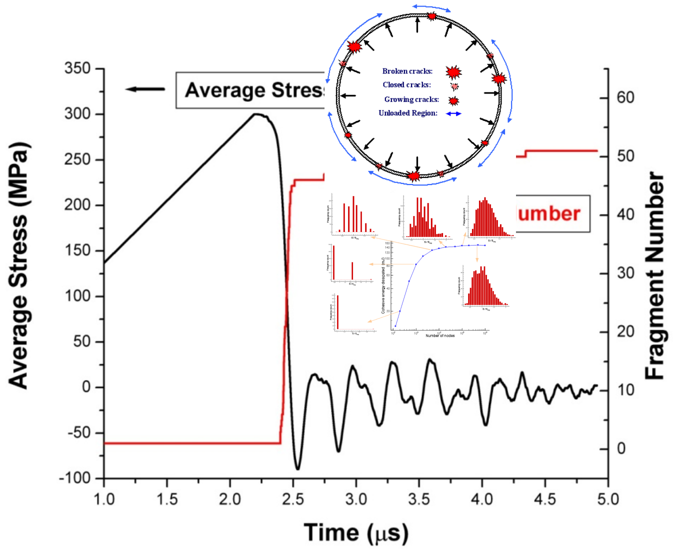
Dynamic fragmentation simulations

G. Anciaux

Civil Enginering, Materials Science, EPFL

Mott’s problem: Zhou Molinari Ramesh 2007, Levy Molinari 2010

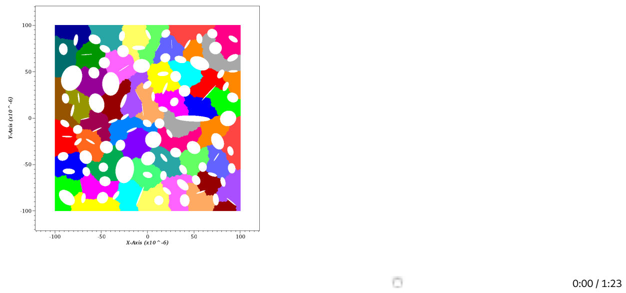


- Ceramic ring length: L= 50 mm
- Elastic parameters: r =2750 Kg/m3, E=250 GPa , c= 10000 m/s
- Fracture parameters: sc =300 MPa, dc= 0.667 mm, Gc=100 N/m
- randomization of σ_c (small variation around mean)

Outcome : definition of a precise law in the low strain rate regime

Secondary waves effect: Illustration with mesh partitioning, M. Vocialta

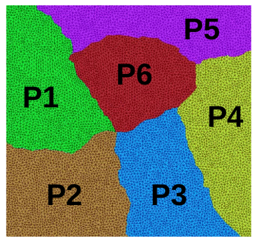
- Ellipsoidal voids, biaxial expansion, initial strain rate: 105 s-1
- E = 320 x 103 MPa ν = 0.237 ρ = 3250 kg/m3 σc = 200 MPa



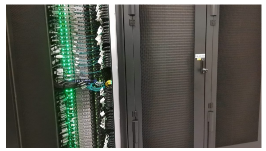
Outcome: Vizualization of secondary waves

Extension to 3D: High Performance Computing

- HPC necessary to achieve convergence:
  e.g. accurate representation of microcracks
- Parallelisation through domain decomposition
- Problem: topological changes for extrinsic cohesive elements



Domain decomposition



Jed at EPFL

- 30’240 cores
- 375 nodes with 512 GB RAM
- 42 nodes with 1 TB RAM
- 2 nodes with 2TB RAM

Fragments shape and orientation

Explosive loading of a sherical container

S. Levy, J.-F. Molinari. *Dynamic Fragmentation of Ceramics, Signature of Defects and Scaling of Fragment Sizes*. Journal of the Mechanics and Physics of Solids. **58**(1),12-26. (2010)

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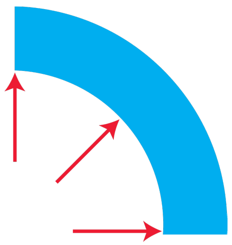
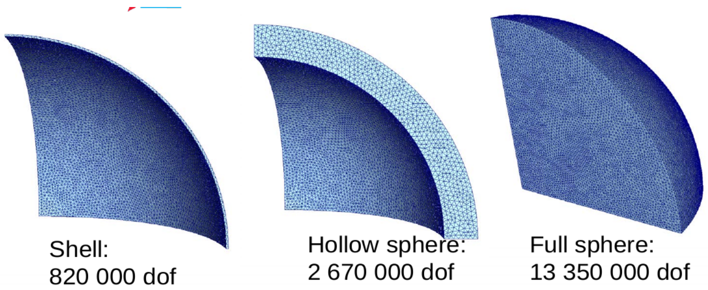
M. Vocialta, J.-F. Molinari. *Influence of Internal Impacts between Fragments in Dynamic Brittle Tensile Fragmentation*. International Journal of Solids and Structures. **58**,247-256. (2015)

