Dynamic fragmentation simulations

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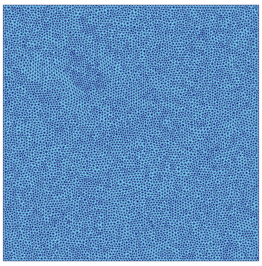
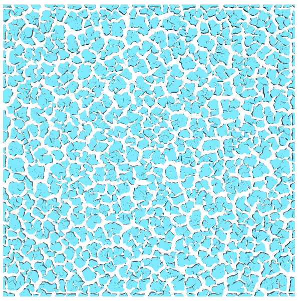
Numerical examples: fragmentation of a plate



Biaxial loading of ceramic plate



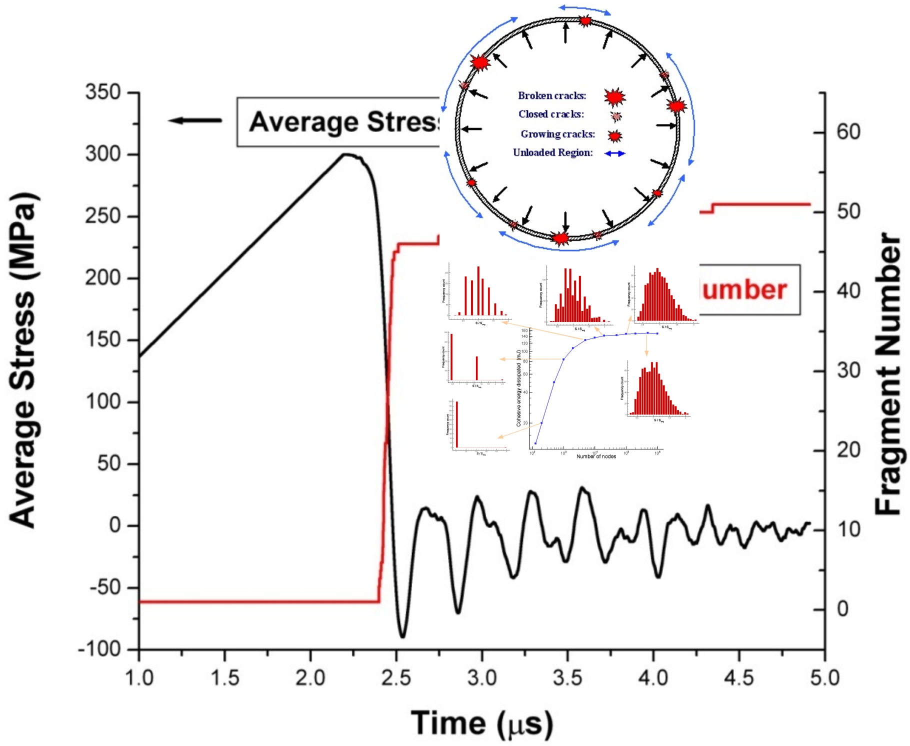
And fragmentation

Mott’s problem

Zhou Molinari Ramesh 2007, Levy Molinari 2010

- Simplest test (expanding ring)
- Nonetheless, complex…

TODO: ask jf is it 3D ? is it parallel ?

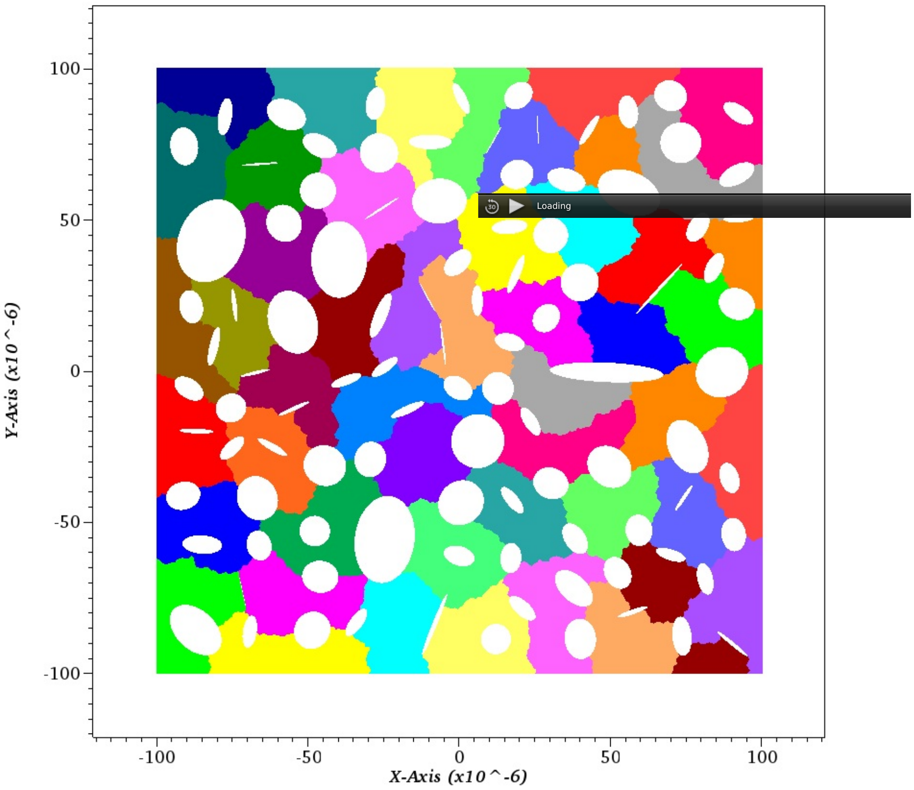


- Ceramic ring length: L= 50 mm
- Elastic parameters: r =2750 Kg/m3, E=250 GPa , c= 10000 m/s
- Fracture parameters: sc =300 MPa, dc= 0.667 mm, Gc=100 N/m

(small variation around mean)

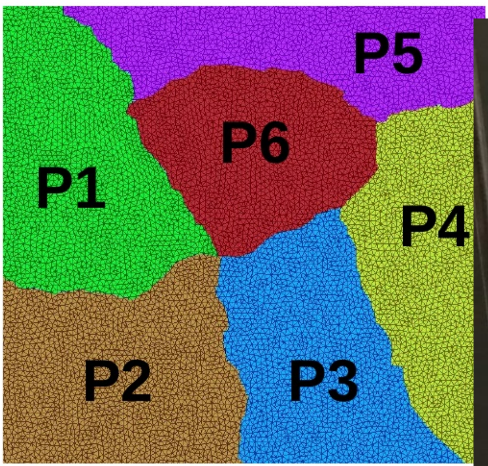
Secondary waves effect: Illustration with mesh partitioning, M. Vocialta

- Ellipsoidal voids, biaxial expansion, initial strain rate: 105 s-1
- E = 320 x 103 MPa ν = 0.237 ρ = 3250 kg/m3 σc = 200 MPa

