Computational Fracture Mechanics with FE method

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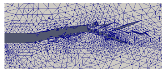
FE Numerical methods for fracture: wide choice

Cohesive element (Camacho, Ortiz, Pandolfi, Needleman ...)

(+) Crack description closest to fract. mech., contact resolution

(-) Mesh dependency



Zhou, Molinari and Shioya, Eng. Fract. Mech. (2005)

G. Camacho, M. Ortiz. *Computational Modelling of Impact Damage in Brittle Materials*. International Journal of Solids and Structures. **33**(20- 22),2899-2938. (1996)

G. Ruiz, M. Ortiz, A. Pandolfi. *Three-Dimensional Finite-Element Simulation of the Dynamic Brazilian Tests on Concrete Cylinders*. International Journal for Numerical Methods in Engineering. **48**(7),963-994. (2000)

X.-P. Xu, A. Needleman. *Numerical Simulations of Fast Crack Growth in Brittle Solids*. Journal of the Mechanics and Physics of Solids. **42**(9),1397-1434. (1994)

X-FEM (Belytschko, Moës, ...)

(+) No mesh dependency

(-) Need ad hoc criterion for crack branching

Song, Wang and Belytschko, Comput. Mech. (2008)

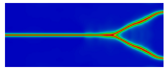
N. Moës, J. Dolbow, T. Belytschko. *A Finite Element Method for Crack Growth without Remeshing*. International Journal for Numerical Methods in Engineering. **46**(1),131-150. (1999)

J.-H. Song, H. Wang, T. Belytschko. *A Comparative Study on Finite Element Methods for Dynamic Fracture*. Computational Mechanics. **42**(2),239-250. (2008)

Phase field (Karma, Miehe, Marigo, Bourdin, ...)

(+) Complex crack description in 2D, 3D

(-) Cost

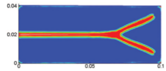


Borden, Verhoosel, Scott, Hughes and Landis, Comput. Methods Appll. Mech. Engrg. (2012)

Peridynamics (Silling, Bobaru, ...)

(+) Meshfree, integro-differential approach

(-) Computational cost ?

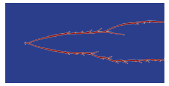


Yho Doh Ha and Bobaru, Int. J. of Fract. (2010)

Classical Non-local continuum damage (Bazant, Pijaudier-Cabot, Jirasek ...)

(+) No mesh dependency, based on realistic damage behavior, easy to implement, well-recognized method but rarely tested in dynamics

(-) Computational cost



Discrete approach: cohesive zone concept, Dugdale Barenblatt, 1960's

- Introduce a cohesive region (process zone) at the crack tip in the form of a length scale
- Eliminate the singularity of stresses at the extended crack tip
- Process zone size l_z not known a priori but **can be estimated in simple cases**

Rice and Palmer estimate:

$$l_z = \frac{9\pi}{32} \frac{EG_c}{(1-\nu^2)\sigma_c^2}$$

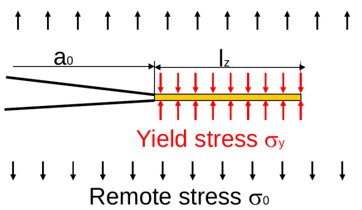
Anderson estimate

$$l_z = \frac{1}{\pi}\left(\frac{K}{\sigma_Y}\right)^2 = \frac{1}{\pi}\frac{EG_c}{\sigma_Y^2}$$

Rq: $G_c \sim K_I^2/E$

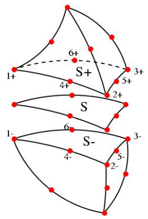
A. Palmer, J. Rice, R. Hill. *The Growth of Slip Surfaces in the Progressive Failure of Over-Consolidated Clay*. Proceedings of the Royal Society of London. A. Mathematical and Physical Sciences. **332**(1591),527-548. (1997)

- Example of constant cohesive stress σ_y



FE implementation: cohesive elements: a popular approach

- Camacho Ortiz (2D) 1996, Pandolfi Ortiz (3D) 1999, Xu Needleman 1993, Rose et al., etc...



