Computational Fracture Mechanics with FE method

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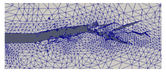
FE Numerical methods for fracture: wide choice

Cohesive element (Camacho, Ortiz, Pandolfi, Needleman ...)

(+) Crack description closest to fract. mech., contact resolution

(-) Mesh dependency



Zhou, Molinari and Shioya, Eng. Fract. Mech. (2005)

G. Camacho, M. Ortiz. *Computational Modelling of Impact Damage in Brittle Materials*. International Journal of Solids and Structures. **33**(20- 22),2899-2938. (1996)

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X.-P. Xu, A. Needleman. *Numerical Simulations of Fast Crack Growth in Brittle Solids*. Journal of the Mechanics and Physics of Solids. **42**(9),1397-1434. (1994)

X-FEM (Belytschko, Moës, ...)

(+) No mesh dependency

(-) Need ad hoc criterion for crack branching

Song, Wang and Belytschko, Comput. Mech. (2008)

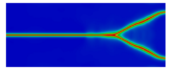
N. Moës, J. Dolbow, T. Belytschko. *A Finite Element Method for Crack Growth without Remeshing*. International Journal for Numerical Methods in Engineering. **46**(1),131-150. (1999)

J.-H. Song, H. Wang, T. Belytschko. *A Comparative Study on Finite Element Methods for Dynamic Fracture*. Computational Mechanics. **42**(2),239-250. (2008)

Phase field (Karma, Miehe, Marigo, Bourdin, ...)

(+) Complex crack description in 2D, 3D

(-) Cost

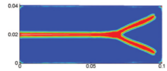


Borden, Verhoosel, Scott, Hughes and Landis, Comput. Methods Appll. Mech. Engrg. (2012)

Peridynamics (Silling, Bobaru, ...)

(+) Meshfree, integro-differential approach

(-) Computational cost ?

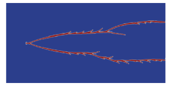


Yho Doh Ha and Bobaru, Int. J. of Fract. (2010)

Classical Non-local continuum damage (Bazant, Pijaudier-Cabot, Jirasek ...)

(+) No mesh dependency, based on realistic damage behavior, easy to implement, well-recognized method but rarely tested in dynamics

(-) Computational cost



Discrete approach: cohesive zone concept, Dugdale Barenblatt, 1960's

- Introduce a cohesive region (process zone) at the crack tip in the form of a length scale
- Eliminate the singularity of stresses at the extended crack tip
- Process zone size l_z not known a priori but **can be estimated in simple cases**

Rice and Palmer estimate:

$$l_z = \frac{9\pi}{32}\frac{EG_c}{(1-\nu^2)\sigma_c^2}$$

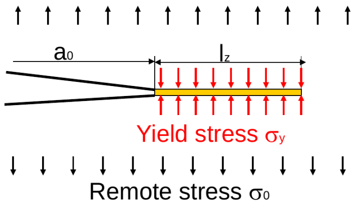
Anderson estimate

$$l_z = \frac{1}{\pi}\left(\frac{K}{\sigma_Y}\right)^2 = \frac{1}{\pi}\frac{EG_c}{\sigma_Y^2}$$

Rq: $G_c \sim K_I^2/E$

A. Palmer, J. Rice, R. Hill. *The Growth of Slip Surfaces in the Progressive Failure of Over-Consolidated Clay*. Proceedings of the Royal Society of London. A. Mathematical and Physical Sciences. **332**(1591),527-548. (1997)

- Example of constant cohesive stress σ_y



FE implementation: cohesive elements: a popular approach

- Camacho Ortiz (2D) 1996, Pandolfi Ortiz (3D) 1999, Xu Needleman 1993, Rose et al., etc...
- Cohesive elements glue two neighboring elements

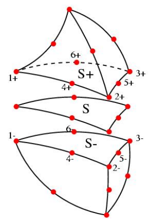
  Cracks created within ordinary elements boundaries (Mesh dependency)