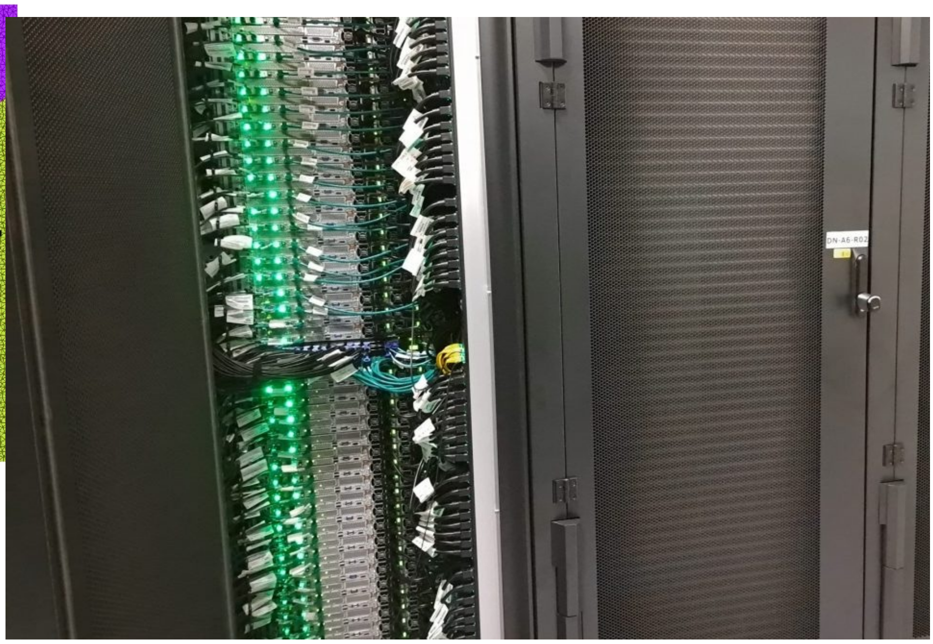


Extension to 3D: High Performance Computing

- HPC necessary to achieve convergence:
  e.g. accurate representation of microcracks
- Parallelisation through domain decomposition
- Problem: topological changes for extrinsic cohesive elements



Domain decomposition



Jed at EPFL

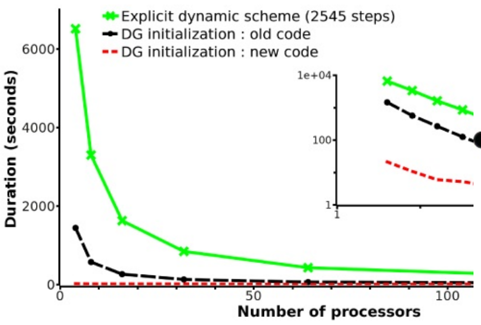
- 30’240 cores
- 375 nodes with 512 GB RAM
- 42 nodes with 1 TB RAM
- 2 nodes with 2TB RAM

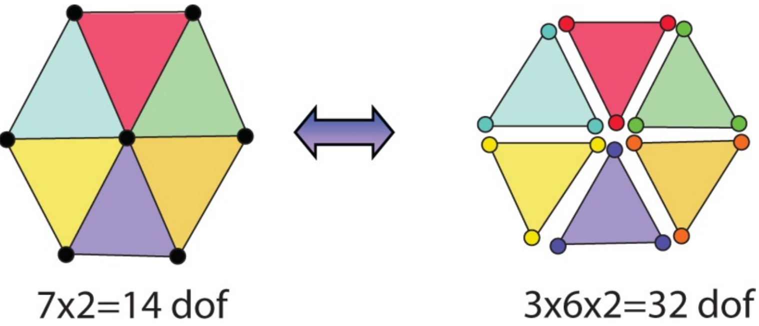
TODO: ask JF why not putting the results of Richart with non local damage ? For the conclusion, why and when should we use non local damage versus cohesive insertion or phase field ? up to him ? Is it still active research ?

An alternative to the FEM: the DGM

Radovitzky and coworkers (MIT)

- Models accurately stress waves
- Includes naturally discontinuities, cracks
- BUT: requires more memory
- BUT: is easily parallelizable and scalable





DGM & cohesive approach

Variational formulation (without failure)

$$\underbrace{\int_{B_{0h}} \left(\rho_0 \ddot{\varphi_h} \delta\varphi_h + P_h : \nabla_0\varphi_h \delta_h \right) dV - \int_{\partial N B_{0h}} \bar{T} \delta \varphi_h dS - \int_{B_{0h}} \rho_0 B \delta\varphi_h dV }_{\text{Continuum term}} + \underbrace{\int_{\partial_I B_{0h}} ⟦\delta \varphi_h⟧\left<P_h\right> dS + \int_{\partial_I B_{0h}} ⟦\varphi_h⟧\otimes N : \left<\frac{\beta_s}{h_s^e}\mathcal{C}\right>:⟦\delta\varphi_h⟧\otimes N dS }_{\text{DG term}} = 0$$

with:

- c^e: wave speed
- β_s: Penalty coefficient
- h_s^e: Characteristic length of the element

- *Jump operator*: $⟦\bullet ⟧ = \bullet^+ - \bullet^-$
- *Mean operator*: $<\bullet> = \frac{\bullet^+ + \bullet^-}{2}$

Noels and Radovitzky, IJNME, 2007

DGM & cohesive approach

Variational formulation (with failure)

$$\underbrace{\int_{B_{0h}} \left(\rho_0 \ddot{\varphi_h} \delta\varphi_h + P_h : \nabla_0\varphi_h \delta_h \right) dV - \int_{\partial N B_{0h}} \bar{T} \delta \varphi_h dS - \int_{B_{0h}} \rho_0 B \delta\varphi_h dV }_{\text{Continuum term}} + \underbrace{(1-\alpha) \int_{\partial_I B_{0h}} ⟦\delta \varphi_h⟧\left<P_h\right> dS + \int_{\partial_I B_{0h}} ⟦\varphi_h⟧\otimes N : \left<\frac{\beta_s}

{h_s^e}\mathcal{C}\right>:⟦\delta\varphi_h⟧\otimes N dS }_{\text{DG term}} + \underbrace{\alpha \int_{\partial_I B_{0h}} T(⟦\varphi_h⟧)⟦\delta\varphi_h⟧ dS}_{\text{Cohesive term}} = 0$$$$\alpha = \left\{ \begin{array}{ll} 0& \quad \text{if } P:⟦N\otimes N⟧ + \beta P: ⟦N\otimes M⟧ - \sigma_c < 0 \\ 1& \quad \text{otherwise} \end{array}\right. $$

