A Review on the Applications of Petri Nets in Modeling, Analysis, and Control of Urban Traffic

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***Abstract*—Urban traffic systems that possess system states that are distributed, parallel, deterministic, stochastic, discrete, and continuous are well suited for a Petri net (PN) approach. The literature survey conducted in this paper shows the vast applications of PNs in modeling and simulation, analyzing and evaluating performances, intelligent control and optimization, and congestion management in urban traffic systems. This paper outlines the related works conducted using PNs and discusses its viability, such as its contributions and limitations. Extendibility and future research potential to further the successful applications of PNs in traffic systems are discussed and proposed in this paper.**

***Index Terms*—Petri net (PN), traffic models, traffic signal optimization, urban traffic systems.**

I. Introduction

URBAN traffic and transportation plays an important part in human civilization, where it has direct impact on social, economic, and environmental aspects of the human race. Various modes of transportation have emerged, and rapid development of transportation infrastructure has enabled major cities to be interconnected, which leads to increases in social and economic activities. However, the rapid increase in the number of vehicles and the increasing demands in all types of transportation modes have resulted in traffic congestion as increasingly more vehicles attempt to use a common transportation infrastructure with limited capacity. This scenario causes traffic congestion and devaluates system performance, which may lead to other environmental and economic problems. Various works have been conducted on the urban traffic systems, such as modeling and simulation, congestion management, and intelligent traffic control, with the purpose of aiding better design and planning, or regulating congestion, and optimizing system performance.

Traffic modeling and simulation aim to study the traffic system that is too complicated to be approached with just numerical and analytical methods. Parameters estimated from these simulation tools provide present and future scenarios that help traffic planners in planning and design work. Some tools function based on some set of rules (microscopic model) that depicts the behavior of vehicles in the network, whereas others are based on mathematical models, such as the Lighthill, Whitham, and Richards (LWR) model, as well as the Payne model [1]. On the other hand, congestion management and intelligent traffic control approaches aim to optimize the performance of urban traffic systems. These performance indexes include total time spent by all vehicles in the network, queues, and the total delays at intersections.

As a matter of fact, traffic models that can accurately describe and predict traffic states are the underlying "engine" of simulation tools. In addition to LWR and Payne models, various traffic models that emulate the behavior of the urban traffic system over time and space to predict traffic states and to analyze system performance have been developed. These models include queuing theory [2], agent-based models [3], activity theory [4], neural networks [5], [6], etc. The state of the traffic system, such as link section speed, flow, travel time, delay, stop time, intersection turnings, etc., can be predicted from these models via simulation. Subsequently, these predictions are used to facilitate performance evaluation and optimization strategies for the traffic operator or the traffic controller. In [7], various traffic control and optimization strategies that can be divided into fixed-time, traffic-responsive, and predictive strategies for road intersections and freeways are comprehensively presented.

In addition to these works, the applications of Petri nets (PNs) in modeling, performance analysis, and control of traffic systems have been conducted for over a decade. It is a graphical tool that provides visual representation and is useful in simulating the dynamic and concurrent activities in systems. As a mathematical tool, it enables systems to be governed by a set of mathematical equations, e.g., state and algebraic equations, hence providing an avenue for system analysis [8]. This net-theoretic approach adds another modeling paradigm to the methods mentioned previously as it can suitably describe the urban traffic and transportation system that possesses system states that are distributed, parallel, deterministic, stochastic, discrete, and continuous. As a result, PN traffic models become useful tools for analyzing system performance and assisting intelligent and optimized traffic control. The literature survey conducted in this paper shows a wide spectrum of works conducted in these aspects.

This paper presents reviews on the applications of PNs in traffic systems modeling, analysis, and control. The review is categorized into 1) modeling and simulation and 2) performance analysis, implementation, optimization, and intelligent

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TABLE I
PN-BASED MICROSCOPIC TRAFFIC MODELS

Research group	Type of Petri Net	Parameters Modeled	Key Features
Di Febbraro and Giglio [12][13][14]	DTPN	Road links and intersections, vehicle occupancy and turning rate at crossing sections and signal timing plan.	Modularity is introduced where the microscopic model is defined as integration of a few DTPN sub-models that can be expressed as state equations.
Di Febbraro, Sacco and Giglio [15]	STPN	Vehicle occupancy and movement are modeled differently from their work in [12]-[13] using stochastic time transitions.	Solving deadlocks in previous DTPN models and estimate queues at intersection.
Badamchizadeh and Joroughi [17] Makela *et. al* [18]	DSPN	Road links and intersections; traffic signal control; vehicles occupancy and travel time modeled with both deterministic and stochastic time.	Estimate queue lengths [18] and vehicles' waiting time [17].
Basile *et. al* [19] Dotoli *et. al* [21] [22]	CTPN	Road links and intersection; traffic signal control, vehicle arrival and crossing rate, vehicle occupancy and vehicle routing	Coloured tokens are used to model vehicle routing and occupancy. Simulate objective functions [19] and queues [21][22].