  Computationally expensive ($h_{FE} < l_Z < W$)
- Cracks explicitly described by cohesive elements

  Easy to handle branching, fragmentation

As close as one can be to fracture mechanics
Can incorporate contact

- The opening/closing properties of cohesive elements are governed by a cohesive law



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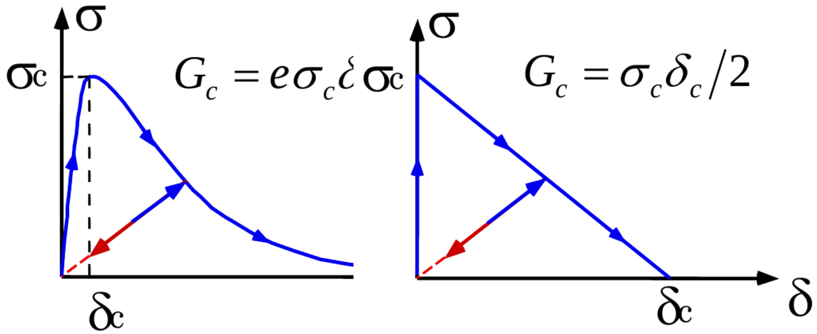
Two examples of cohesive laws: amongst a wide choice...

$$\Rightarrow \text{Opening} : d\delta > 0$$
$$\Rightarrow \text{Closing} : d\delta < 0$$



Exponential Irreversible Model

Physically reasonable but modifies structual response

Linear Irreversible Model

Does not modify elastic properties

Needs to be introduced dynamically

Determination of cohesive law: cohesive parameters

From a potential?

$$\mathbf{T} = \frac{\partial \Phi}{\partial \boldsymbol{\Delta}}$$

Examples

- Xu, Needleman (1993)

$$\Phi(\Delta) = \sigma_c \delta_n \exp\left[1 - \left(1 + \frac{\Delta}{\delta_n}\right) e^{-\frac{\Delta}{\delta_n}}\right]$$
$$\mathbf{T} = \sigma_c \frac{\Delta}{\delta_n} e^{1-\frac{\Delta}{\delta_n}} \mathbf{n}$$

- Camacho, Ortiz (1996)

$$\Phi(\Delta) = \frac{1}{2}\sigma_c \delta \left(2 - \frac{\delta}{\delta_c}\right)$$

Effective opening displacement

$$\delta = \sqrt{\beta^2 \Delta_t^2 + \Delta_n^2}$$
$$\mathbf{T} = \frac{T}{\delta}\left(\beta^2 \Delta_t \mathbf{t} + \Delta_n \mathbf{n}\right)$$

X.-P. Xu, A. Needleman. *Numerical Simulations of Fast Crack Growth in Brittle Solids*. Journal of the Mechanics and Physics of Solids. **42**(9),1397-1434. (1994)

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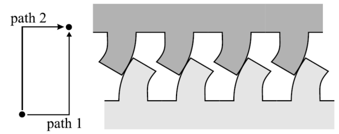
Determination of cohesive law: not from potentials?

- Potential-derived laws can have anomalies

  Ex: Xu - Needleman (wrong predicted crack growth speed)

- And path dependence can be physically rationalized

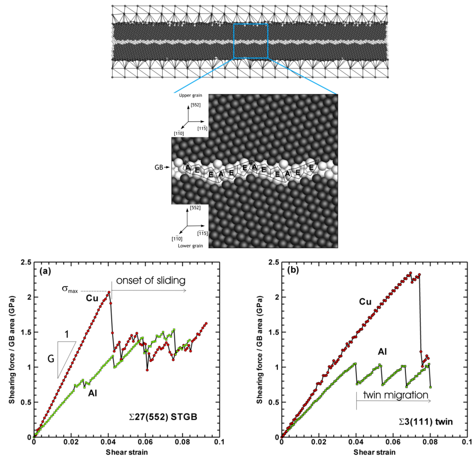


Energy dissipated path 1 > Energy dissipated path 2

- Alternative: direct construction of a cohesive law
  - Bosch, Schreurs, Geers, EFM, 2006