Noels and Radovitzky, IJNME, 2007

Numerical implementation

- Equations of motion:

$$\forall \text{node} i, \forall t_n$$$$M\ddot{\varphi}^n_i + f^b(\varphi_i^n) + f_I(\varphi_i^n) = f^{ext}(\varphi_i^n)$$$$f^I = \left\{ \begin{array}{ll} \text{Before failure initiation:}& \text{FEM or DGM}\\ \text{After failure initiation:} & \text{cohesive law} \end{array}\right.$$

Stability condition:

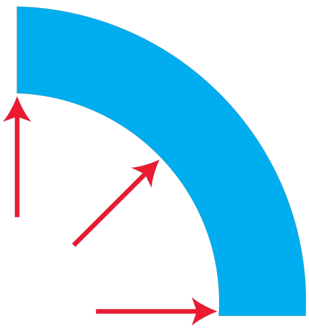
$$\Delta t \leq \left\{ \begin{array}{ll} min_e \frac{h_s^e}{c^e} & \text{for FEM}\\ min_e \frac{h_s^e}{\sqrt{\beta_s}c^e} & \text{for DGM} \end{array}\right.$$

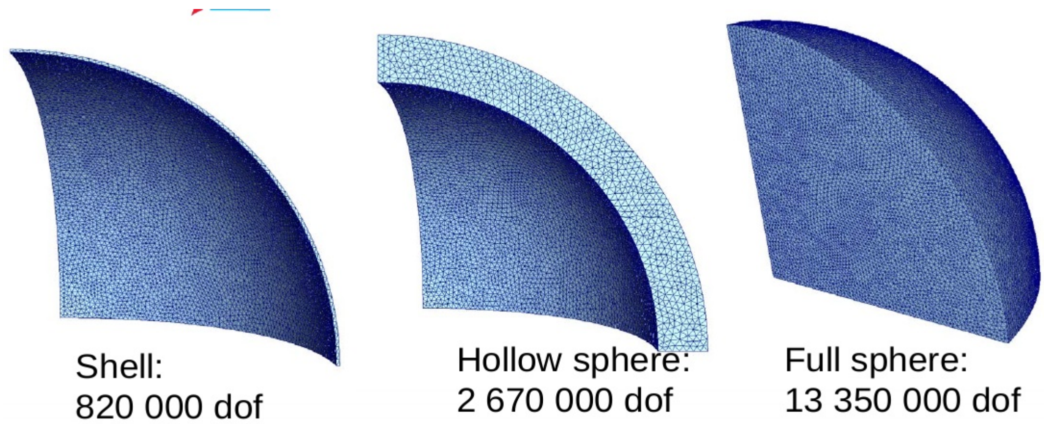
Fragments shape and orientation

Explosive loading of a sherical container

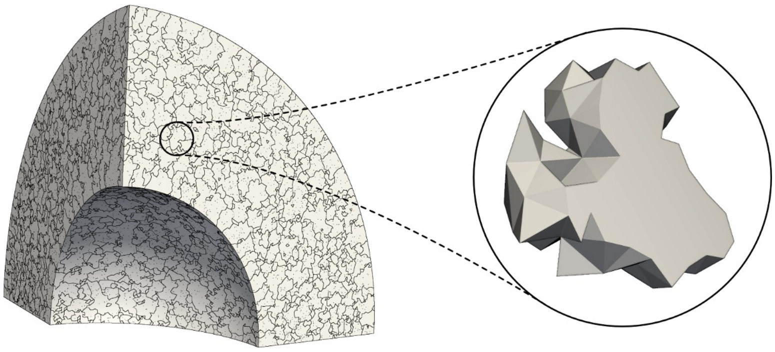
Material parameters:

- E=370 GPa
- ρ=3900 kg/m3
- Gc=1N/m
- Weibull distribution of σ_c (mw=2 and λ=50MPa)

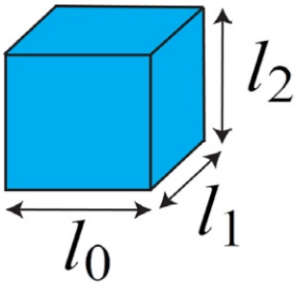


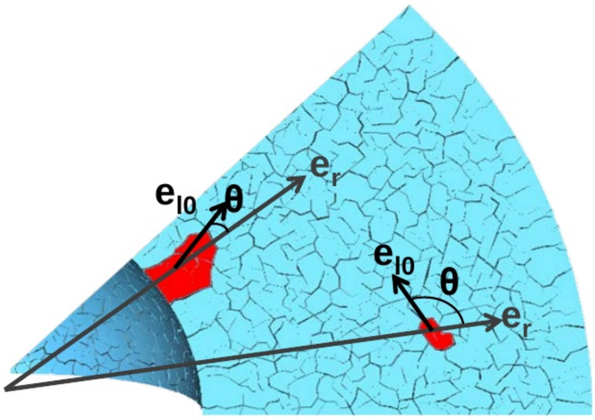


Fragments shape and orientation



Fragment shape? Fragment orientation?





Fragments shape and orientation: Effect of membrane thickness

Shell:

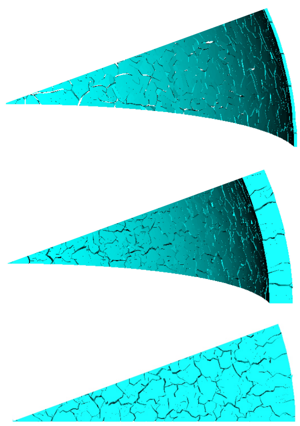
- 2D fragments ($l_2 \approx 0$ & l_0, l_1 random)
- $\theta \approx \pi/2$

Thin membrane:

- 3D fragments (l_0, l_1, l_2 random)
- $\theta \approx 0$
- Crack branching if thickness sufficiently large

Full sphere:

- 3D fragments (l_0, l_1, l_2 random)
- θ random



TODO: ask JF what is the idea you wanted to show ?

simplify t osay there are effects of the thickness

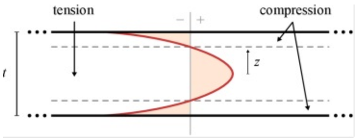
Back to CG cohesive-element approach: Tempered glass fragmentation;