traffic control. Works on these areas are comprehensively explained in Sections II and III, respectively. These works are discussed with a proposal of future works in the end of this paper.

II. MODELING AND SIMULATION

From the perspective of traffic engineers, traffic models and their properties can be classified according to physical interpretation: level of detail, discrete or continuous, and deterministic or stochastic [1]. Physical interpretation of the traffic situations is approached with system theories [9]. As traffic models can be represented at different levels of detail, namely microscopic, macroscopic, and mesoscopic models, PN has been promising in encapsulating such detail in its net structure that comes with useful structural and mathematical properties. Here, traffic states can be modeled as continuous, stochastic, discrete, and hybrid systems. In addition, the PN models description of the traffic situation could deterministically or stochastically simulate some traffic situations faster than in real time. In the following, the aim is to outline the contributions of PNs in describing traffic situations at different levels of detail. Different types of PNs have been used by different researchers, and the simulations conducted reveal the viability of this net-theoretic approach.

A. Microscopic Model

Microscopic models capture the behavior of every single vehicle, such as driver parameters (e.g., aggressiveness, reaction time, etc.) and parameters of each vehicle (e.g., mass, acceleration, etc.) moving in the traffic stream. These parameters are sampled based on a stochastic distribution function derived from measurements of real traffic data. Existing models that microscopically describe the traffic situation include the car following model [10] and the famous cellular automaton by Nagel [11]. In the area of PNs, Table I summarizes some of the studies reported in the literature. The table shows the research group, the type of PN used, the parameters described by the PN model, and the key features of these works.

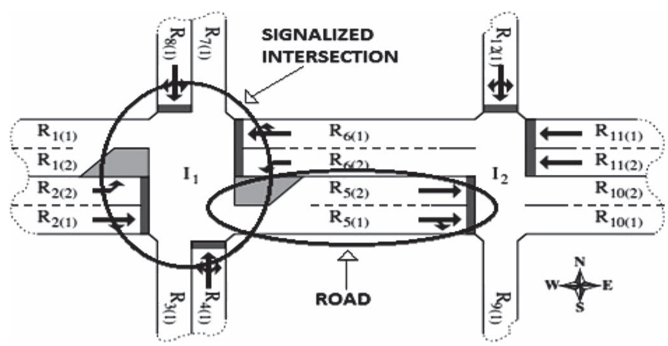

Fig. 1. Signalized intersections and roads.

Febbraro and Giglio [12] applied deterministic-timed PN (DTPN) for a microscopic model of a signalized traffic urban area consisting of signalized intersections and roads, as shown in Fig. 1. Roads that are characterized by the number of lanes and capacity describe the physical space that could be occupied by a vehicle when moving from one intersection to another. Crossing sections at signalized intersections comprised of finite places that model the crossing and occupancy behavior of vehicles. Vehicles entering an intersection can exit toward more than one direction. Hence, the vehicle turning rate is also described. The DTPN models of roads and intersections are integrated with a DTPN describing a signal timing plan and a PN handling structural conflict to pursue models useful for traffic management and signal control strategies. The dynamics of these DTPNs are expressed by their respective state equations, which can be merged into an overall DTPN model that resemble a "switching" system that can be switched to different modules and level of details [13], [14].