Determination of cohesive law

From atomistic simulations



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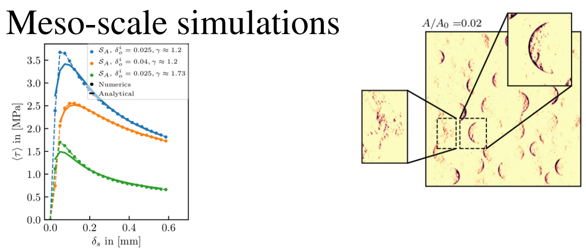
Determination of cohesive law

Elasto-plastic shear resistance in concrete

- Employ the mortar-aggregate structure



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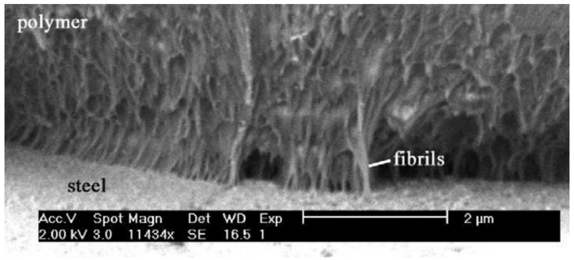
Determination of cohesive law: from experiments

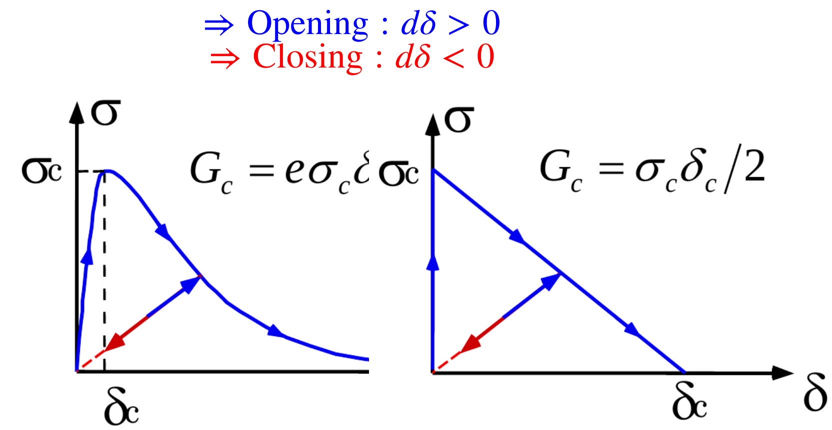
Vast literature: in composites, polymers, metals, ceramics, concrete...



Identification and characterization of delamination in polymer coated metal sheet, van den Bosch, Schreurs, Geers, JMPS, 2008

M. van den Bosch, P. Schreurs, M. Geers. *Identification and Characterization of Delamination in Polymer Coated Metal Sheet*. Journal of the Mechanics and Physics of Solids. **56**(11),3259-3276. (2008)

Back to our two examples: intrinsic versus extrinsic law



Exponential Irreversible Model

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Linear Irreversible Model

Does not modify elastic properties

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Dynamic insertion of cohesive elements: Extrinsic law

- Introduce a cohesive element when
  - the facet opening tractions is larger than a critical stress
  - Mode-I

$$\mathbf{n} \cdot \sigma^{\pm} \cdot \mathbf{n} > \sigma_c$$

- Camacho Ortiz

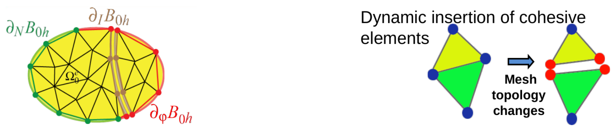
$$\sqrt{\left(\mathbf{n} \cdot \sigma^{\pm} \cdot \mathbf{n}\right)^2 + \frac{1}{\beta^2}\left(\mathbf{n} \cdot \sigma^{\pm} \cdot \mathbf{t}\right)^2} > \sigma_c$$