Vocialta Corrado Molinari; Eng. Frac. Mech., 2018

Tempered glass must shatter in small fragments for safety reasons

tonischildersbedrijf.nl

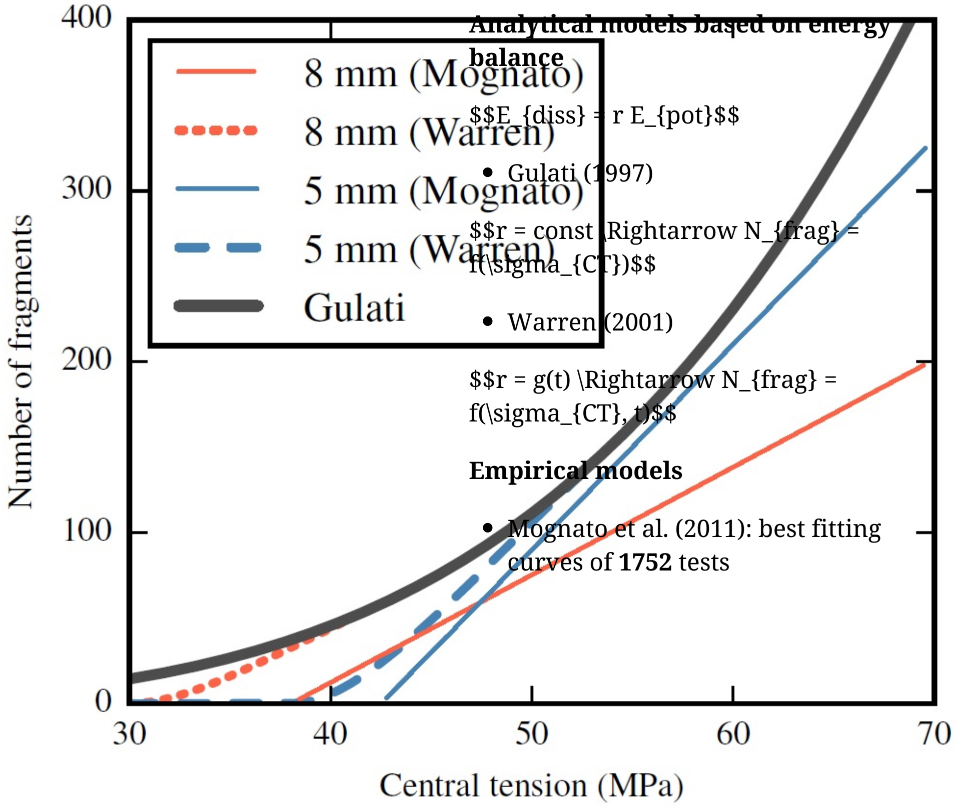
Thermal temper:



$$\sigma_x = \sigma_y = f(z) = \sigma_{CT} \left[1 - 12(\frac{z}{t})^2\right]$$

Motivation

Discrepancy between analytical formulas and experiments



Analytical models based on energy balance

$$E_{diss} = r E_{pot}$$

- Gulati (1997)

$$r = const \Rightarrow N_{frag} = f(\sigma_{CT})$$

- Warren (2001)

$$r = g(t) \Rightarrow N_{frag} = f(\sigma_{CT}, t)$$

Empirical models

- Mognato et al. (2011): best fitting curves of **1752** tests

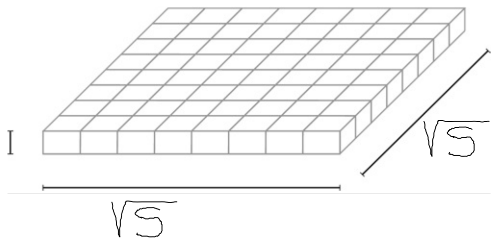
- Gulati S (1997) Proceedings of Glass Performance Days, 13-15.
- Warren P. (2001) Fractography of Glasses and Ceramics IV, 389-400.
- Mognato E., Barbieri A., Schiavonato M., Pace M. (2011) Proceedings of Glass Performance Days, 115-118

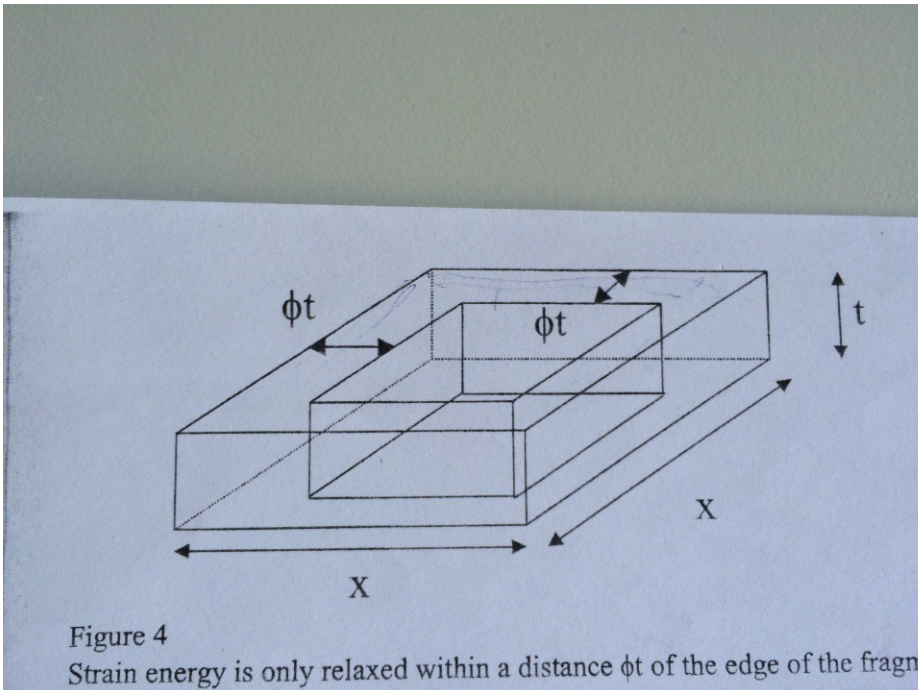
TODO ask JF about conclusions that should be retained ?

- mognato: experience
- warren: analytique (ratio de conversion energie en fragment)

Analytical model

Energy balance in statics assuming squared fragments





$$E_{diss} = 2 G_c t \sqrt{S} \left(\sqrt{N_{frag} - 1}\right) \quad \text{and}\quad E_{pot} = \frac{4}{5}\frac{1-\nu}{E} \sigma_{CT}^2 tS$$

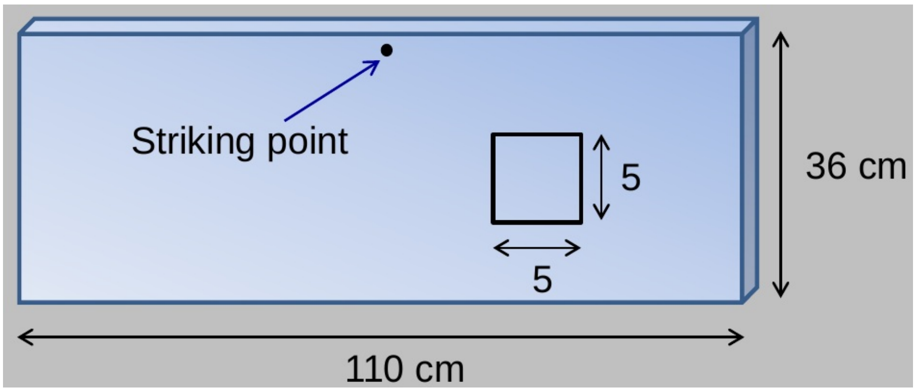
$$\Rightarrow E_{diss} = rE_{pot} \Rightarrow N_{frag} = \left(\frac{2r}{5G_c}\frac{1-\nu}{E}\sigma_{CT}^2 \sqrt{S} +1\right)^2$$