Undesirable deadlock states occurred in [12]–[14] when the nets are tested on some case studies. This motivated the application of stochastic-timed PNs (STPNs) instead of DTPNs to model the interarrival time of vehicles entering the traffic area, the minimum time necessary to pass through a crossing section,

TABLE II
PN-BASED MACROSCOPIC TRAFFIC MODELS

Research group	Type of Petri Net	Parameters Modeled	Key Features
Tolba *et. al* [23]	VCPN	Average speed of vehicle, density and output flow rate of freeways	Approximate fundamental diagram that can be used to approximate traffic parameters such as vehicle speed [24].
Kutil and Hanzalek [25]	CCPN	Traffic signal control, free space at crossing section, vehicle turning rates, delay and average speed	Instead of employing TPN, traffic signal control is incorporated into the CCPN. A conflict resolution algorithm handle conflicts in net and flow rate is simulated.
Di Febbraro *et. al* [26][27]	HPN	Traffic flows at road stretches and intersections, turning rates and traffic signal control	A modular framework that comprised of PN-subnets representing different part of the traffic system. Traffic flow and queue dynamics can be simulated.
Dotoli *et. al* [32]	FOHPN and TPN	Vehicle speeds, lane interruption, turning rates and traffic signal control	Reduced the state space dimension in previous work [21] for queue estimation using MATLAB.
Zhang and Jia [33]	Hybrid CPN	Traffic flows, vehicle speed and traffic signal control	A hybrid model CPN with TPN to model traffic macroscopically.
Di Febbraro and Sacone [35]	HPN	Traffic flows and events happening in freeway	Simulate the effect of events on the traffic flow in freeway.
Xuan *et. al* [34]	HPN	Traffic flows at non-signalized	Simulate dynamic traffic flows at non-signalized intersection by establishing relationships between traffic flows, passing capacity and queues.
Farhi *et. al* [36]	Ordinary PN and min plus algebra	Traffic flows and density	Estimating fundamental diagram from PN model.
Wang *et. al* [37]	STPN	Traffic flows, vehicle arrival rates and traffic signal control	Simulate queue lengths and delay time.

and the minimum time to travel along a road. Hence, traffic streams are separately considered, and deadlock occurrence can be prevented [15]. Queues can be estimated from the simulation of this new model. Similar queue estimation has been conducted using STPN in [16]. They model free speed travel time, headway distribution, and control signal of traffic lights using stochastic p-timed PNs. Stochastic time is associated to places instead of transitions to enable the dynamics of the token flow, which represents vehicles.

A combination of both stochastic and deterministic characteristics of traffic behavior is modeled using deterministic and stochastic PNs (DSPNs) [17], [18]. DSPN contains three classes of transitions, namely, exponential, immediate, and deterministic transitions, which give a comprehensive description of vehicle arrival and traveling time across an intersection. Here, vehicle arrival time is defined with exponential distributed firing (stochastic) or deterministic firing depending on road lengths [18]. Simulations performed on these models successfully calculate queue lengths [18] and average waiting time of vehicles at intersections [17].

In microscopic models, each vehicle moving in the traffic stream carries unique behavior and parameters where "uncolored" PNs are unable to describe. Hence, a colored TPN (CTPN) fills in this gap to represent these vehicles with distinct attributes using colored tokens, which are not easily represented by a simple PN token. Basile *et al.* [19] described an intersection using a CTPN to represent particularly the routing behavior of a vehicle at an intersection (e.g., turning right or left or going straight). The CTPN describes the crossing area of the intersection in a finite number of zones, where each zone can be occupied with only one vehicle at a time. The time required of a vehicle to move from one zone to another is modeled by stochastic-timed transitions. Tokens in the model are given a stochastic value, which represents the direction that a particular vehicle intends to take. A transition with appropriate guards in the CTPN and a two-phase traffic light modeled using a TPN regulate the firing of tokens (movement of vehicles) in the net. However, road links between intersections is modeled using non-PN method called the hybrid stochastic model [20]. Objective functions are generated when the model is simulated.

A similar approach using a CTPN for an intersection in Bari, Italy, is conducted by Dotoli and Fanti [21]. In this model, tokens with different colors are used to describe vehicles' routing. In contrast to [19], road link cells are described with places, whereas the time for a vehicle to transit between cells is deterministic. Simulations are conducted to estimate queue lengths under different traffic scenarios and the results validated with real traffic data [21], [22].

B. Macroscopic Model

Macroscopic traffic models do not describe the traffic situation on the level of independent vehicles. Variables such as traffic flow, density, and average velocity are used to provide aggregated information on multiple vehicles. PN contributions to this type of traffic model are listed in Table II.