- General case

$$f(\sigma^+, \sigma^-) > \sigma_c$$

- This is computationally challenging (particularly in parallel): 3D dynamic insertion $\Rightarrow$ keep track of facets, edges, duplicated nodes,..
- Can create stress perturbations
- But results in computational savings

FEM implementation: explicit dynamics, extrinsic approach



Space discretization:

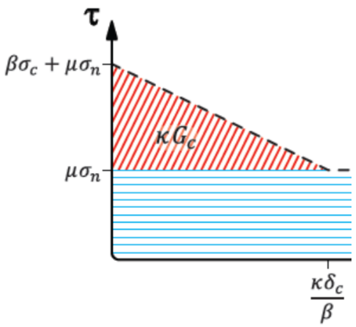
$$B_{0h} = \bigcup_{e=0}^{N} \Omega_0^e$$

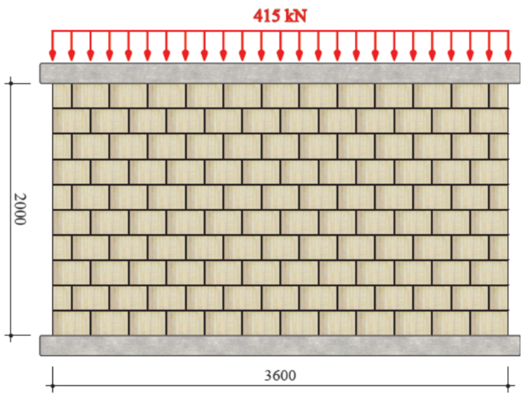
$$\underbrace{\int_{B_{0h}} \left(\rho_0 \ddot{\mathbf{x}}_h \delta \mathbf{x}_h + P_h : \nabla_0 \mathbf{x}_h \delta \mathbf{x}_h\right) dV - \int_{\partial N B_{0h}} \bar{\mathbf{T}} \delta \mathbf{x}_h dS - \int_{B_{0h}} \rho_0 B \delta \mathbf{x}_h dV}_{\text{Continuum term}}$$

$$+ \underbrace{\int_{\partial_I B_{0h}} \mathbf{T}(\llbracket \mathbf{x}_h \rrbracket) \llbracket \delta \mathbf{x}_h \rrbracket dS}_{\text{Cohesive term}} = 0$$

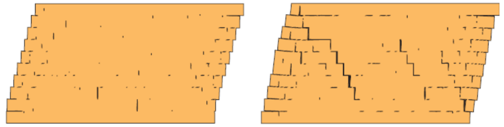
with the *jump operator*: $\llbracket \bullet \rrbracket = \bullet^+ - \bullet^-$

Application 1: coupling with contact: cohesive/Friction law



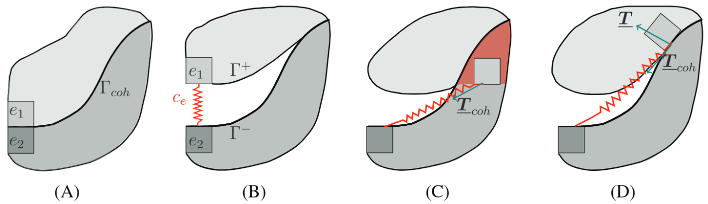


Snozzi, Molinari, IJNME, 2013
Masonry wall loaded in shear at two loading rates



L. Snozzi. *A meso-scale computational approach to dynamic failure in concrete*. EPFL (Lausanne), 2013.

Mesh refinement and contact



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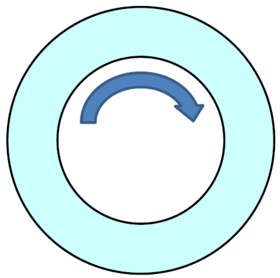
Application 2: ring fragmentation