- r = const (Gulati)
- r = g(t) (Warren)

Epot : loi parabolique Ediss: energie de fracture sur les carrés

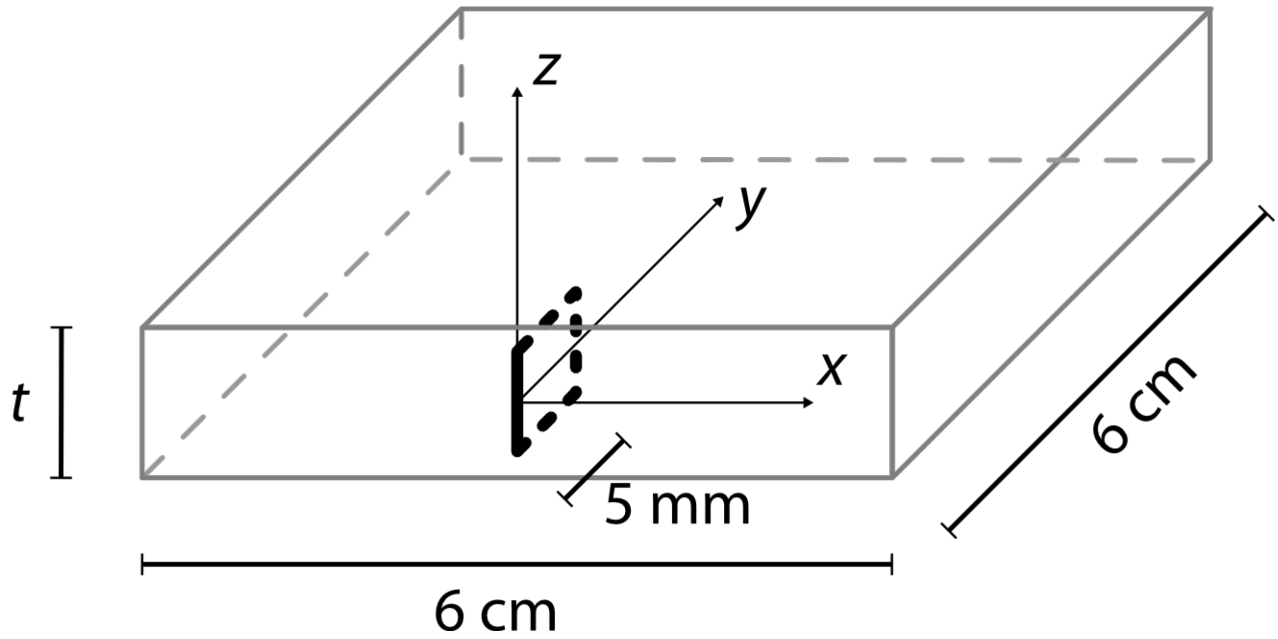
Numerical model

Standard fragmentation test (EN 12150-1)



Finite element model

- Phase 1: parabolic eigenstresses are applied
- Phase 2: a notch is created, to simulate a crack coming from the impact point
- Phase 3: fragmentation driven by the eigenstresses



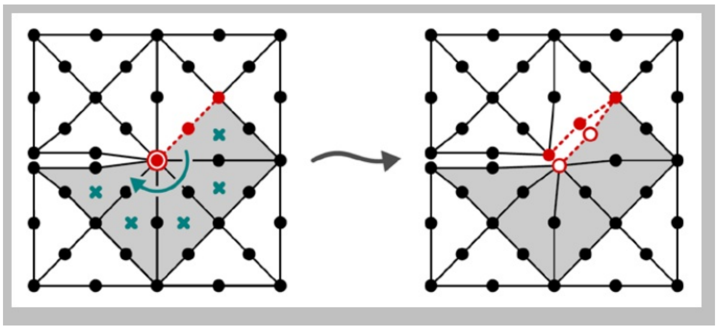
Numerical model

Available in Akantu

Crack propagation is simulated with the **cohesive element method**

$$\Downarrow$$

Cohesive elements are dynamically inserted along the borders of the standard elements

