CTFEDA

$$y(n) = a(n) * s(n)$$

$$y_{p,k} \approx \sum_{p'=0}^{L-1} s_{p-p',k}\, a_{p',k} = s_{p,k} * a_{p,k}$$

$$y^i_{p,k} = \sum_{p'=0}^{L-1} s_{p-p',k}\, a^i_{p',k}$$

$$\underbrace{\begin{bmatrix} y^1_{p,k} \\ y^2_{p,k} \\ \vdots \\ y^M_{p,k} \end{bmatrix}}_{\mathbf{y}_d} = \underbrace{\begin{bmatrix} a^1_{0,k} & a^1_{1,k} & \cdots & a^1_{L-1,k} \\ a^2_{0,k} & a^2_{1,k} & \cdots & a^2_{L-1,k} \\ \vdots & \vdots & \ddots & \vdots \\ a^M_{0,k} & a^M_{1,k} & \cdots & a^M_{L-1,k} \end{bmatrix}}_{\mathbf{A}} \underbrace{\begin{bmatrix} s_{p,k} \\ s_{p-1,k} \\ \vdots \\ s_{p-(L-1),k} \end{bmatrix}}_{\mathbf{s}}$$

$$\mathbf{y}_{d,p} = \mathbf{A} \cdot \mathbf{s}_p$$

$$y_p^i = [\; y_p^i \quad y_{p-1}^i \quad \cdots \quad y^i_{p-(L-1)} \;]$$

$$a^i = [\; a_0^i \quad a_1^i \quad \cdots \quad a_{L-1}^i \;]$$

$M \times L$

$$M-1 \quad \begin{bmatrix} y_p^2 & -y_p^1 & 0 & 0 \\ y_p^3 & 0 & -y_p^1 & 0 \\ y_p^4 & 0 & 0 & -y_p^1 \end{bmatrix} \begin{bmatrix} a^1 \\ a^2 \\ a^3 \\ a^4 \end{bmatrix} = 0$$

IMC FLMS

$$\boldsymbol{\mathcal{W}}_{L\times 2L}^{01} = \mathbf{F}_{L\times L}\mathbf{W}_{L\times 2L}^{01}\mathbf{F}_{2L\times 2L}^{-1}$$
$$\boldsymbol{\mathcal{W}}_{2L\times L}^{10} = \mathbf{F}_{2L\times 2L}\mathbf{W}_{2L\times L}^{10}\mathbf{F}_{L\times L}^{-1}$$

where $\boldsymbol{\mathcal{D}}_{x_i}(m)$ is a diagonal matrix whose diagonal elements are given by the DFT of the first column of $\mathbf{C}_{x_i}(m)$, i.e., the

$$\boldsymbol{\mathcal{S}}_{x_i}(m) \stackrel{\triangle}{=} \boldsymbol{\mathcal{W}}_{L\times 2L}^{01}\boldsymbol{\mathcal{D}}_{x_i}(m)\boldsymbol{\mathcal{W}}_{2L\times L}^{10},\ i = 1, 2, \ldots, M.$$

$$\tilde{\boldsymbol{\mathcal{R}}}_{ij}(m) \stackrel{\triangle}{=} \boldsymbol{\mathcal{S}}_{x_i}^{H}(m)\boldsymbol{\mathcal{S}}_{x_j}(m),\ i, j = 1, 2, \ldots, M.$$

$$\tilde{\boldsymbol{\mathcal{R}}}(m) = \begin{bmatrix} \sum_{i\neq 1}\tilde{\boldsymbol{\mathcal{R}}}_{ii}(m) & -\tilde{\boldsymbol{\mathcal{R}}}_{21}(m) & \cdots & -\tilde{\boldsymbol{\mathcal{R}}}_{M1}(m) \\ -\tilde{\boldsymbol{\mathcal{R}}}_{12}(m) & \sum_{i\neq 2}\tilde{\boldsymbol{\mathcal{R}}}_{ii}(m) & \cdots & -\tilde{\boldsymbol{\mathcal{R}}}_{M2}(m) \\ \vdots & \vdots & \ddots & \vdots \\ -\tilde{\boldsymbol{\mathcal{R}}}_{1M}(m) & -\tilde{\boldsymbol{\mathcal{R}}}_{2M}(m) & \cdots & \sum_{i\neq M}\tilde{\boldsymbol{\mathcal{R}}}_{ii}(m) \end{bmatrix}_{ML\times ML}.$$

$$\hat{\underline{\boldsymbol{h}}}(m+1) = \frac{\hat{\underline{\boldsymbol{h}}}(m) - \mu_f\tilde{\boldsymbol{\mathcal{R}}}(m)\hat{\underline{\boldsymbol{h}}}(m)}{\sqrt{L}\left\|\hat{\underline{\boldsymbol{h}}}(m) - \mu_f\tilde{\boldsymbol{\mathcal{R}}}(m)\hat{\underline{\boldsymbol{h}}}(m)\right\|}.$$

NMCFLMS

$$(\mathbf{F}_{L\times L})_{p,q} = e^{-j2\pi pq/L},\ p,q = 0,1,\ldots,L-1.$$

$$\begin{aligned}\mathbf{W}^{01}_{L\times 2L} &= [\,\mathbf{0}_{L\times L} \quad \mathbf{I}_{L\times L}\,]\\ \mathbf{W}^{10}_{2L\times L} &= [\,\mathbf{I}_{L\times L} \quad \mathbf{0}_{L\times L}\,]^T\end{aligned}$$

