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मण्डल

Ancient Science Publishers

Decode The Myth

अन्तर्चक्षुः

भगवान् महर्षि हिरण्यगर्भ

Beacons of Light

Edsger Wybe Dijkstra

Richard Phillips Feynman

Leonhard Euler

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$x^2 \triangleq$

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Excerpt from the Chapter Algebraic Concepts

Concept *$\mathcal{C}$ is a predicate describing a set of syntactic and semantic requirements on related types ($< T_i >$) together with a collection of similar procedures $(f : T^i \rightarrow T^j)$ stated in terms of the properties, attributes and type functions $(F : \mathcal{C}^i \rightarrow \mathcal{C}^j)$ defined on the types.*

$$\therefore \mathcal{C}\,(< T_i >) \triangleq \wedge < \Psi_j >$$

where $\triangleq$ stands for *is defined by* and the Ψ_j represent independent clauses defining the concept.

```
template<class T>
    concept integral = is_integral_v<T>;
```

If a type T fulfills all the requirements of a concept $\mathfrak{C}$, then T ***models*** $\mathfrak{C}$, i.e. $T \vDash \mathfrak{C}$.

$$\text{int8\_t and uint8\_t} \vDash \text{integral}.$$

Concept $\mathfrak{C}^i$ is a ***refinement*** *of concept $\mathfrak{C}^j$ if it subsumes the latter, i.e. if $\mathfrak{C}^i$ is true for a set of types, then $\mathfrak{C}^j$ is also true for the same set.*

In other words, $\mathfrak{C}^i$ *refines* $\mathfrak{C}^j$ ($\mathfrak{C}^i \looparrowright \mathfrak{C}^j$) by addition of more requirements to $\mathfrak{C}^j$, i.e. $\mathfrak{C}^j$ *weakens* $\mathfrak{C}^i$ ($\mathfrak{C}^j \looparrowleft \mathfrak{C}^i$).

```
template<class T>
    concept signed_integral = integral<T> && is_signed_v<T>;
```

$$\text{signed\_integral} \looparrowright \text{integral}$$
$$\text{int8\_t} \vDash \text{signed\_integral}$$

```
template<class T>
concept unsigned_integral = integral<T> && !signed_integral<T>;
```

$$\text{unsigned\_integral} \looparrowright \text{integral}$$
$$\text{uint8\_t} \vDash \text{unsigned\_integral}$$

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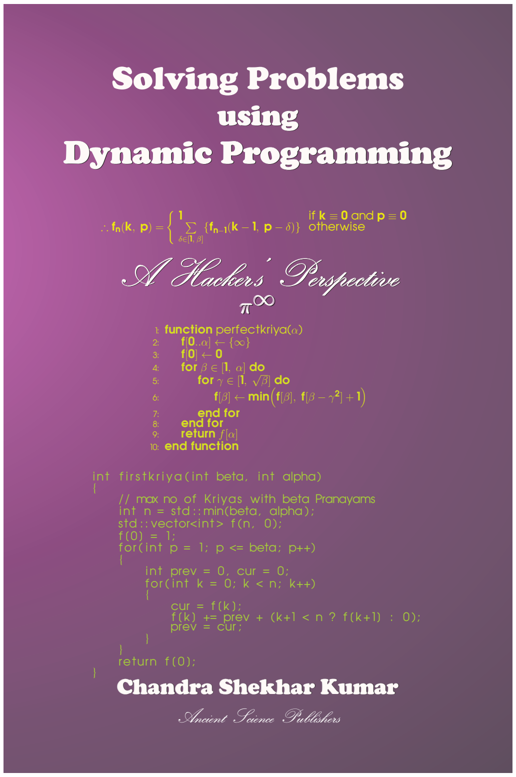
Solving Problems using Dynamic Programming : A Hacker's Perspective

A hacker's approach to a coding problem is beyond the foundational aspect of underlying genetic and computational structures, often termed as π^{∞}.