Illustration of mesh dependency, Zhou Molinari IJNME 2004

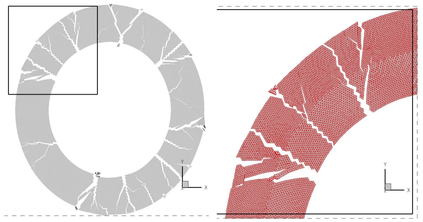
Fragmentation of a rotating brittle ceramic (SiN) ring under centrifugal force



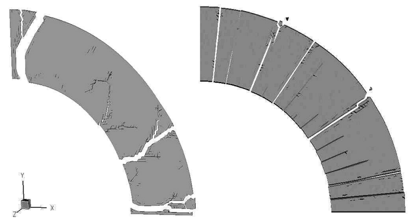
- Material properties: r =3300 Kg/m3 E=320GPa n= 0.3
- Wave Velocities: CL= 9847 m/s, Cs=6107 m/s, CR=5620 m/s
- Fracture Properties: sc =450 Mpa, dc= 0.889 mm, Gc=200 N/m

Computationally challenging! Symmetry ⇒ Mesh dependency of fracture paths likely

Fragmentation patterns: OK results

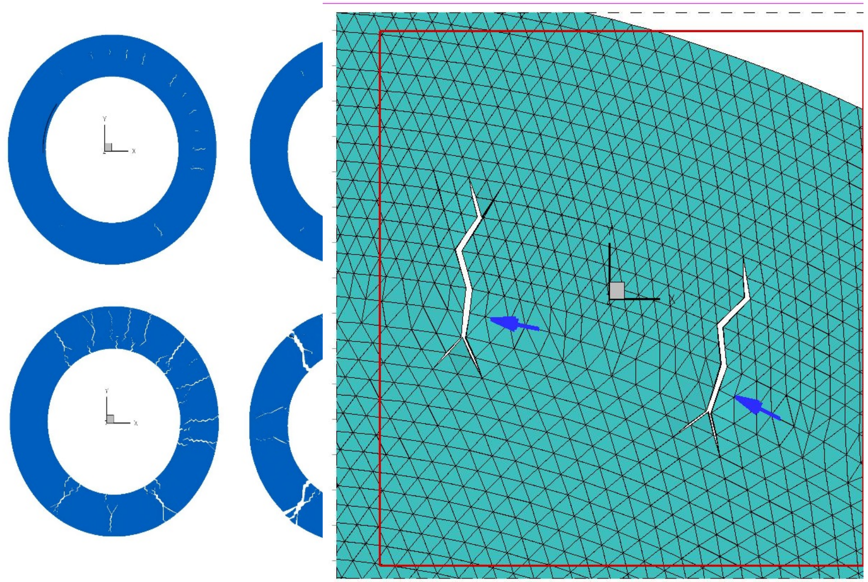


Degenerate fragmentation patterns: not OK results



- Mesh influenced by boundaries
- Partitioned Mesh
- Cracks may propagate in certain preferred orientations

Degenerate fragmentation patterns: not OK results



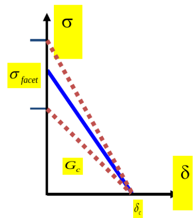
Cracks initiate at small elements perpendicular to loading

Addressing mesh dependency: stochastic cohesive elements (Zhou, Molinari,IJNME, 2004)

- All facets between tetrahedral elements are defects
- A defect is activated as a micro-crack if the tensile stress applied on the facet is larger than the facet strength

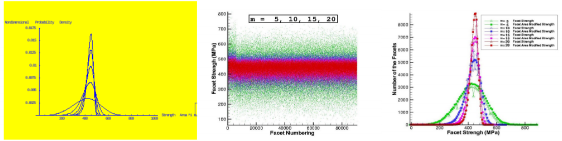
$$\sigma_{local} > \sigma_{facet}$$

- The mechanical behavior of a micro-crack is modeled by a cohesive element
- Facet strength distribution follows *Weibull* theory