$\begin{bmatrix} 1 & 0 \end{bmatrix}$ $\begin{bmatrix} 1 \\ 0 \end{bmatrix}$

$$\begin{aligned}\boldsymbol{\mathcal{W}}^{01}_{L\times 2L} &= \mathbf{F}_{L\times L}\mathbf{W}^{01}_{L\times 2L}\mathbf{F}^{-1}_{2L\times 2L}\\ \boldsymbol{\mathcal{W}}^{10}_{2L\times L} &= \mathbf{F}_{2L\times 2L}\mathbf{W}^{10}_{2L\times L}\mathbf{F}^{-1}_{L\times L}\end{aligned}$$

$$\begin{aligned}\boldsymbol{\mathcal{W}}^{10}_{L\times 2L} &= \mathbf{F}_{L\times L}\mathbf{W}^{10}_{L\times 2L}\mathbf{F}^{-1}_{2L\times 2L} = \frac{1}{2}\left(\boldsymbol{\mathcal{W}}^{10}_{2L\times L}\right)^H\\ \boldsymbol{\mathcal{W}}^{01}_{2L\times L} &= \mathbf{F}_{2L\times 2L}\mathbf{W}^{01}_{2L\times L}\mathbf{F}^{-1}_{L\times L} = 2\left(\boldsymbol{\mathcal{W}}^{01}_{L\times 2L}\right)^H\end{aligned}$$

$$\hat{\mathbf{h}}_j(m) = [\,\hat{h}_{j,0}(m) \quad \hat{h}_{j,1}(m) \quad \cdots \quad \hat{h}_{j,L-1}(m)\,]^T.$$

$$\underline{\hat{\boldsymbol{h}}}_i(m) = \mathbf{F}_{L\times L}\hat{\mathbf{h}}_i(m).$$

$$\underline{\hat{\boldsymbol{h}}}^{10}_k(m) = \boldsymbol{\mathcal{W}}^{10}_{2L\times L}\underline{\hat{\boldsymbol{h}}}_k(m) = \mathbf{F}_{2L\times 2L}\begin{bmatrix}\hat{\mathbf{h}}_k(m)\\ \mathbf{0}\end{bmatrix}$$

$\underline{\hat{h}}_k(m) = W^{10}_{L\times 2L}\ \underline{\hat{h}}_k^{10}(m)$

$$\mathbf{C}_{x_i}(m) = \begin{bmatrix} x_i(mL-L) & x_i(mL+L-1) & \cdots & x_i(mL-L+1)\\ x_i(mL-L+1) & x_i(mL-L) & \cdots & x_i(mL-L+2)\\ \vdots & \vdots & \ddots & \vdots\\ x_i(mL) & x_i(mL-1) & \cdots & x_i(mL+1)\\ \vdots & \vdots & \ddots & \vdots\\ x_i(mL+L-1) & x_i(mL+L-2) & \cdots & x_i(mL-L)\end{bmatrix}$$

where $\boldsymbol{\mathcal{D}}_{x_i}(m)$ is a diagonal matrix whose diagonal elements are given by the DFT of the first column of $\mathbf{C}_{x_i}(m)$, i.e., the

$$\begin{aligned}\underline{\boldsymbol{e}}_{ij}(m) &= \boldsymbol{\mathcal{W}}^{01}_{L\times 2L}\\ &\times \left[\boldsymbol{\mathcal{D}}_{x_i}(m)\boldsymbol{\mathcal{W}}^{10}_{2L\times L}\underline{\hat{\boldsymbol{h}}}_j(m) - \boldsymbol{\mathcal{D}}_{x_j}(m)\boldsymbol{\mathcal{W}}^{10}_{2L\times L}\underline{\hat{\boldsymbol{h}}}_i(m)\right]\end{aligned}$$

$$\underline{\boldsymbol{e}}^{01}_{ik}(m) = \boldsymbol{\mathcal{W}}^{01}_{2L\times L}\underline{\boldsymbol{e}}_{ik}(m) = \mathbf{F}_{2L\times 2L}\begin{bmatrix}\mathbf{0}\\ \mathbf{F}^{-1}_{L\times L}\underline{\boldsymbol{e}}_{ik}(m)\end{bmatrix}.$$

$$\begin{aligned}\boldsymbol{\mathcal{P}}_k(m) &= \lambda\boldsymbol{\mathcal{P}}_k(m-1) + (1-\lambda)\\ &\times \sum_{i=1,i\neq k}^{M}\boldsymbol{\mathcal{D}}^*_{x_i}(m)\boldsymbol{\mathcal{D}}_{x_i}(m),\ k = 1,2,\ldots,M\end{aligned}$$

$$\begin{aligned}\underline{\hat{\boldsymbol{h}}}^{10}_k(m+1) &= \underline{\hat{\boldsymbol{h}}}^{10}_k(m) - \rho\left[\boldsymbol{\mathcal{P}}_k(m) + \delta\mathbf{I}_{2L\times 2L}\right]^{-1}\\ &\times \sum_{i=1}^{M}\boldsymbol{\mathcal{D}}^*_{x_i}(m)\underline{\boldsymbol{e}}^{01}_{ik}(m),\ k = 1,2,\ldots,M.\end{aligned}$$