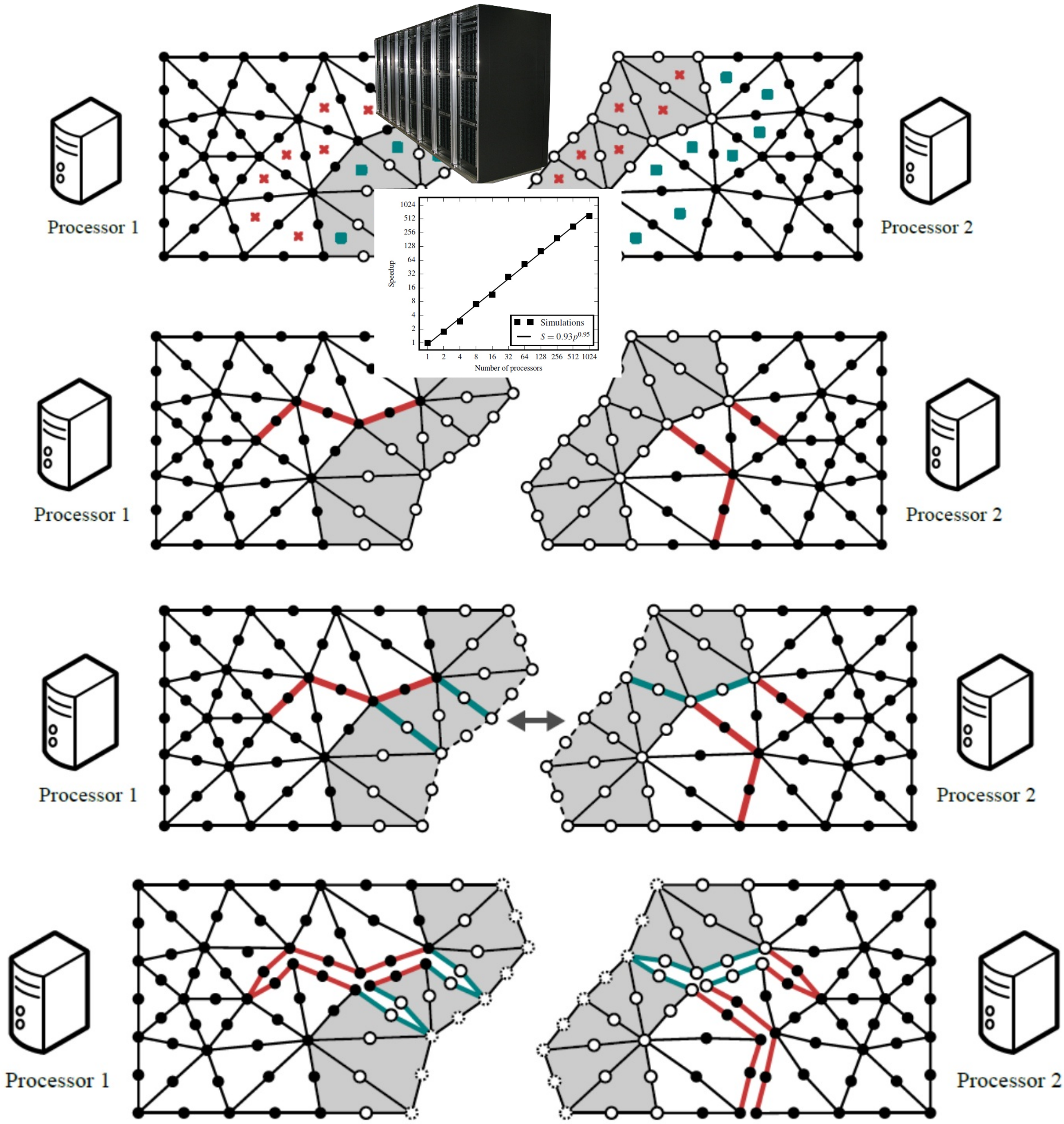


- Mesh: quadratic tetrahedral elements, average size = 0.3 mm
- Explicit central difference integration scheme
- Time step: $\Delta_{crit} = 0.2 min_{el} \frac{h_{el}}{c} \simeq 10^{-8}s$
- Algorithm implemented in the open-source FE library **Akantu** (http://lsms.epfl.ch/akantu)
- Simulations were run in parallel on **192 processors**

Vocialta M (2015) High Performance Computing Simulations of Dynamic Fragmentation in Brittle Materials. PhD Thesis, EPFL

Parallel dynamic insertion of cohesive el.

Vocialta, thesis EPFL



Constitutive laws

Soda-Lime-Silicate glass

Linear elastic bulk:

$E = 70$ GPa

$\nu = 0.22$ $\rho = 2500$ kg/m3

Cohesive law:

$\sigma_c = 70$ MPa

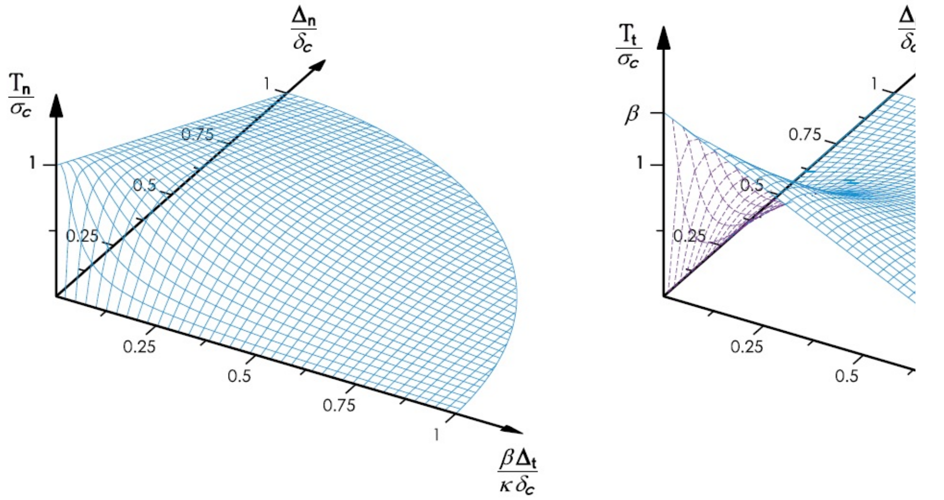
$G_c = 7.6$ J/m2 (KIC = 0.75 MPa m1/2)
$\beta = 3$

$$\vec T = \left(\frac{\beta^2}{\kappa}\Delta_t \vec t + \Delta_n \vec n\right) \frac{\sigma_c}{\delta} \left(1 - \frac{\delta}{\delta_c}\right)$$

$$\delta = \sqrt{\beta^2\Delta_t^2 + \Delta_n^2}$$

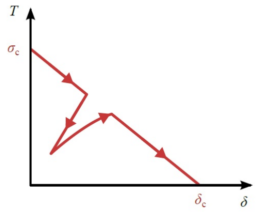
$$\beta = \frac{\tau_c}{\sigma_c}$$

$$\sigma_{eq} = \sqrt{\frac{t_s^2}{\beta^2} + t_n^2}$$



Camacho G, Ortiz M (1996) Int J Solids Struct, 33:2899-2938. Nguyen O, Repetto EA, Ortiz M, Radovitzky RA (2001) Int J Fract, 110:351-369.

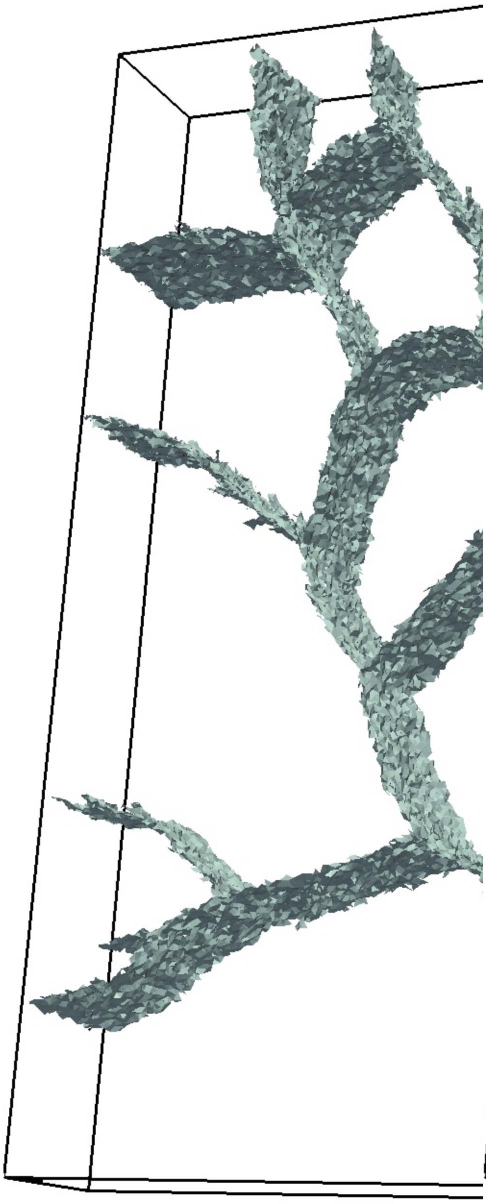
Fatigue irreversible behavior during unloading-reloading cycles:



