Backpropagation

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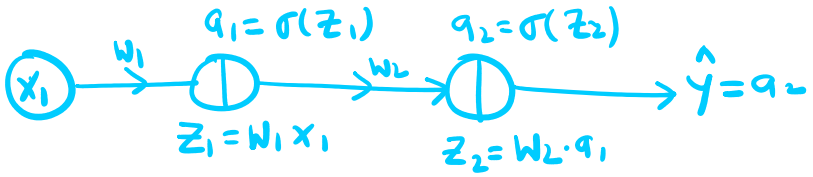

Error $= (y - \hat{y})^2 = (y - a_2)^2$

Update Rule $\Rightarrow$ $w_N = w_0 - \eta \frac{\partial E}{\partial w}$

$\frac{\partial E}{\partial w} = ??$ Let's start with w_2

Error $= (y - a_2)^2$, $a_2 = \sigma(z_2)$, $z_2 = w_2 \cdot a_1$

$$\frac{\partial E}{\partial w_2} = \underbrace{\frac{\partial E}{\partial a_2}}_{\text{I}} \cdot \underbrace{\frac{\partial a_2}{\partial z_2}}_{\text{II}} \cdot \underbrace{\frac{\partial z_2}{\partial w_2}}_{\text{III}} = \underbrace{-2(y - a_2)}_{\text{I}} \cdot \underbrace{\sigma(z_2)(1 - \sigma(z_2))}_{\text{II}} \cdot \underbrace{a_1}_{\text{III}}$$

$$w_2^N = w_2^0 - \eta \frac{\partial E}{\partial w_2}$$

Similarly for w_1

$E = (y - a_2)^2$, $a_2 = \sigma(z_2)$, $z_2 = w_2 \cdot a_1$, $a_1 = \sigma(z_1)$, $z_1 = w_1 x_1$

$$w_1^N = w_1^0 - \eta \frac{\partial E}{\partial w_1}$$

$$\frac{\partial E}{\partial w_1} = \underbrace{\frac{\partial E}{\partial a_2}}_{\text{I}} \cdot \underbrace{\frac{\partial a_2}{\partial z_2}}_{\text{II}} \cdot \frac{\partial z_2}{\partial a_1} \cdot \frac{\partial a_1}{\partial z_1} \cdot \frac{\partial z}{\partial w_1}$$

$\frac{\partial z_2}{\partial a_1} \rightarrow w_2$

$\frac{\partial a_1}{\partial z_1} \rightarrow \sigma(z_1) \cdot (1 - \sigma(z_1))$

$\frac{\partial z}{\partial w_1} \rightarrow x_1$

$w_2 \cdot \sigma(z_1) \cdot (1 - \sigma(z_1)) \cdot x_1$

$w_2 \cdot \sigma(z_1) \cdot (1 - \sigma(z_1)) \cdot a_0$

let's take $x_1 = a_0$

$$w_1^N = w_1^0 - \eta \frac{\partial E}{\partial w_1}$$