- Cohesive elements glue two neighboring elements

  Cracks created within ordinary elements boundaries (Mesh dependency)

  Computationally expensive ($h_{FE} < l_Z < W$)

- Cracks explicitly described by cohesive elements

  Easy to handle branching, fragmentation

As close as one can be to fracture mechanics
Can incorporate contact

- The opening/closing properties of cohesive elements are governed by a cohesive law

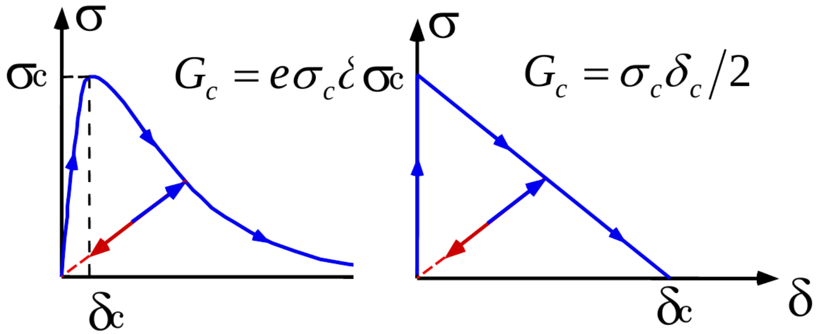
G. Camacho, M. Ortiz. *Computational Modelling of Impact Damage in Brittle Materials*. International Journal of Solids and Structures. **33**(20- 22),2899-2938. (1996)

X.-P. Xu, A. Needleman. *Numerical Simulations of Fast Crack Growth in Brittle Solids*. Journal of the Mechanics and Physics of Solids. **42**(9),1397-1434. (1994)

M. Ortiz, A. Pandolfi. *Finite-Deformation Irreversible Cohesive Elements for Three-Dimensional Crack-Propagation Analysis*. International Journal for Numerical Methods in Engineering. **44**(9),1267-1282. (1999)

Two examples of cohesive laws: amongst a wide choice...

$$\Rightarrow \text{Opening} : d\delta > 0$$
$$\Rightarrow \text{Closing} : d\delta < 0$$



Exponential Irreversible Model

Physically reasonable but modifies structual response

Linear Irreversible Model

Does not modify elastic properties

Needs to be introduced dynamically

Determination of cohesive law: cohesive parameters

From a potential?

$$\mathbf{T} = \frac{\partial \Phi}{\partial \boldsymbol{\Delta}}$$

Examples

- Xu, Needleman (1993)

$$\Phi(\Delta) = \sigma_c \delta_n \exp\left[1 - \left(1 + \frac{\Delta}{\delta_n}\right) e^{-\frac{\Delta}{\delta_n}}\right]$$
$$\mathbf{T} = \sigma_c \frac{\Delta}{\delta_n} e^{1-\frac{\Delta}{\delta_n}} \mathbf{n}$$

- Camacho, Ortiz (1996)

$$\Phi(\Delta) = \frac{1}{2}\sigma_c \delta \left(2 - \frac{\delta}{\delta_c}\right)$$

Effective opening displacement

$$\delta = \sqrt{\beta^2 \Delta_t^2 + \Delta_n^2}$$
$$\mathbf{T} = \frac{T}{\delta}\left(\beta^2 \Delta_t \mathbf{t} + \Delta_n \mathbf{n}\right)$$

X.-P. Xu, A. Needleman. *Numerical Simulations of Fast Crack Growth in Brittle Solids*. Journal of the Mechanics and Physics of Solids. **42**(9),1397-1434. (1994)

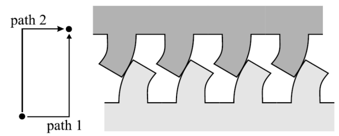
G. Camacho, M. Ortiz. *Computational Modelling of Impact Damage in Brittle Materials*. International Journal of Solids and Structures. **33**(20- 22),2899-2938. (1996)

Determination of cohesive law: not from potentials?