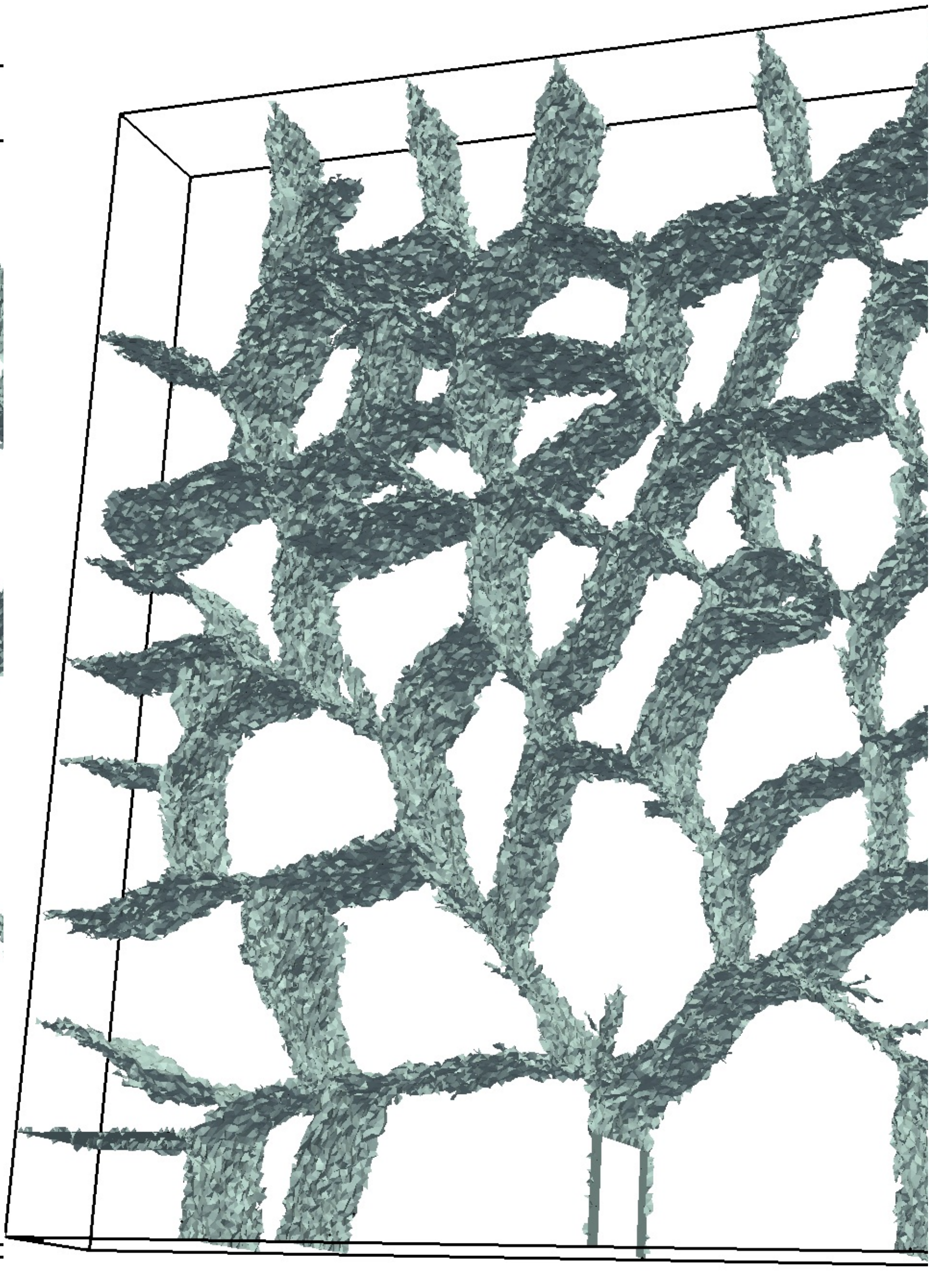
Results of the numerical simulations

Results of the numerical simulations

The number of fragments is proportional to the eigenstress



$$\sigma_{CT} = 37.5 MPa$$



$$\sigma_{CT} = 42.5 MPa$$

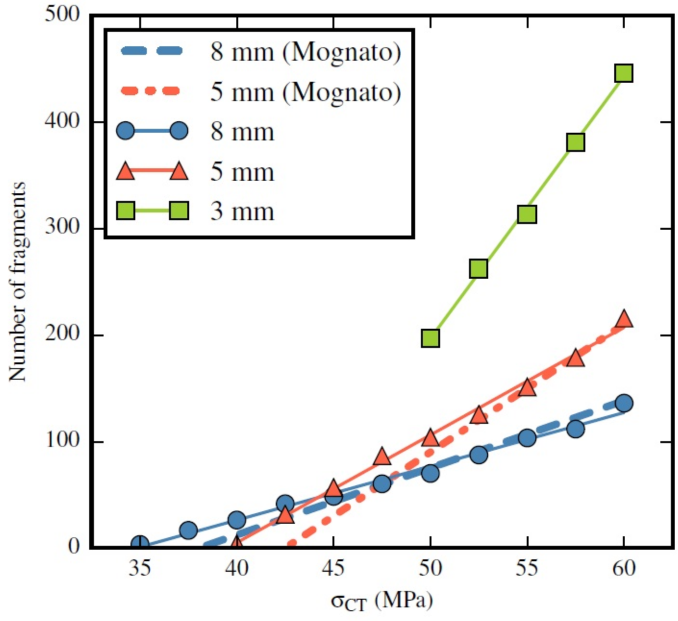
Number of fragments

Numerical results match reasonably well experimental data

σ_c is the only parameter tuned to fit the experimental data

$$\Downarrow$$

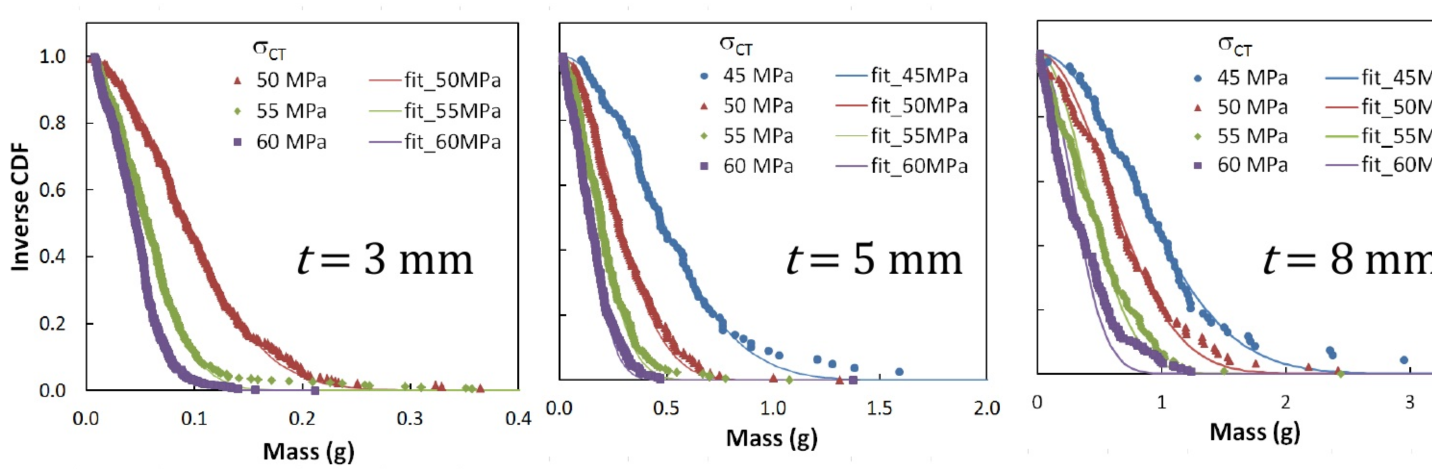
$\sigma_c = 70$ MPa for all the simulations



https://doi.org/10.1016/j.engfracmech.2017.09.015
or vocialta thesis

Fragments mass distribution

Inverse cumulative distribution function (CDF) (Grady & Kipp, 1985)



$$N(m) = \exp\left[- \left(\frac{m}{\mu}\right)^2\right]$$

Fitting parameter $\mu \simeq m_{average}$

Weibull Probability Density Function with modulus 2:

$$pdf(m) = \frac{2m}{\mu^2} \exp\left[- \left(\frac{m}{\mu}\right)^2\right]$$

Analysis of the energy fields

The simulations are run up to a steady-state condition

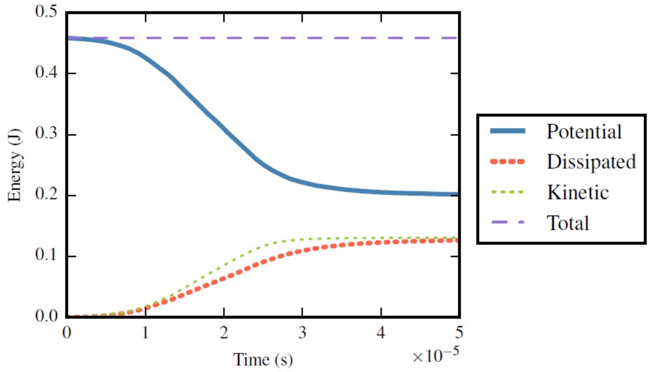
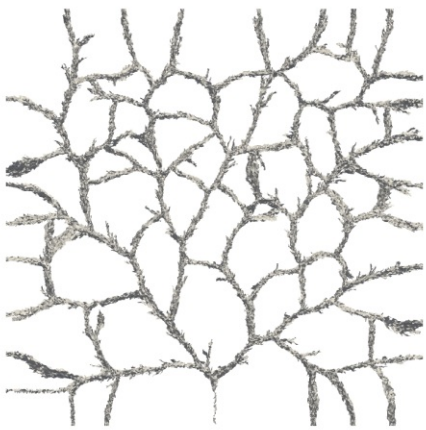


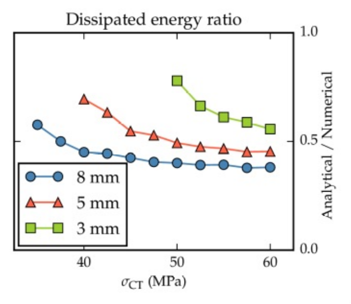
Figure 1β: Energies over time for the plate with thickness 8 mm and σ_{CT} = 45 MPa. $$t = 8 mm, \sigma_{CT} = 45 MPa$$

https://doi.org/10.1016/j.engfracmech.2017.09.015 or vocialta thesis

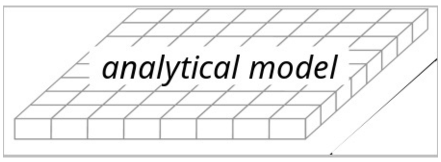
Analysis of the energy fields



Nfrag from simulations



The analytical dissipated energy is half of the numerical one