A concept becomes *not difficult* because the *complexities* built into it are clarified. In a bid to reach the *core* of the problem, the concept is split-broken into fragments, *complexities* are exposed and *delicate* points are examined. Then the concept is *recomposed* to make it integral and as a result, this reintegrated concept becomes sufficiently simple and comprehensible.

This helps build a hacker's insight to reveal the internal structure and internal logic of the concepts, algorithms and mathematical theorems.

This book provides a hacker's perspective to solving problems using dynamic programming. Written in an extremely lively form of problems and solutions (including code in modern C++ and pseudo style), this leads to extreme simplification of optimal coding with great emphasis on unconventional and integrated science of dynamic Programming. Though aimed primarily at serious programmers, it imparts the knowledge of deep internals of underlying concepts and beyond to computer scientists alike.

Ancient Science Publishers | *Chandra Shekhar Kumar*
July, 2020. | 256 pages | ISBN 9781722497170

Beautiful (C++) code snippets. Unique yogic exposition to coding.

Ancient Science Hackers

Excerpt from the Chapter (Optimal Loot Partition):

§ Problem. *The head of a gang of robbers embarks on distribution of the looted amount $l(>0)$, starting with division into two parts : x and $l-x$ for $0 \leq x \leq l$. From x : they get a return of $u(x)$ such that they are left with a lesser amount αx : $0 < \alpha < 1$ and from $l-x$: a return of $v(l-x)$ such that they are left with a lesser amount $\beta(l-x)$: $0 < \beta < 1$. So the total amount left after the first step of division is $\alpha x + \beta(l-x)$ and the process continues. Devise the partition strategy to help them maximize the return obtained in a finite n or infinite number of steps.* ♢

§§ Solution. Let $y(x)$ denote the return after the first step:

$$\therefore y(x) = u(x) + v(l-x)$$

Assuming u and v to be continuous functions, it is trivial to find the maximum of $y(x)$ over $x \in [0, l]$ using calculus (or graphical approach) :

$$\frac{dy}{dx} = \frac{d}{dx}u(x) + \frac{d}{dx}v(l-x) = 0 \text{ (for extrema)}.$$

Solve for x and $y(x)$ is maximum for that x for which $\dfrac{d^2y}{dx^2} < 0$.

Suppose $u(x) = x$ and $v(l-x) = -(l-x)^2$, then

$$y = x - (l-x)^2$$
$$\therefore \frac{dy}{dx} = 1 + 2(l-x) = 0,$$

$$\therefore x = l + \frac{1}{2}.$$
$$\frac{d^2y}{dx^2} = -2 < 0.$$
$$\therefore y_{max} = l + \frac{1}{2} - \frac{1}{4} = l + \frac{1}{4}.$$

After the first step, the initial amount l is reduced to l_1(say):

$$\therefore l_1 = \alpha x + \beta(l - x)$$

In the second step, l_1 is partitioned into x_1 (say) and $(l_1 - x_1)$ for $0 \leq x_1 \leq l_1$. Hence, the return from the second step is $u(x_1) + v(l_1 - x_1)$. Therefore, the total return after the two steps is:

$$\therefore y(x, x_1) = u(x) + v(l - x) + u(x_1) + v(l_1 - x_1).$$

Maximum of the function $y(x, x_1)$ over the 2-dimensional space (x, x_1) yields the maximum return, such that $x \in [0, l]$ and $x_1 \in [0, l_1]$.

Similarly, the total return after n steps is :

$$\therefore y(x, x_1, x_2, \ldots, x_{n-1}) = u(x) + v(l - x) + \sum_{i=1}^{n-1} \left[u(x_i) + v(l_i - x_i)\right]. \tag{21.1}$$

Here $x_i \in [0, l_i]$.

Using this *enumerative* approach to maximize the n-dimensional return, the computation procedure soon becomes cumbersome, error-prone and exponential in nature.

Any choice of $x, x_1, x_2, \ldots$ is a *policy*.

The policy maximizing $y(x, x_1, x_2, \ldots)$ is an *optimal policy*.