Variables such as traffic flow, density, and velocity are aggregated time-varying parameters of the traffic situation, which resemble a continuous-time (CTS) flow system. As a consequence, CTS PN is applied by some researchers to represent the traffic situation. Tolba *et al.* [23] macroscopically model a freeway section using continuous PNs with variable speeds (VCPN). In their model, the freeway section is divided into n segments of different lengths. The VCPN model is able to describe parameters such as the average speed of the vehicle, density, and the output flow rate of each segment. The markings and marking invariants in the VCPN is directly related to density and the flow rate, whereas the firing frequencies depict average speed. A fundamental diagram is approximated from the model and compared with the supply-and-demand model. Simulation on the model using least squares estimation successfully estimated parameters, such as vehicle speed, that are applicable for traffic monitoring [24].

A light controlled intersection model based on constant-speed continuous PNs (CCPNs) is proposed in [25]. Similar to other intersection models, signal control, free space at the crossing section, vehicle turning rates, delay, and average speed are described. However, instead of separately employing a TPN to model signal alternations, signal control is incorporated in the same model being described by transitions with appropriate time intervals. Due to the conflicting nature of the CCPN, an algorithm for conflict resolution is developed using linear programming. Vehicle flow rates simulated from the model compared with real data shows that the model is accurate.

Application of a hybrid PN (HPN), which is a combination of both discrete and continuous places and transitions, has been found instrumental in describing macroscopic models. This type of a PN treats the traffic network as a hybrid system that comprised of discrete-event and CTS components. Febbraro *et al.* [26] proposed a modular framework that was made up of an overall HPN that consists of HPN subnets called HPN modules (HPNMs) representing urban traffic network such as intersections and road stretches. These HPNMs structures are independent of each other and can be expressed as state equations. HPNMs representing nonsignalized intersections and signalized intersections are developed. For instance, a four-way intersection with a two-phase traffic light is modeled in this paper. Continuous transitions and places of the HPN take into account the traffic flow as fluids, where input and output flows, turning rates of vehicles through the intersection, and the vehicle flow crossing the section are represented. The discrete part of the HPN models the traffic light that rules the intersection. The authors later applied HPN to describe a two-phase signalized intersection to simulate traffic flow and queue dynamics, which provide a basis to solve some control problems, such as traffic light plan optimization, dynamic routing, and special vehicle control [27]. Other similar works on using HPNs to model intersections and road stretches are found in [28]–[31].

An extended model of the HPN, such as first-order HPNs (FOHPNs) that describes traffic flow as fluids, is developed in [32]. The FOHPN models lane cells and intersection cells. These submodels are interconnected to describe the whole traffic network. Two significant features of the model, in addition to the modeling traffic flow, is the capability to model vehicle speeds and lane interruption events in lane cells. Vehicle firing speeds are modeled by transitions with minimum and maximum firing speeds to describe traffic at peak hours, i.e., weekday during the day and weekend. The FOHPN intersection model contains arc weights that describe a vehicle path and turning rates regulated by a TPN model resembling the traffic signal plan. The advantage of the FOHPN is its aggregated formulation, which reduces the dimension of the state space compared with their CTPN models in [21]. Hence, simulation can be easily implemented using Matlab for a real traffic network and queue at estimated road links. Hybrid models may not necessary comprised of a continuous and discrete PNs. In [33], a colored PN (CPN) is combined with a discrete PN to model an intersection macroscopically.

In addition to modeling signalized intersections, nonsignalized intersections [34] and freeway [35] have also been modeled using the HPN approach. Xuan *et al.* [34] proposed an HPN model for a nonsignalized T-intersection to represent dynamic traffic flows. The proposed model can effectively identify the relationships between traffic flows, the passing capacity, and vehicle queues. On the other hand, Febbraro and Sacone [35] proposed an HPN for a freeway system. The CTS component of the HPN is used to describe the behavior of traffic flow in the freeway. The discrete-time part (DES), which *regulates* the CTS, is a representation of events that modify the behavior of the traffic flow. The DES has stochastic firing times (with a given probability distributions) to represent occurrences of events. The length of time that an event occurs is monitored by a message-passing algorithm. The work established a promising framework to the implementation of traffic control strategies for freeways.

In addition to HPNs, macroscopic study on traffic conditions has been conducted using ordinary PNs, and the dynamics of the traffic is derived using min-plus algebra [36]. A fundamental diagram linking the average flow and the density can be computed from the model as trajectory dynamics can be estimated using the method. The model is applied to a system with two circular roads, and an intersection shows fundamental diagrams with different phases. Adding to the types of PNs in macroscopic models, STPNs have found their roles in specifying motion of vehicles in a road segment between two intersections, i.e., specifying traffic flow and traffic control at signalized intersections [37]. The phases of the traffic lights are determined by deterministic transitions, whereas stochastic-timed transitions with exponential distributed firing times are used to describe vehicle arrival rates at intersections. A simulation algorithm implemented on the STPN models produced performance indexes for the traffic control, such as queue lengths and time delay.