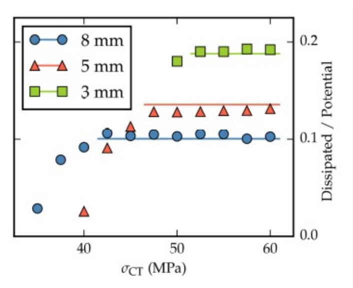


analytical model

Mauro Corrado

Energy conversion factor

The ratios of analytical dissipated and potential energies reach a plateau



$$r = \frac{E_{diss}}{E_{pot}} \propto t^{-\alpha} = \frac{4.6 \cdot 10^{-3}}{t^{0.639}}$$

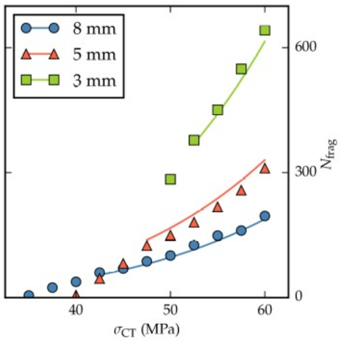
$t \simeq 2/3$

r = conversion mais il y a des ondes.

Energy conversion factor

With this ratio an accurate number of fragments is estimated

$$E_{diss} = E_{pot} r$$
$$\Downarrow$$
$$N_{frag} = \left(\frac{2r}{5G_c} \frac{1-\nu}{E} \sigma_{CT}^2 \sqrt{S}+1\right)^2$$

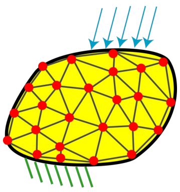


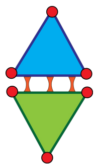
Summary: Fragmentation with HPC-FEM in highly non-linear damage problems

Reliable and efficient numerical framework:

Continuous response simulated with the FEM

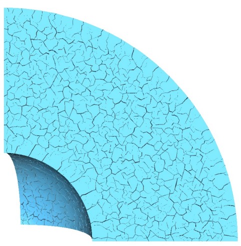
Material heterogeneity and failure through cohesive approach
Parallel simulations and significant computational power


Discretization of a continuum body


Cohesive methodology

Summary: Fragment analysis

- Converged results
- Average fragment size depends on material parameters, defects, and loading: can be quantified
- Two regimes (quasi-static and dynamic): transition quantified
- Smaller fragments than Grady's prediction, but -2/3 scaling law accurate
- Membrane explosion:
  - 2D fragments only if shell
  - random size and orientation of cracks when far from boundaries


Fragmented sphere

- Tempered glass fragmentation example