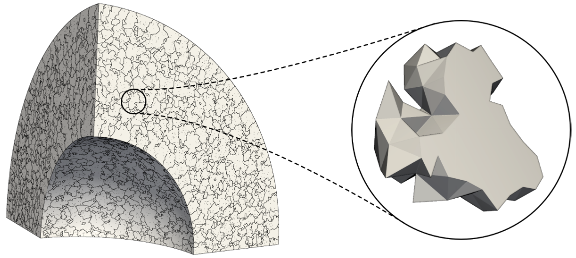
Material parameters:

- E=370 GPa
- ρ=3900 kg/m3
- Gc=1N/m
- Weibull distribution of σc (mw=2 and λ=50MPa)

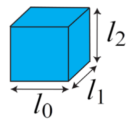


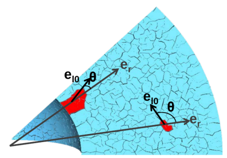


Fragments shape and orientation



Fragment shape? Fragment orientation?





Fragments shape and orientation: Effect of membrane thickness

Shell:

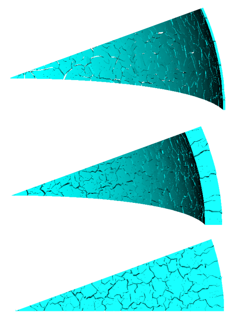
- 2D fragments ($l_2 \approx 0$ & l_0, l_1 random)
- θ ≈ π/2

Thin membrane:

- 3D fragments (l_0, l_1, l_2 random)
- θ ≈ 0
- Crack branching if thickness sufficiently large

Full sphere:

- 3D fragments (l_0, l_1, l_2 random)
- θ random



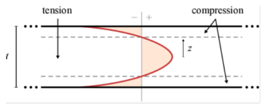
Tempered glass fragmentation;

Vocialta Corrado Molinari; Eng. Frac. Mech., 2018

Tempered glass must shatter in small fragments for safety reasons

tonischildersbedrijf.nl

Thermal temper:

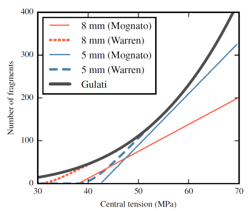


$$\sigma_x = \sigma_y = f(z)$$
$$= \sigma_{CT}\left[1 - 12\left(\frac{z}{t}\right)^2\right]$$

M. Vocialta, M. Corrado, J.-F. Molinari. *Numerical Analysis of Fragmentation in Tempered Glass with Parallel Dynamic Insertion of Cohesive Elements*. Engineering Fracture Mechanics. **188**,448-469. (2018)

Motivation

Discrepancy between analytical formulas and experiments



Analytical models based on energy balance

$$E_{diss} = rE_{pot}$$

- Gulati (1997)

$$r = const \Rightarrow N_{frag} = f(\sigma_{CT})$$

- Warren (2001)

$$r = g(t) \Rightarrow N_{frag} = f(\sigma_{CT}, t)$$

Empirical models

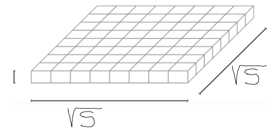
- Mognato et al. (2011): best fitting curves of **1752** tests
- Gives the ration of energy conversion into fragments

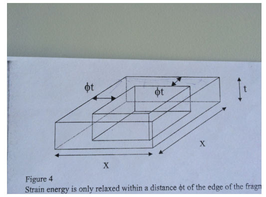
- Gulati S (1997) Proceedings of Glass Performance Days, 13-15.
- Warren P. (2001) Fractography of Glasses and Ceramics IV, 389-400.
- Mognato E., Barbieri A., Schiavonato M., Pace M. (2011) Proceedings of Glass Performance Days, 115-118

Analytical model

Energy balance in statics assuming squared fragments

Numerical model



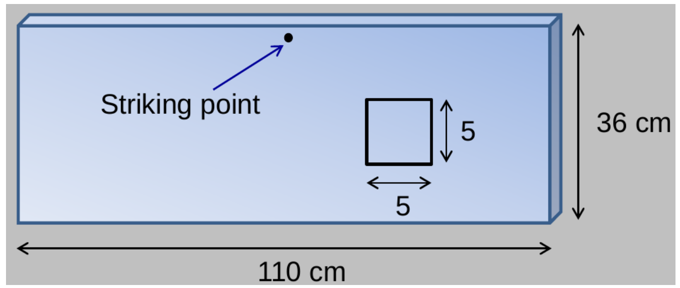


Idea:

- E_{pot}: parabolic law based on pre-stressed profile
- E_{diss}: fracture energy integrated over all squares

$$E_{diss} = 2G_c t\sqrt{S}\left(\sqrt{N_{frag}} - 1\right) \quad \text{and} \quad E_{pot} = \frac{4}{5}\frac{1-\nu}{E}\sigma_{CT}^2 tS$$
$$\Rightarrow E_{diss} = rE_{pot} \Rightarrow N_{frag} = \left(\frac{2r}{5G_c}\frac{1-\nu}{E}\sigma_{CT}^2\sqrt{S} + 1\right)^2$$

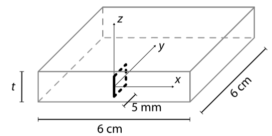
Standard fragmentation test (EN 12150-1)



Finite element model

Up to 4 million elements

- Phase 1: parabolic eigenstresses are applied
- Phase 2: a notch is created, to simulate a crack coming from the impact point
- Phase 3: fragmentation driven by the eigenstresses

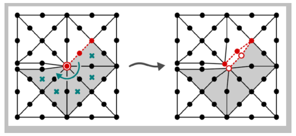


Insertion strategy

Crack propagation is simulated with the **cohesive element method**

⇓

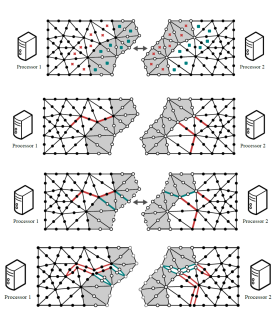
Cohesive elements are dynamically inserted along the borders of the standard elements

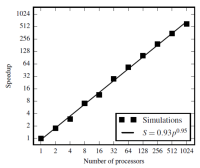


- Mesh: quadratic tetrahedral elements, average size = 0.3 mm
- Explicit central difference integration scheme
- Time step: $\Delta_{crit} = 0.2 min_{el} \frac{h_{el}}{c} \simeq 10^{-8} s$
- Algorithm implemented in the open-source FE library **Akantu** (http://lsms.epfl.ch/akantu)
- Simulations were run in parallel on **192 processors**

Vocialta M (2015) High Performance Computing Simulations of Dynamic Fragmentation in Brittle Materials. PhD Thesis, EPFL

Parallel dynamic insertion of cohesive el.





M. Vocialta. *High Performance Computing Simulations of Dynamic Fragmentation in Brittle Materials*. EPFL (Lausanne), 2015.

Constitutive laws

Soda-Lime-Silicate glass

Linear elastic bulk:

$E = 70GPa$

$\nu = 0.22$ $\rho = 2500 kg/m3$

Cohesive law:

$\sigma_c = 70MPa$

$G_c = 7.6 J/m2 (KIC = 0.75 MPam1$
$/2)$

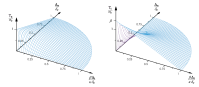
$\beta = 3$

$$\vec{T} = \left(\frac{\beta^2}{\kappa}\Delta_t \vec{t} + \Delta_n \vec{n}\right)\frac{\sigma_c}{\delta}\left(1 - \frac{\delta}{\delta_c}\right)$$

$$\delta = \sqrt{\beta^2 \Delta_t^2 + \Delta_n^2}$$

$$\beta = \frac{\tau_c}{\sigma_c}$$

$$\sigma_{eq} = \sqrt{\frac{t_s^2}{\beta^2} + t_n^2}$$



G. Camacho, M. Ortiz. *Computational Modelling of Impact Damage in Brittle Materials*. International Journal of Solids and Structures. **33**(20- 22),2899-2938. (1996)

Results of the numerical simulations

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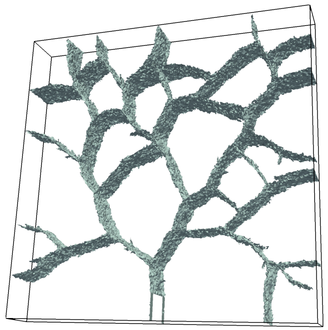
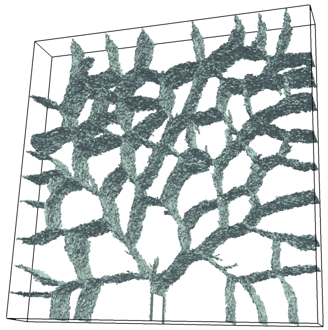
Results of the numerical simulations