It can be noted that each step depends on the respective policy only. Hence at the $(i+1)^{th}$ step, the corresponding *one-dimensional* choice is made : a choice of $x_i \in [0, l]$.

Hence an optimal policy leads to the corresponding maximum return.

Let $y_n(l)$ denote the maximum total return, given the initial amount l and n steps.

$$\therefore y_1(l) = \underset{x \in [0,l]}{\text{Max}} \left[u(x) + v(l-x) \right].$$

After the first step, l becomes $\alpha x + \beta(l-x)$:

$$\therefore y_2(l) = \underset{x \in [0,l]}{\text{Max}} \left[u(x) + v(l-x) + y_1\left(\alpha x + \beta(l-x)\right) \right].$$

This leads to a recurrence relation :

$$\therefore y_n(l) = \underset{x \in [0,l]}{\text{Max}} \left[u(x) + v(l-x) + y_{n-1}\left(\alpha x + \beta(l-x)\right) \right]. \tag{21.2}$$

Hence a single n-dimensional problem is reduced to a sequence of n one-dimensional problems.

Here, the optimal return depends on the initial amount l and initial decision of division into the parts l and $l - x$ only.

This is possible due to **the Principle of Optimality** :

> *An optimal policy has the property that whatever the initial state and initial decision are, the remaining decisions must constitute an optimal policy with regard to the state resulting from the first decision.*

Hence Eq. (21.2) is the required optimal strategy. ■

Excerpt from the Chapter (Constrained Subsequence):

Maximum Sum

§ Problem. Given a sequence of $n \in (-\infty,\ \infty)$ integers, determine the largest possible sum of the contiguous subsequence. ♢

§§ **Solution**. Let $f_n(i)$ be the maximum sum of a contiguous subsequence ending at index i, obtained using an optimal policy and n steps.

Let s_i be the value of the element at index i, i.e. s_i is used at the n^{th} step. The we can use an optimal policy starting with previously accumulated maximum sum of a contiguous subsequence ending at index $i - 1$.

Hence the required optimal procedure is

$$\therefore f_n(i) = \underset{i \in [0,\, n-1]}{\text{Max}} \left[f_{n-1}(i-1) + s_i \right]$$

At each step (with addition of s_i), there are 2 options :

1. leverage the previous accumulated maximum sum if $f_{n-1}(i-1) + s_i > 0$, because it is better to continue with a positive running sum or

2. start afresh with a new range (with the starting sum as 0) if $f_{n-1}(i-1) + s_i < 0$, because it is better to start with 0 than continuing with a negative running sum.

Also note that:

- If all the elements are negative, then there is no such subsequence, i.e. the required sum is 0.

- If all the elements are positive, then the entire sequence is the required subsequence, i.e. the required sum is the sum of all the elements of the sequence.

- The required subsequence (if any) starts at and ends with a positive value.

Time complexity is $\mathcal{O}(n)$. Space complexity is $\mathcal{O}(1)$.

```
int maxseq(std::vector<int> & s)
{
    int current_sum = 0;
    int max_sum = 0;

    for(int x : s)
```

Maximum sum contiguous subsequence : compute sum

function maxseq($s[0..n-1]$)
  $currentsum \leftarrow 0$
  $maxsum \leftarrow 0$
  for $x \in s[0..n-1]$ **do**
    $currentsum \leftarrow$ **max**$(currentsum + x, 0)$
    $maxsum \leftarrow$ **max**$(maxsum, currentsum)$
  end for
  return $maxsum$
end function

```
    {
        current_sum = std::max(current_sum + x, 0);
        max_sum = std::max(max_sum, current_sum);
    }
    return max_sum;
}
```

■

Circular Sequence

§ Problem. Given a circular sequence s of $n \in (-\infty, \infty)$ integers, find the maximum possible sum of a non-empty contiguous subsequence of s. ◊

§§ Solution. The end of a circular sequence wraps around the start of the sequence itself, i.e.