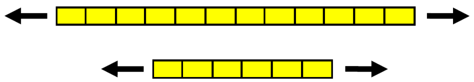
Addressing mesh dependency: Weibull distribution of cohesive strengths

$$P(\bar{\sigma}_{facet}) = 1 - \exp\left[-\left(\frac{\bar{\sigma}_{facet}}{\bar{\sigma}_s}\right)^m\right]$$
$$\frac{dP}{d\bar{\sigma}_{facet}} = \frac{m}{\bar{\sigma}_s}\left(\frac{\bar{\sigma}_{facet}}{\bar{\sigma}_s}\right)^{m-1}\exp\left[-\left(\frac{\bar{\sigma}_{facet}}{\bar{\sigma}_s}\right)^m\right]$$
$$\bar{\sigma}_{facet} = A_{facet}^{1/m}\sigma_{facet}, \qquad \bar{\sigma}_s = A_s^{1/m}\sigma_s$$

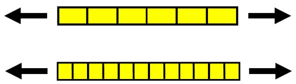


Physical interpretation

Weibull distribution of cohesive strengths



A larger component is easier to break (same element size): physical!

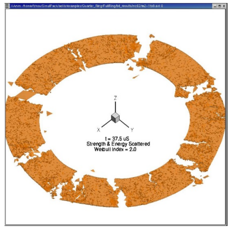


A finer mesh provides more possibilities for the component to break (same component size): numerical artifact!

Mesh-size dependency of the simulation results!
→ Need area dependence in WLWD
(removes ring fragmentation abnormalities)

WLWD ≡ weakest link weibul distribution

Monte Carlo simulations: Weibull distribution of cohesive strengths, m=2;

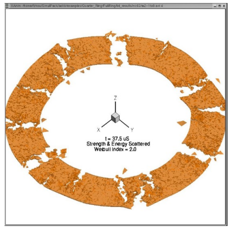


Test #1

Test #2

Test #3

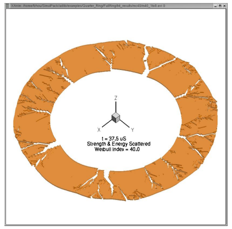
Test #4



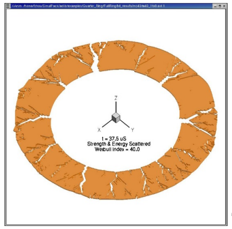
Test #5

Test #6

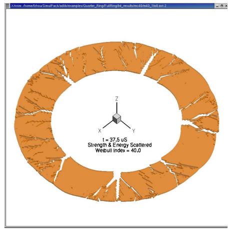
Monte Carlo simulations: Weibull distribution of cohesive strengths, m=40



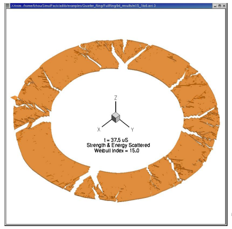
Test #1



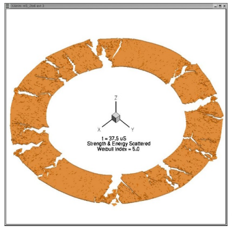
Test #2



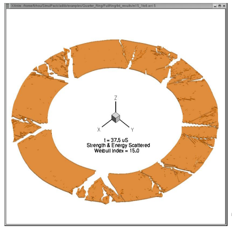
Test #3



Test #4



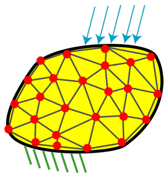
Test #5



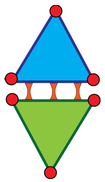
Test #6

Summary: showed examples using FEM in highly non-linear damage problems

- Reliable and efficient numerical framework:
  - Continuous response simulated with the FEM
  - Material heterogeneity and failure through cohesive approach
  - Requires significant computational power: parallel simulations and HPC? After break: dynamic fragmentation



Discretization of a continuum body



Cohesive methodology

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