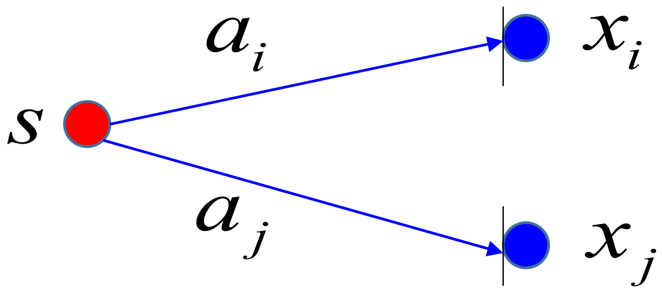
Blind system ID and adaptive eigenvalue decomposition

Blind system ID: ATF

$A_1 X_2 = A_2 X_1$

$\times X^T$

$\begin{bmatrix} X_2 & -X_1 \end{bmatrix} \begin{bmatrix} A_1 \\ A_2 \end{bmatrix} = 0$



In the time domain, consider a single-source scenario.

$a_i(n) * a_j(n) * s(n) = a_j(n) * a_i(n) * s(n) \Rightarrow a_i(n) * x_j(n) = a_j(n) * x_i(n)$ (cross-relation)

$\Rightarrow -a_i(n) * x_j(n) + a_j(n) * x_i(n) = 0 \qquad (1)$

$\mathbf{x}_i(n) = \begin{bmatrix} x_i(n) & x_i(n-1) & \cdots & x_i(n-N+1) \end{bmatrix}^T, \quad \mathbf{a}_i = \begin{bmatrix} a_i(0) & a_i(1) & \cdots & a_i(N-1) \end{bmatrix}^T, \quad i = 1, 2$

$\mathbf{x}(n) = \begin{bmatrix} -\mathbf{x}_1^T(n) & \mathbf{x}_2^T(n) \end{bmatrix}^T, \quad \mathbf{a} = \begin{bmatrix} \mathbf{a}_1^T & \mathbf{a}_2^T \end{bmatrix}^T$

Using the above vector definitions, we may rewrite (1) into $\mathbf{x}^T(n)\mathbf{a} = 0$

$E\left[\mathbf{x}(n)\mathbf{x}^T(n)\mathbf{a}(n)\right] = \mathbf{0} \Rightarrow \mathbf{R}_{xx}\mathbf{a} = \mathbf{0} \qquad (2)$

$\Rightarrow \mathbf{a} \in N(\mathbf{R}_{xx}) \because \mathbf{R}_{xx}$ singular, $\mathbf{a}$ pertains to zero eigenvalues.

Blind system ID

Instead of the above equation, a more robust formulation against noise is to solve the following constrained optimization problem:

$$\min_{\mathbf{a}} \left[\mathbf{a}^H \mathbf{R}_{xx} \mathbf{a} \right] \quad st. \quad \mathbf{a}^H \mathbf{a} = M \rightarrow \text{to avoid } \mathbf{a} = \mathbf{0}$$

Thus, the optimizer is the eigenvector of $\mathbf{R}_{xx}$ correponding to

the minimum eigenvalue normalized to $\|\mathbf{a}\| = \sqrt{M}$.

Note, however, that $\mathbf{a}$ is not unique up to a scalar filter. An EQ may be needed to avoid artifacts.

$\mathbf{R}_{xx} = E(\mathbf{x}\mathbf{x}^T)$ can be calculated recursively by using

$$\mathbf{R}_{xx}(n+1) = \alpha \mathbf{R}_{xx}(n) + (1-\alpha)\mathbf{x}(n)\mathbf{x}^T(n), \quad n: \text{ time index}$$

Extension to the multi-mic case

Consider an M-microphone array setting.

$$\mathbf{x}_i(n) = \left[x_i(n) \quad x_i(n-1) \quad \cdots \quad x_i(n-N+1)\right]^T,$$

$$\mathbf{a}_i(n) = \left[a_i(0) \quad a_i(1) \quad \cdots \quad a_i(N-1)\right]^T, \quad i = 1, 2, \ldots, M$$

For $M = 4$,

$$\begin{bmatrix} \mathbf{x}_2^T(n) & -\mathbf{x}_1^T(n) & 0 & 0 \\ \mathbf{x}_3^T(n) & 0 & -\mathbf{x}_1^T(n) & 0 \\ \mathbf{x}_4^T(n) & 0 & 0 & -\mathbf{x}_1^T(n) \end{bmatrix} \begin{bmatrix} \mathbf{a}_1 \\ \mathbf{a}_2 \\ \mathbf{a}_3 \\ \mathbf{a}_4 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \Rightarrow \mathbf{X}^T\mathbf{a} = \mathbf{0}$$

$$E\left[\mathbf{X}(n)\mathbf{X}^T(n)\mathbf{a}\right] = \mathbf{0} \Rightarrow \mathbf{R}_{xx}\mathbf{a} = \mathbf{0} \qquad (2)$$

$\Rightarrow \mathbf{a} \in N(\mathbf{R}_{xx})$ $\because \mathbf{R}_{xx}$ singular, $\mathbf{a}$ pertains to zero eigenvalues.