- Potential-derived laws can have anomalies

  Ex: Xu - Needleman (wrong predicted crack growth speed)

- And path dependence can be physically rationalized

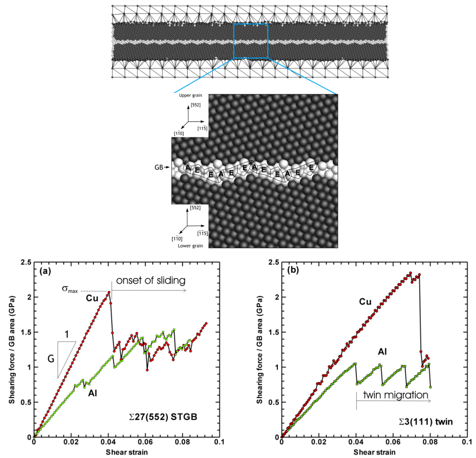


Energy dissipated path 1 > Energy dissipated path 2

- Alternative: direct construction of a cohesive law
  - Bosch, Schreurs, Geers, EFM, 2006

Determination of cohesive law

From atomistic simulations

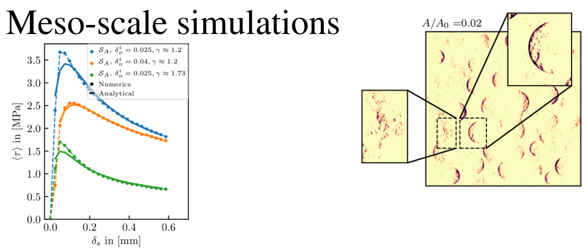


F. Sansoz, J. Molinari. *Mechanical Behavior of Σ Tilt Grain Boundaries in Nanoscale Cu and Al: A Quasicontinuum Study*. Acta Materialia. **53**(7),1931-1944. (2005)

Determination of cohesive law

Elasto-plastic shear resistance in concrete

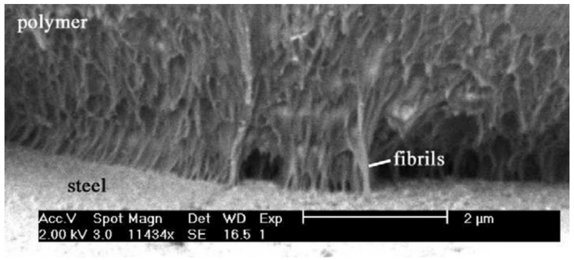
- Employ the mortar-aggregate structure



M. Pundir, G. Anciaux. *Numerical Generation and Contact Analysis of Rough Surfaces in Concrete*. Journal of Advanced Concrete Technology. **19**(7),864-885. (2021)

Determination of cohesive law: from experiments

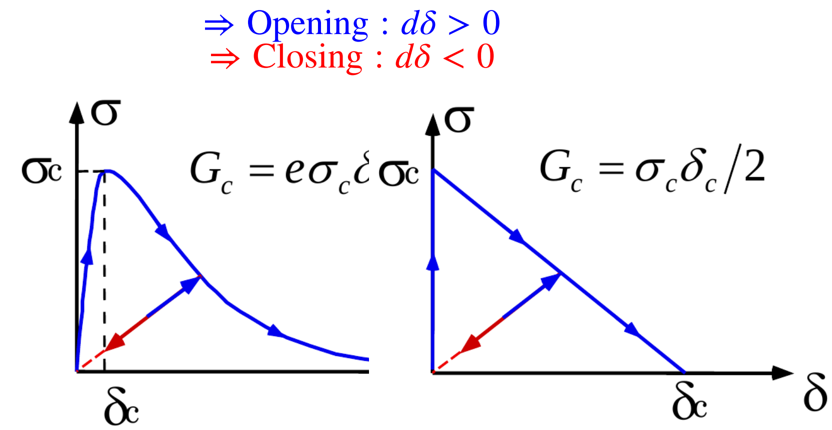
Vast literature: in composites, polymers, metals, ceramics, concrete…



Identification and characterization of delamination in polymer coated metal sheet, van den Bosch, Schreurs, Geers, JMPS, 2008

M. van den Bosch, P. Schreurs, M. Geers. *Identification and Characterization of Delamination in Polymer Coated Metal Sheet*. Journal of the Mechanics and Physics of Solids. **56**(11),3259-3276. (2008)

Back to our two examples: intrinsic versus extrinsic law



Exponential Irreversible Model

Physically reasonable but modifies structural response

Linear Irreversible Model

Does not modify elastic properties

Needs to be introduced dynamically

Dynamic insertion of cohesive elements: Extrinsic law

- Introduce a cohesive element when
  - the facet opening tractions is larger than a critical stress
  - Mode-I

$$\mathbf{n} \cdot \sigma^{\pm} \cdot \mathbf{n} > \sigma_c$$

- Camacho Ortiz

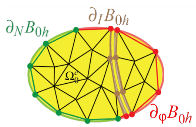
$$\sqrt{\left(\mathbf{n} \cdot \sigma^{\pm} \cdot \mathbf{n}\right)^2 + \frac{1}{\beta^2}\left(\mathbf{n} \cdot \sigma^{\pm} \cdot \mathbf{t}\right)^2} > \sigma_c$$

