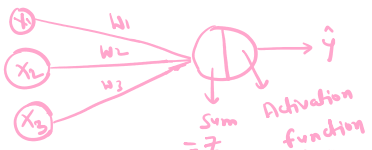
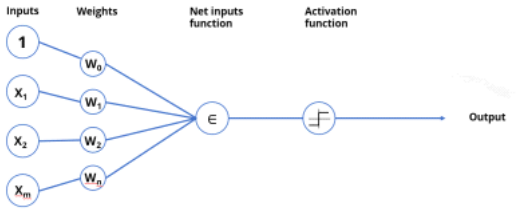
Perceptron Model

Sunday, September 26, 2021 6:39 AM





$W^T \cdot X = \begin{bmatrix} W_1 & W_2 & W_3 \end{bmatrix}_{1\times3} \cdot \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}_{3\times1} = \begin{bmatrix} W_1x_1 + W_2x_2 + W_3x_3 \end{bmatrix}_{1\times1} = z$

$= f(z) = \sigma(W_1x_1 + W_2x_2 + W_3x_3)$

↓ Sigmoid function

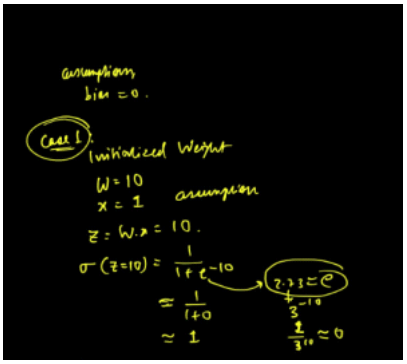
domain for Sigmoid

$[-\infty, \infty] \rightarrow [0, 1]$

$\sigma = \frac{1}{1+e^{(-z)}}$

Prediction $= \hat{y} = \frac{1}{1+e^{-(W_1x_1+W_2x_2+W_3x_3)}}$

↓ Sum



Randomly

Output $Y = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$ Input $X = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 1 & 1 \end{bmatrix}_{3\times3}$ $W = \begin{bmatrix} 0.50 \\ 0.50 \\ 0.50 \end{bmatrix}_{3\times1}$

$W^T X = \begin{bmatrix} 0.50\times1 + 0 + 0.50\times1 \\ 0 + 0.5\times1 + 0 \\ 0.5\times1 + 0.5\times1 + 0.5\times1 \end{bmatrix} = \begin{bmatrix} 1 \\ 0.5 \\ 1.5 \end{bmatrix}_{3\times1} = z = f(z)$

$\begin{bmatrix} 0.5 & 0.5 & 0.5 \end{bmatrix}_{1\times3} \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 1 & 1 \end{bmatrix}_{3\times3}$

$f(z) = \hat{y} = \begin{bmatrix} \frac{1}{1+e^{(-1)}} \\ \frac{1}{1+e^{(-0.5)}} \\ \frac{1}{1+e^{(-1.5)}} \end{bmatrix} = \begin{bmatrix} 0.73 \\ 0.62 \\ 0.81 \end{bmatrix}$ ($\hat{y}$)

Error $= y - \hat{y} = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} - \begin{bmatrix} 0.73 \\ 0.62 \\ 0.81 \end{bmatrix} = \begin{bmatrix} -0.73 \\ -0.62 \\ 0.19 \end{bmatrix}$

$\sigma = \frac{1}{1+e^{-z}}$

$\sigma \begin{bmatrix} 1 \\ 0.5 \\ 1.5 \end{bmatrix} \rightarrow \begin{bmatrix} \frac{1}{1+e^{-1}} \\ \frac{1}{1+e^{-0.5}} \\ \frac{1}{1+e^{-1.5}} \end{bmatrix} \rightarrow \begin{bmatrix} 0.73 \\ 0.62 \\ 0.81 \end{bmatrix} = \hat{y}$

Error $= y - \hat{y}$

forward Propagation $= \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} - \begin{bmatrix} 0.73 \\ 0.62 \\ 0.81 \end{bmatrix} = \begin{bmatrix} -0.73 \\ -0.62 \\ 0.19 \end{bmatrix}$