C. *Mesoscopic Model*

Mesoscopic models combine the properties of both microscopic and macroscopic models. In mesoscopic models, individual vehicles are taken into account, but their interactions and behavior are based on macroscopic parameters. The use of PNs in this area of modeling is insubstantial. The survey found PNs being applied in [38] and [39].

The VCPN in [23] is combined with a TPN to form a hybrid model for an isolated intersection in [38]. The model is able to macroscopically describe the arrival and departure average flow

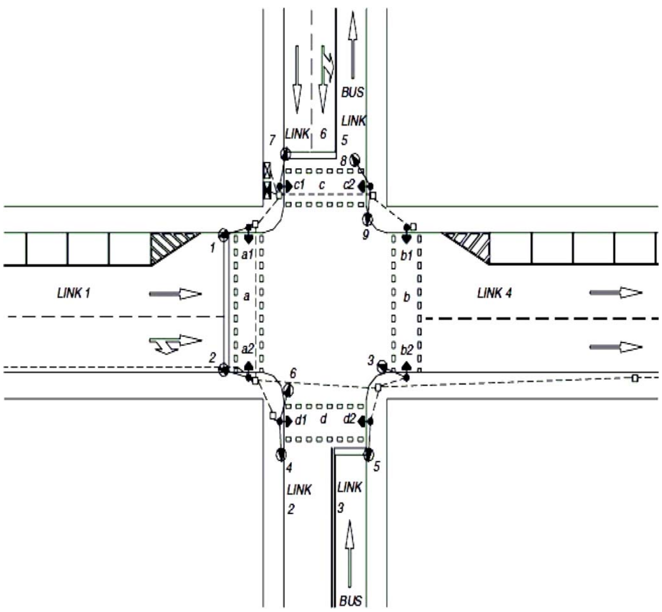

Fig. 2. Layout of a real traffic intersection located in the urban area of Bari, Italy.

rates and queues at a signalized intersection. In addition, the VCPN can be transformed into a TPN model to study the traffic performance microscopically. The TPN is able to describe the road link in homogenous cells that depicts the movement of vehicles base on time occupancy. Both macroscopic (VCPN) and microscopic TPN models are used to evaluate vehicle queue lengths at an intersection with fixed cycle time and vehicle interval traffic signal control, respectively.

Demongodin [39] applied a batch PN (BPN), which is an extension of the HPN, to model variable delays on continuous flows in highways. The BPN is applied to model the motion of vehicles moving along a highway. The model describes the highway by decomposing it into sections. This extended model is comprised of batch nodes that model vehicle motion on sections of the highways in batches. Hence, a group of vehicles moving on different sections can be simulated with a discrete-event approach on continuous-varying parameters such as density, speed, position, and length. The dynamic evolution of batches such as meeting of two batches, accumulation of batches, and destruction and overtaking of batches, are taken into account by the BPN. Evaluation is conducted on the model to evaluate highway traffic situation under different control policies for speed regulation, which serves to prove the importance of variable speed limit control laws on highways.

D. Example of Microscopic Model

Here, the aim is to further illustrate the application of PNs as a modeling and simulation tool for a signalized crossroad intersection. For this purpose, the work of Dotoli *et al.* [21], [22] on microscopic modeling will be used as an example. The authors applied a CTPN to describe a real traffic intersection in Bari, Italy (see Fig. 2). The real traffic intersection in Bari t is composed of six links (L_1 to L_6) with length $l_1 = 40$ m, $l_2 = l_3 = 45$ m, and $l_4 = l_5 = l_6 = 50$ m, respectively. The capacity for link 1 C_1 is 16 passenger car units (PCUs). The other link capacities are $C_2 = C_3 = 9$ PCUs, $C_4 = C_6 = 24$ PCUs, and $C_5 = 15$ PCUs, assuming that one PCU is 5 m long.

Fig. 3 shows the CTPN model of the intersection. Places p_i and p'_i with $i = 1, \ldots, 9$ are the subsets of places modeling cell links. L_1 is modeled by 16 link places, which is equivalent to 16 PCUs capacity, L_2 by 9 places, and L_4 and L_6 by 24 places. L_3 and L_5 are dedicated as bus routes. Considering that each bus is equivalent to three PCUs, L_3 and L_5 are modeled by three and four places, respectively. Places p_i and p'_i with $i = 10, 11, 12$ model the intersection crossing area composed of six cells.