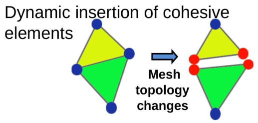
- General case

$$f(\sigma^+, \sigma^-) > \sigma_c$$

- This is computationally challenging (particularly in parallel):
  3D dynamic insertion ⇒ keep track of facets, edges, duplicated nodes,..
- Can create stress perturbations
- But results in computational savings

FEM implementation: explicit dynamics, extrinsic approach





Space discretization:

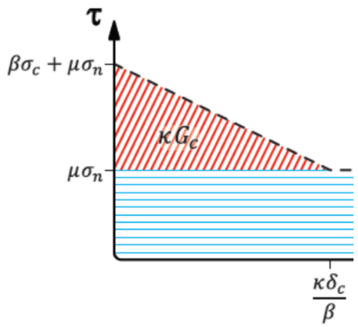
$$B_{0h} = \bigcup_{e=0}^{N} \Omega_0^e$$

$$\underbrace{\int_{B_{0h}} (\rho_0 \ddot{\mathbf{x}}_h \delta\mathbf{x}_h + P_h : \nabla_0 \mathbf{x}_h \delta\mathbf{x}_h)\, dV - \int_{\partial N B_{0h}} \bar{\mathbf{T}} \delta\mathbf{x}_h dS - \int_{B_{0h}} \rho_0 B \delta\mathbf{x}_h dV}_{\text{Continuum term}}$$

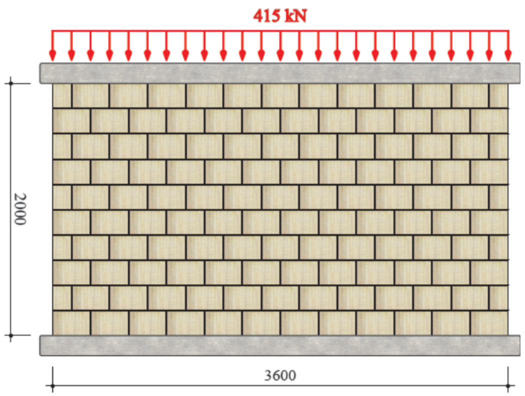
$$+ \underbrace{\int_{\partial_I B_{0h}} \mathbf{T}([\![\mathbf{x}_h]\!]) [\![\delta\mathbf{x}_h]\!] dS}_{\text{Cohesive term}} = 0$$

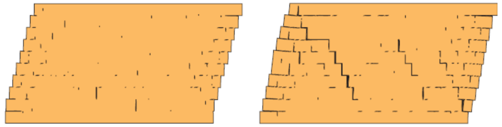
with the *jump operator*: $[\![\bullet]\!] = \bullet^+ - \bullet^-$

Application 1: coupling with contact: cohesive/Friction law



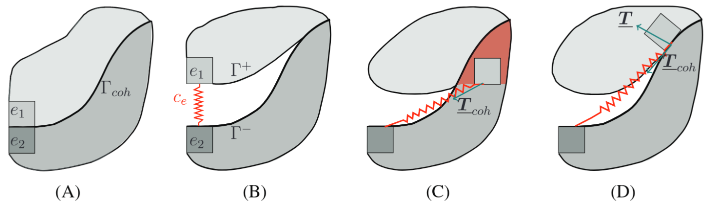
Snozzi, Molinari, IJNME, 2013
Masonry wall loaded in shear at two loading rates





L. Snozzi. *A meso-scale computational approach to dynamic failure in concrete*. EPFL (Lausanne), 2013.

Mesh refinement and contact

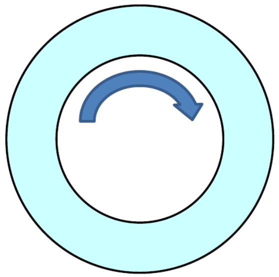


M. Pundir, G. Anciaux. *Coupling between Cohesive Element Method and Node-to-Segment Contact Algorithm: Implementation and Application*. International Journal for Numerical Methods in Engineering. **122**(16),4333-4353. (2021)

