12 Lesson 12 (September 20, 2024)

12.1 Higher Order Derivatives

Let $y = f(x)$. Then, the first derivative of y with respect to x is

$$y' = f'(x) = \frac{dy}{dx} = \frac{d}{dx}(f(x)) = f^{(1)}(x).$$

The second derivative of y with respect to x is

$$y'' = f''(x) = \frac{d^2y}{dx^2} = \frac{d^2}{dx^2}(f(x)) = f^{(2)}(x).$$

The n^{th} derivative of y with respect to x is

$$\frac{d^ny}{dx^n} = \frac{d^n}{dx^n}(f(x)) = f^{(n)}(x).$$

Example 1: Let $f(x) = 3x^3 + 4x^2 + 5x + 6$. Find $f'''(x)$. What is $\cancel{f'''(0)}$?

$f'''(0) - f''(0)$

$f'(x) = 9x^2 + 8x + 5$

$f''(x) = 18x + 8$

$f'''(x) = 18$

$f'''(0) - f''(0) = 18 - (18 \cdot 0 + 8) = 18 - 8 = 10$

Example 2: Let $f^{(3)}(x) = 8\csc(7x - 2)$. Find $f^{(4)}(x)$.

$f^{(4)}(x) = 8 \cdot -\csc(7x-2)\cot(7x-2) \cdot 7$ (chain rule)

$= -56\csc(7x-2)\cot(7x-2)$

12.2 Acceleration

Let $s(t)$ denote the position function. Recall that $v(t) = s'(t)$ denotes the velocity function. The acceleration function can be denoted by $a(t)$, and $a(t) = v'(t) = s''(t)$.

Example 3: A particle is traveling on a straight line with a position function of

$$s(t) = \frac{2}{3}t^3 + 6t^2,$$

where t is time in seconds and $s(t)$ is position in feet. What is the acceleration when the velocity of the particle is 54 feet per second?

$v(t) = s'(t) = 2t^2 + 12t$

$a(t) = v'(t) = 4t + 12$

$54 = 2t^2 + 12t$

$\Rightarrow 2t^2 + 12t - 54 = 0$

$\Rightarrow t^2 + 6t - 27 = 0$

$\Rightarrow (t-3)(t+9) = 0$

$\Rightarrow t = 3$ or $\underbrace{t = -9}_{\text{time can't be negative}}$

$a(3) = 4 \cdot 3 + 12 = 24 \text{ ft/s}^2$

12.3 Practice Problems

1. Find the second derivative of $f(x) = 3x^2 \ln(10x)$.

$f'(x) = 6x\ln(10x) + \frac{1}{10x} \cdot 10 \cdot 3x^2$ (product and chain rule)

$= 6x\ln(10x) + 3x$

$f''(x) = 6\ln(10x) + \frac{1}{10x} \cdot 10 \cdot 6x + 3$ (product and chain rule)

$= 6\ln(10x) + 9$

2. Find the second derivative of $\frac{x}{x-1}$. $= f(x)$

$$f'(x) = \frac{\overbrace{1(x-1) - 1(x)}^{x-1-x}}{(x-1)^2} \quad \text{(quotient rule)}$$

$$= \frac{-1}{(x-1)^2}$$

$$f''(x) = \frac{0\cdot(x-1)^2 - \overbrace{2(x-1)\cdot 1\cdot(-1)}^{2(x-1)}}{(x-1)^4} \quad \text{(quotient and chain rule)}$$

$$= \frac{2x-2 \}\, 2(x-1)}{(x-1)^4}$$

$$= \frac{2}{(x-1)^3}$$

3) Suppose $f(x) = 4\sqrt{x} + \frac{8}{x}$

Find $f''(1)$.

Note $f(x) = 4x^{1/2} + 8x^{-1}$

$$f'(x) = \frac{1}{2}\cdot 4\cdot x^{1/2 - 2/2} + 8\cdot(-1)\cdot x^{-1-1}$$

$$= 2x^{-1/2} - 8x^{-2}$$

$$f''(x) = 2\cdot\left(-\frac{1}{2}\right)\cdot x^{-1/2-2/2} - 8\cdot(-2)\cdot x^{-2-1}$$

$$= -x^{-3/2} + 16x^{-3}$$

$$= -x^{-3/2}\cdot\left(\frac{x^3}{x^3}\right) + \frac{16}{x^3}$$

$$= \frac{-x^{3/2} + 16}{x^3}$$

$$f''(1) = \frac{-1+16}{1} = 15$$