§5.13. Assignment 4 (Fall 2020)

Exercise 5.52. THE BIRTH OF A PATH OF LENGTH ℓ. In a $G_{n,p}$, we say that a_n is a threshold for a path of length ℓ if

$$\lim_{n\to\infty} \mathbf{P}\{\text{there exists a path of length } \ell\} = \begin{cases} 0 & \text{if } p = o(a_n) \\ 1 & \text{if } p = \omega(a_n). \end{cases}$$

Find a_n by appropriate use of the first and second moment methods.

Exercise 5.53. A RANDOM MAP. Consider a graph on n nodes in which each node independently picks one other node to connect to. Double edges are collapsed into one edge. Obtain first order asymptotics for the expected number of connected components.

In addition, answer questions 5.34 and 5.43.