Application 2: ring fragmentation

Illustration of mesh dependency, Zhou Molinari IJNME 2004

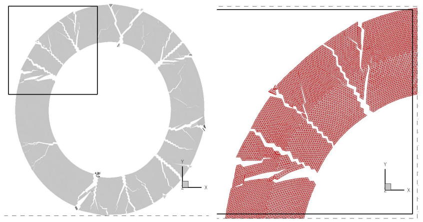
Fragmentation of a rotating brittle ceramic (SiN) ring under centrifugal force



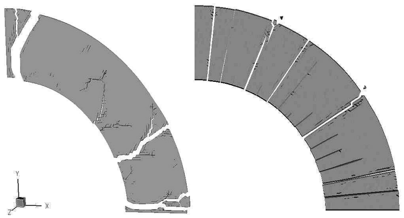
- Material properties: r =3300 Kg/m3 E=320GPa n= 0.3
- Wave Velocities: CL= 9847 m/s, Cs=6107 m/s, CR=5620 m/s
- Fracture Properties: sc =450 Mpa, dc= 0.889 mm, Gc=200 N/m

Computationally challenging! Symmetry $\Rightarrow$ Mesh dependency of fracture paths likely

Fragmentation patterns: OK results

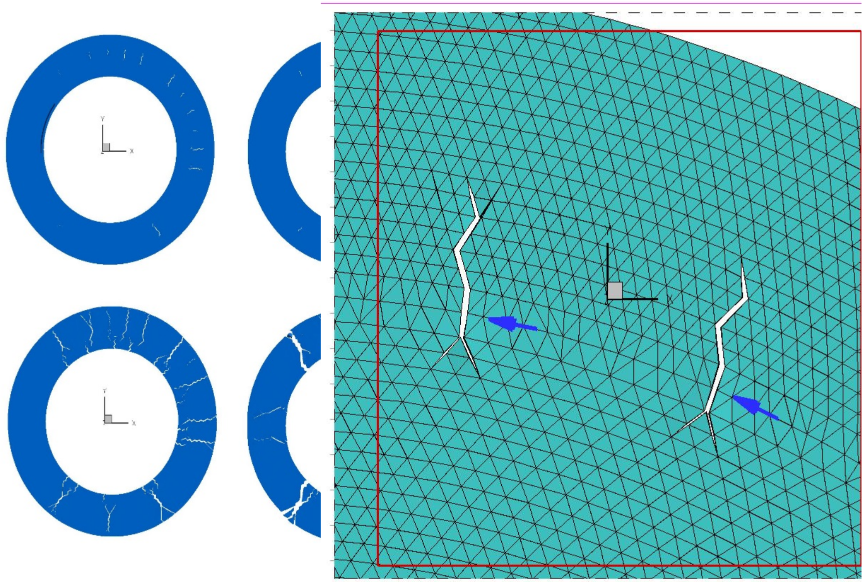


Degenerate fragmentation patterns: not OK results



- Mesh influenced by boundaries
- Partitioned Mesh
- Cracks may propagate in certain preferred orientations

Degenerate fragmentation patterns: not OK results



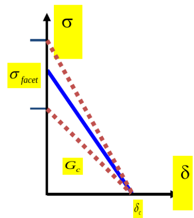
Cracks initiate at small elements perpendicular to loading

Addressing mesh dependency: stochastic cohesive elements (Zhou, Molinari,IJNME, 2004)

- All facets between tetrahedral elements are defects
- A defect is activated as a micro-crack if the tensile stress applied on the facet is larger than the facet strength

$$\sigma_{local} > \sigma_{facet}$$

- The mechanical behavior of a micro-crack is modeled by a cohesive element
- Facet strength distribution follows *Weibull* theory

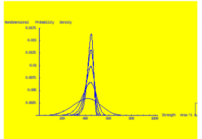


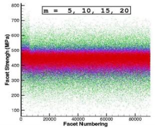
Addressing mesh dependency: Weibull distribution of cohesive strengths