The flow of tokens (vehicle) between two consecutive link cells is modeled by flow transitions, which have deterministic firing times. In addition to flow transitions, the CTPN model also contains input transitions that model the arrival of vehicle in a link and output transitions that model the departure of a vehicle from a link to the outside of the considered intersection. Transitions $t_{0,1}$, $t_{0,2}$, $t_{0,3}$, $t_{0,4}$, and $t_{0,5}$ are input transitions, whereas transitions $t_{1,0}$, $t_{2,0}$, $t_{3,0}$, and $t_{4,0}$ are output transitions. Both input and output transitions have deterministic firing times that model the interarrival time of vehicles. Transitions t_j with $j = 12, \ldots, 21$ model vehicles that are changing lanes in two-lane intersection links such as L_1, L_4, and L_6. A set of transitions, namely, t_1, t'_1, t_3, t_6, and t'_6, are intersection transitions that model vehicle entering a cross section. These transitions are connected to a TPN that models the traffic light of the intersection, as shown in Fig. 4.

Color tokens are labeled as a_1, a_2, a_3, a_4, and a_5, which represent vehicles traveling through the intersection and their associated routings. Color tokens a_1 and a_2 are associated with vehicles following the route $(\mathrm{L}_1, \mathrm{L}_4)$ and $(\mathrm{L}_1, \mathrm{L}_2)$, respectively. Moreover, colors a_3 and a_4 refer to vehicles following the route $(\mathrm{L}_6, \mathrm{L}_2)$ and $(\mathrm{L}_6, \mathrm{L}_4)$, respectively. Finally, color a_5 represents vehicles following the route $(\mathrm{L}_3, \mathrm{L}_5)$.

The TPN that models the traffic light describes the signal timing plan for the intersection. This TPN model regulates the firing of the intersection transitions (e.g., t_1, t'_1, t_3, t_6, and t'_6); hence, the allowance of the vehicle from the respective links to enter the intersection is dependent on the enabling of these transitions by the TPN. Color token routings are also governed by the definitions of matrices $\mathrm{Post}(p_i, t_j)$ and $\mathrm{Pre}(p_i, t_j)$ that define the arc weights from a place to a transition with respect to a particular color.

The overall CTPN and TPN models are implemented and simulated using Matlab based on matrix formulation of the marking updates. Simulation is conducted for a few scenarios, namely, scenarios 1 (weekdays around 3:00 P.M.), 2 (weekdays around 6:00 P.M.), and 3 (oversaturated situation), to obtain the number of vehicles in the links at certain time period. The time instants of the vehicle arrivals are obtained for three scenarios from the direct measures at the input links that are registered at the considered time of the day. For simplicity, only simulation results of link 1 for scenario 1 is shown (see Fig. 5). Simulated results are compared with real data from the links to ascertain the validity of the model. The results in Fig. 5 and the results for the other links that are not shown here confirmed the capability of the model to correctly predict traffic performance under different timing plans.