The number of fragments is proportional to the eigenstress



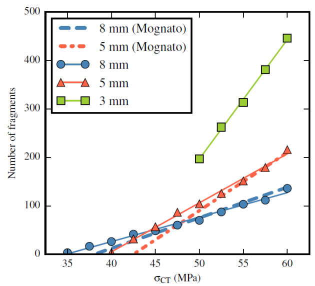
$\sigma_{CT} = 37.5MPa$



$\sigma_{CT} = 42.5MPa$

Number of fragments

Numerical results match reasonably well experimental data



https://doi.org/10.1016/j.engfracmech.2017.09.015 or vocialta thesis

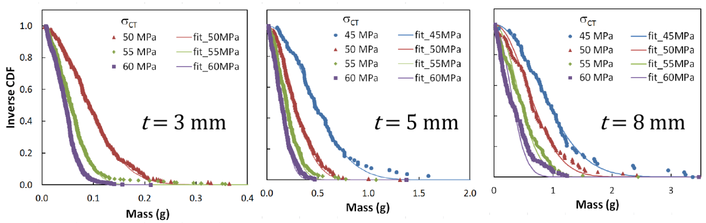
σ_c is the only parameter tuned to fit the experimental data

$\Downarrow$

$\sigma_c = 70$ MPa for all the simulations

Fragments mass distribution

Inverse cumulative distribution function (CDF) (Grady & Kipp, 1985)



$$N(m) = \exp\left[-\left(\frac{m}{\mu}\right)^2\right]$$

Fitting parameter $\mu \simeq m_{average}$

Weibull Probability Density Function with modulus 2:

$$pdf(m) = \frac{2m}{\mu^2}\exp\left[-\left(\frac{m}{\mu}\right)^2\right]$$

Analysis of the energy fields

The simulations are run up to a steady-state condition

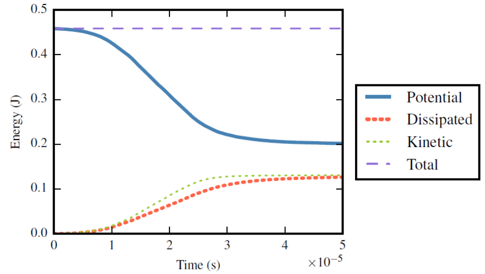
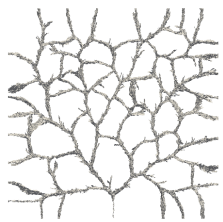


Figure 1β: Energies over time for the plate with thickness 8 mm and σ_{CT} = 45 MPa.

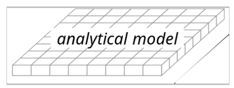
$$t = 8mm, \sigma_{CT} = 45MPa$$

https://doi.org/10.1016/j.engfracmech.2017.09.015 or vocialta thesis

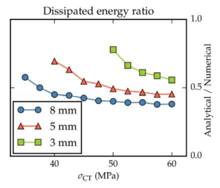
Analysis of the energy fields



Nfrag from simulations



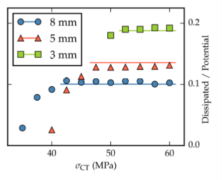
analytical model



The analytical dissipated energy is half of the numerical one

Energy conversion factor

The ratios of analytical dissipated and potential energies reach a plateau



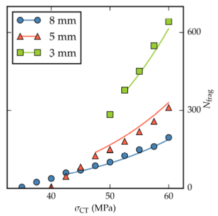
$$r = \frac{E_{diss}}{E_{pot}} \propto t^{-\alpha}$$

$$t \simeq 2/3$$

Energy conversion factor

With this ratio an accurate number of fragments is estimated

$$E_{diss} = E_{pot} r$$
$$\Downarrow$$
$$N_{frag} = \left(\frac{2r}{5G_c} \frac{1-\nu}{E} \sigma_{CT}^2 \sqrt{S} + 1 \right)^2$$

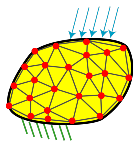


Summary: Fragmentation with HPC-FEM in highly non-linear damage problems

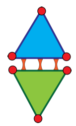
Reliable and efficient numerical framework:

Continuous response simulated with the FEM

Material heterogeneity and failure through cohesive approach
Parallel simulations and significant computational power



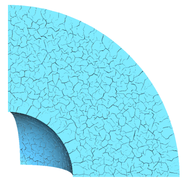
Discretization of a continuum body



Cohesive methodology

Summary: Fragment analysis

- Converged results
- Average fragment size depends on material parameters, defects, and loading: can be quantified
- Two regimes (quasi-static and dynamic): transition quantified
- Smaller fragments than Grady's prediction, but -2/3 scaling law accurate
- Membrane explosion:
  - 2D fragments only if shell
  - random size and orientation of cracks when far from boundaries
- Tempered glass fragmentation example



Fragmented sphere

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