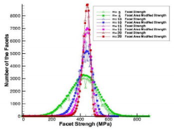
$$P(\bar{\sigma}_{facet}) = 1 - \exp\left[-\left(\frac{\bar{\sigma}_{facet}}{\bar{\sigma}_s}\right)^m\right]$$

$$\frac{dP}{d\bar{\sigma}_{facet}} = \frac{m}{\bar{\sigma}_s}\left(\frac{\bar{\sigma}_{facet}}{\bar{\sigma}_s}\right)^{m-1} \exp\left[-\left(\frac{\bar{\sigma}_{facet}}{\bar{\sigma}_s}\right)^m\right]$$

$$\bar{\sigma}_{facet} = A_{facet}^{1/m}\sigma_{facet}, \qquad \bar{\sigma}_s = A_s^{1/m}\sigma_s$$

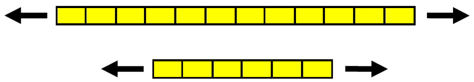




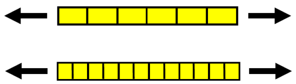


Physical interpretation

Weibull distribution of cohesive strengths



A larger component is easier to break (same element size): physical!

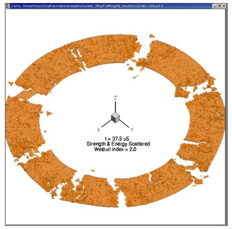


A finer mesh provides more possibilities for the component to break (same component size): numerical artifact!

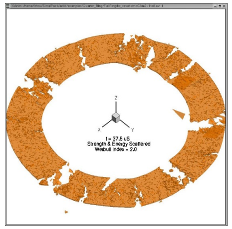
Mesh-size dependency of the simulation results!
→ Need area dependence in WLWD
(removes ring fragmentation abnormalities)

WLWD ≡ weakest link weibul distribution

Monte Carlo simulations: Weibull distribution of cohesive strengths, m=2;

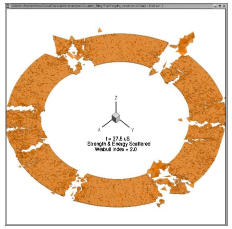


Test #1

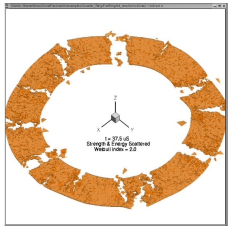


Test #2

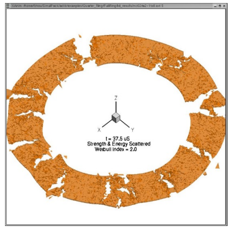
Test #3



Test #4



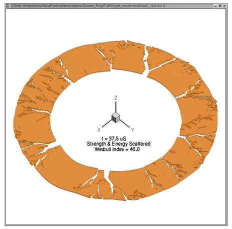
Test #5



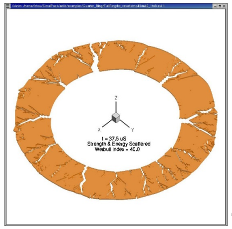
Test #6

Monte Carlo simulations: Weibull

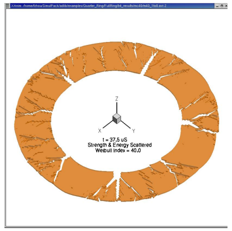
distribution of cohesive strengths, m=40



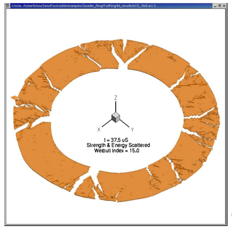
Test #1



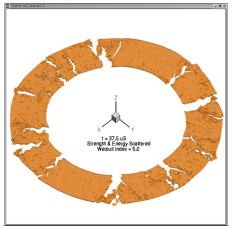
Test #2



Test #3



Test #4



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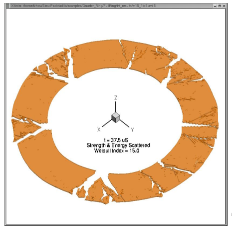
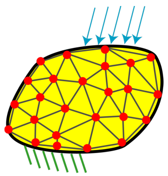


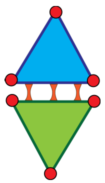
Test #6

Summary: showed examples using FEM in highly non-linear damage problems

- Reliable and efficient numerical framework:
  - Continuous response simulated with the FEM
  - Material heterogeneity and failure through cohesive approach
  - Requires significant computational power: parallel simulations and HPC? After break: dynamic fragmentation



Discretization of a continuum body



Cohesive methodology

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