$$\because i \equiv (i+n) \textbf{ mod } n \quad \forall i \in [0, n)$$
$$\therefore s_i \equiv s_{(i+n) \textbf{ mod } n} \quad \forall i \in [0, n).$$

s_0	s_1	$\cdots$	s_i	$\cdots$	s_{n-1}
$\uparrow$	$\uparrow$	$\uparrow$	$\uparrow$	$\uparrow$	$\uparrow$
s_n	s_{n+1}	$\cdots$	s_{n+i}	$\cdots$	s_{2n-1}

For a maximum contiguous subsequence $[s_i \cdots s_j]$, the solution of Dialogue 21 can be used.

$$s_0 \quad \cdots \quad \underbrace{s_i \quad s_{i+1} \quad \cdots \quad s_{j-1} \quad s_j}_{\text{max subsequence}} \quad \cdots \quad s_{n-1}$$

For a maximum contiguous subsequence $[s_j \cdots s_{n-1}, s_0 \cdots s_i]$, the left-over part $[s_{i+1} \cdots s_{j-1}]$ forms a minimum contiguous subsequence.

$$\underbrace{s_0 \quad \cdots \quad s_i}_{\text{max subseq part 2}} \quad \overbrace{s_{i+1} \quad \cdots \quad s_{j-1}}^{\text{min subsequence}} \quad \underbrace{s_j \quad \cdots \quad s_{n-1}}_{\text{max subseq part 1}}$$

Summation of the contiguous subsequence $[s_j \cdots s_{n-1}, s_0 \cdots s_i]$ is

$$\begin{aligned} &= s_j + \cdots + s_{n-1} + s_0 + \cdots + s_i \\ &= s_0 + \cdots + s_{n-1} - [s_{i+1} + \cdots + s_{j-1}] \end{aligned}$$

This is maximum when $[s_{i+1} + \cdots + s_{j-1}]$ is minimum.

$$\therefore \mathrm{Max}[s_j + \cdots + s_{n-1} + s_0 + \cdots + s_i] = \sum_{k=0}^{k=n-1} s_k - \mathrm{Min} \sum_{k=i+1}^{k=j-1} s_k$$

$$\begin{aligned} \therefore \text{Maximum sum subsequence } &= \text{ Total sum of the sequence} \\ &- \text{ Minimum sum subsequence} \end{aligned}$$

Time complexity is $\mathcal{O}(n)$. Space complexity is $\mathcal{O}(1)$.

```
int maxsum_circular(std::vector<int> & s)
{
    int current_max = 0, max_sum = std::numeric_limits<int>::min();
    int current_min = 0, min_sum = std::numeric_limits<int>::max();
    int total_sum = 0;

    for(int x : s)
    {
        current_max = std::max(current_max + x, x);
```

Maximum sum circular subsequence

```
function maxcircularseq(s[0..n − 1])
    currentmax ← 0
    maxsum ← −∞
    currentmin ← 0
    minsum ← ∞
    totalsum ← 0

    for x ∈ s[0..n − 1] do
        currentmax ← max(currentmax + x, x)
        maxsum ← max(maxsum, currentmax)

        currentmin ← min(currentmin + x, x)
        minsum ← min(minsum, currentmin)

        totalsum ← totalsum + x
    end for

    if totalsum == minsum then            ▷ All elements are -ve
        return maxsum          ▷ Value of the least -ve element
    else
        return max(maxsum, totalsum − minsum)
    end if
end function
```

```
        max_sum = std::max(max_sum, current_max);

        current_min = std::min(current_min + x, x);
        min_sum = std::min(min_sum, current_min);

        total_sum += x;
    }
    // when all elements are -ve => total_sum == min_sum,
    // i.e. total_sum - min_sum becomes 0 => empty subsequence
    // but max_sum still holds the value of the least -ve element,
    // hence return this singleton than an empty one
```

```
    return total_sum == min_sum ? max_sum : std::max(max_sum, total_su
}
```

■

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