 4, S. 215–223
- [Col12] COLBOURNE, John R.: *The geometry of involute gears*. Springer Science & Business Media, 2012
- [CXHL18] CHEN, Zongbin ; XU, Rongwu ; HE, Lin ; LIAO, Jian: Modeling an internal gear pump. In: *AIP Conference Proceedings* Bd. 1967 AIP Publishing LLC, 2018, S. 040049
- [Den19] DENGEL, Brian: *Internal ring gears design and considerations*. <https://gearsolutions.com/departments/tooth-tips/internal-ring-gears-design-and-considerations/>. Version: Dezember 2019. – [Online; Stand December 15, 2019]
- [Gmb] GMBH, SAUER B.: *HIGH PERFORMANCE INTERNAL GEAR PUMPS*. https://www.sauerbibus.de/fileadmin/editors/countries/sab/Produkte/Sumitomo/PDF/SAB_Sumitomo_7-2010_A4_GB.pdf. – [Online; Stand 09 May 2021]
- [GMC07] GAMEZ-MONTERO, PJ ; CODINA, E: Flow characteristics of a trochoidal-gear pump using bond graphs and experimental measurement. Part 2. In: *Proceedings of the Institution of Mechanical Engineers, Part I: Journal of Systems and Control Engineering* 221 (2007), Nr. 3, S. 347–363
- [Ich59] ICHIKAWA, Tsuneo: Characteristics of internal gear pump. In: *Bulletin of JSME* 2 (1959), Nr. 5, S. 35–39
- [Jen12] JENNER, Christopher: *Gear Pumps*. <http://processprinciples.com/2012/07/gear-pumps/>. Version: Juli 2012. – [Online; Stand 2 July, 2012]
- [LF04] LITVIN, Faydor L. ; FUENTES, Alfonso: *Gear geometry and applied theory*. Cambridge University Press, 2004
- [sci18] SCIENCE tec: *Calculation of involute gears*. <https://www.tec-science.com/mechanical-power-transmission/>

- `involute-gear/calculation-of-involute-gears/`.
Version: Oktober 2018. – [Online; Stand October 31, 2018]
- [SCZ16] SONG, Wei ; CHEN, Yinglong ; ZHOU, Hua: Investigation of fluid delivery and trapped volume performances of Truninger gear pump by a discretization approach. In: *Advances in Mechanical Engineering* 8 (2016), Nr. 10, S. 1687814016672365
- [SZZ13] SONG, Wei ; ZHOU, Hua ; ZHAO, Yonggang: Design of the conjugated straight-line internal gear pairs for fluid power gear machines. In: *Proceedings of the Institution of Mechanical Engineers, Part C: Journal of Mechanical Engineering Science* 227 (2013), Nr. 8, S. 1776–1790
- [Yan07] YANG, Shyue-Cheng: Study on an internal gear with asymmetric involute teeth. In: *Mechanism and Machine Theory* 42 (2007), Nr. 8, S. 977–994
- [ZDXY17] ZHOU, Hua ; DU, Ruilong ; XIE, Anhuan ; YANG, Huayong: Theoretical analysis for the flow ripple of a tandem crescent pump with index angles. In: *Applied Sciences* 7 (2017), Nr. 11, S. 1148
- [ZS13] ZHOU, Hua ; SONG, Wei: Theoretical flowrate characteristics of the conjugated involute internal gear pump. In: *Proceedings of the Institution of Mechanical Engineers, Part C: Journal of Mechanical Engineering Science* 227 (2013), Nr. 4, S. 730–743