MDM-2024 Homework N

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Present the plots of minimum, maximum and mean distances for each L_p and discuss the results. How the curves are behaving when k increases? Can you characterize the form of curves? What is the effect of p? If a curve seems to be converging, tell also the value that it is approaching.

2.1 Min-Max-Mean Distance

P values lower than 1 grow polynomamially. The following shows the proportionality.

$$\forall p < 1, \lim_{k \to \infty} \left(\sum_{i=1}^{k} |x_i|^p \right)^{\frac{1}{p}} \propto k^{\frac{1}{p}} \tag{1}$$

and p value 1 grows linearly.

$$\forall p = 1, \lim_{k \to \infty} \left(\sum_{i=1}^{k} |x_i|^p \right)^{\frac{1}{p}} \propto k \tag{2}$$

P values greater than 1 grow as p roots of k. The reasoning comes from the norm equation:

$$\forall p > 1, \lim_{k \to \infty} \left(\sum_{i=1}^{k} |x_i|^p \right)^{\frac{1}{p}} \propto \sqrt[p]{k} \tag{3}$$

For all the curves, they can be formulated as curves according to their growth rate, polynomial, linear, or p-root. For the ∞-norm, the curve can be formulated as a multiplicative inverse curve approaching 1 as k approaches infinity.

All the norms except for L_∞ diverge as k increases. However, the L_∞ converges to number one. Mathematically, this follows from the equation for the L_∞ norm. The values generated are between 0 and 1 and thus the maximum value will be one as k approaches infinity.

$$\lim_{k \to \infty} \max_{i=1,\ldots,k} |x_i| = 1 \tag{4}$$

3 Variance of Distance

Present the variance plots for each L_p and discuss the results. How the curves are behaving when k increases? Can you characterize the form of curves? What is the effect of p? If a curve seems to be converging, tell also the value that it is approaching.

For all the quasinorms, i.e, norms p ¡ 1. The variance grows polynomially. Since the data is a sample of random datapoints, the sample variance is used.

$$\operatorname{Var}(X) = \frac{1}{n-1}\sum_{i=1}^{n}(X_i - \mu)^2$$

$$\operatorname{Var}_{\text{sample}}(\|\mathbf{x}\|_p) = \lim_{k\to\infty}\left(\frac{1}{N-1}\sum_{i=1}^{k}\left(\left(\sum_{i=1}^{k}|x_i|^p\right)^{\frac{1}{p}} - \mathbb{E}\left[\left(\sum_{i=1}^{k}|x_i|^p\right)^{\frac{1}{p}}\right]\right)^2\right) \tag{5}$$

Similar to the min-max-mean distance, the variance grows polynomially for all p ¡ 1. When p = 1, the variance grows linearly. And when p =¿ 2, the variance grows proportionally to a $\frac{1}{\log(k)}$ function. (or polynomial??).

The interesting case was the norms p between 1 and 2. For those norms, the variance goes to infinity but much slower than for the p=1 linear growth.

All variances larger or equal to 2 converge to 0 as k increases.

In summary, the curves with p ¡ 1 can be characterized as polynomial curves. The curves with $1 < p < 2 \propto k^{1/p}$

Should we add images??