Alternatively, $\min_{\mathbf{a}} \mathbf{a}^T\mathbf{R}_{xx}\mathbf{a} \quad st. \, \mathbf{a}^T\mathbf{a} = M$

$$\begin{bmatrix} X_2 & -X_1 & 0 & 0 \\ X_3 & 0 & -X_1 & 0 \\ X_4 & 0 & 0 & -X_1 \end{bmatrix} \begin{bmatrix} A_1 \\ A_2 \\ A_3 \\ A_4 \end{bmatrix} = 0$$

is hermitian

$R_{xx} = XX^T$

$R_{xx}^H = (XX^T) = XX^T = R_{xx}$

$R_{xx}^T = (XX^T)^T = XX^T$

Adaptive Eigenvalue Decomposition Algorithm (AEDA)

$\min_{\mathbf{w}} \mathbf{w}^T \mathbf{R}_{xx} \mathbf{w} \quad st. \, \mathbf{w}^T \mathbf{w} = M$

Using the gradient descent approach

$\mathbf{w}_{k+1} = \mathbf{w}_k - \mu \nabla_{\mathbf{w}} J,$ where the Lagrangian

$J = \mathbf{w}^T \mathbf{R}_{xx} \mathbf{w} - \lambda(\mathbf{w}^T \mathbf{w} - M) \qquad (1)$

$\nabla_{\mathbf{w}} J = \mathbf{R}_{xx} \mathbf{w} - \lambda \mathbf{w} \qquad (2)$

(2): $\mathbf{w}_{k+1} = \mathbf{w}_k - \mu(\mathbf{R}_{xx} \mathbf{w}_k - \lambda \mathbf{w}_k) \qquad (3)$

can be computed by using recursive averaging. Here, μ and λ must be specified in advance.

Alternatively, we may introduce the stochaistic gradient ($\mathbf{R}_{xx} \approx \mathbf{x}_k \mathbf{x}_k^T$) and rewrite the update equation as follows:

$\mathbf{w}_{k+1} \approx \mathbf{w}_k - \mu(\mathbf{x}_k \mathbf{x}_k^T \mathbf{w}_k - \lambda \mathbf{w}_k) = (1 + \mu\lambda)\mathbf{w}_k - \mu y_k \mathbf{x}_k \qquad (4)$

Lastly, normalize $\mathbf{w}_{k+1}$ such that $\mathbf{w}_{k+1}^T \mathbf{w}_{k+1} = M,$ i.e.,

$$\mathbf{w}_{k+1} \leftarrow \mathbf{w}_{k+1} \sqrt{\frac{M}{\mathbf{w}_{k+1}^T \mathbf{w}_{k+1}}} \qquad (5)$$

Remarks

Gannot suggested to use the identified filtered ATFs in the MINT approach for dereverberation. However, the performance could be limited because the information of source input is inaccessible. In this case, we can at best obtain the RTFs by setting the scalar filter to be a1^-1 and a1 becomes δ[n].

This can be viewed as the following LCMV problem:

Alternatively, $\min_{\mathbf{a}} \mathbf{a}^T \mathbf{R}_{xx} \mathbf{a} \quad st.\, \mathbf{C}^T \mathbf{a} = \mathbf{g}$

$$\mathbf{C}^T = \begin{bmatrix} \mathbf{I}_{N\times N} & \mathbf{0}_{N\times N} & \cdots & \mathbf{0}_{N\times N} \end{bmatrix}$$

$$\mathbf{g}^T = \begin{bmatrix} 1 & 0 & \cdots & 0 \end{bmatrix} \sim \delta[n]$$

LCMV solution: $\mathbf{a}_{LCMV} = \mathbf{R}_{xx}^{-1}\mathbf{C}(\mathbf{C}^T\mathbf{R}_{xx}^{-1}\mathbf{C})^{-1}\mathbf{g}$

Adaptive implementation by using the Frost's approach:

$$\mathbf{w}_{k+1} = \left(\mathbf{I} - \mathbf{C}\mathbf{C}^{+}\right)\left(\mathbf{w}_k - \mu \mathbf{R}_{xx}\mathbf{w}_k\right) + \mathbf{C}\left(\mathbf{C}^T\mathbf{C}\right)^{-1}\mathbf{g}$$

$$\approx \left(\mathbf{I} - \mathbf{C}\mathbf{C}^{+}\right)\left(\mathbf{w}_k - \mu y_k^* \mathbf{x}_k\right) + \mathbf{C}\left(\mathbf{C}^T\mathbf{C}\right)^{-1}\mathbf{g}$$

$$= \left(\mathbf{I} - \mathbf{C}\mathbf{C}^{T}\right)\left(\mathbf{w}_k - \mu y_k^* \mathbf{x}_k\right) + \mathbf{C}\mathbf{g}$$

$$= \mathbf{w}_{adaptive} + \mathbf{w}_{fixed} = \text{GSC}$$

References

1. O. L. Frost, III, “An Algorithm for Linearly Constrained Adaptive Array Processing,” Proc. IEEE, vol. 60, no. 8, pp.926-935, 1972.
2. Gay and Benesty, Acoustic signal processing for telecommunication, p.244.
3. Naylor (ed), Speech dereverberation, Chap. 5 by Gannot.
4. Vicent’s book, Chap.15