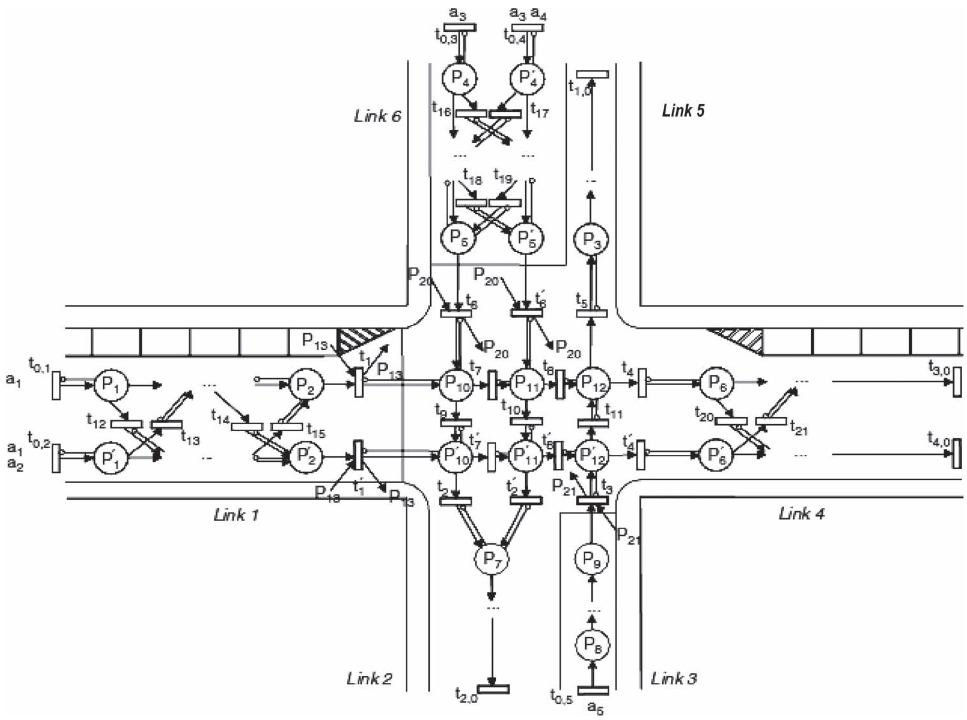

Fig. 3. CTPN model of the intersection shown in Fig. 2.

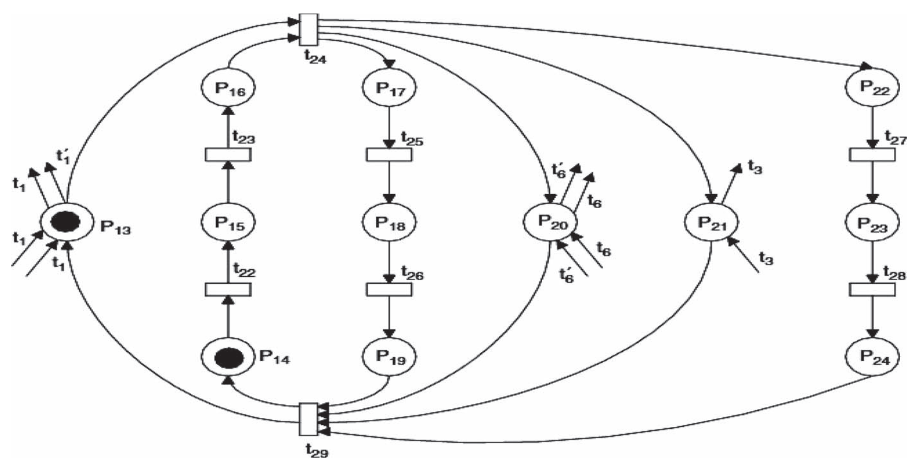

Fig. 4. TPN modeling the traffic light of the intersection in Fig. 2.

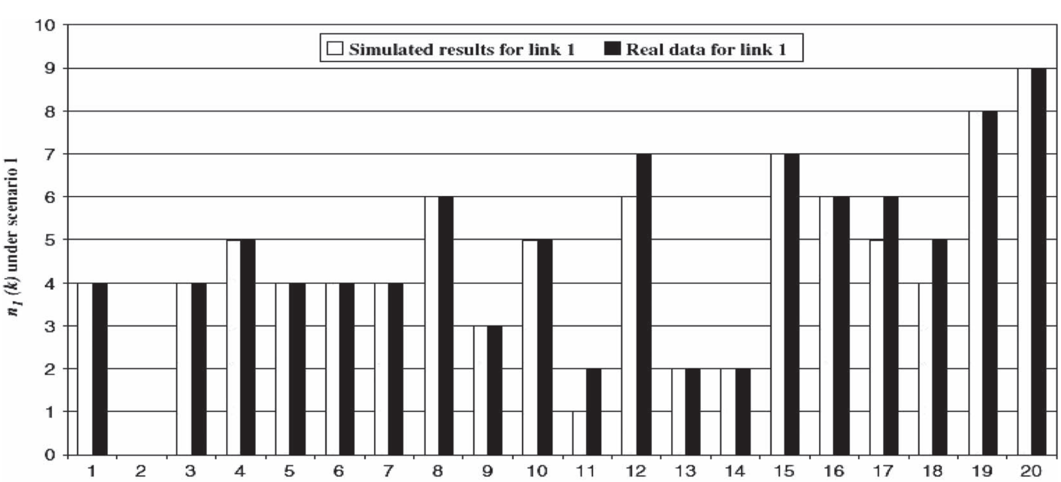

Fig. 5. Simulated and measured number of vehicles in link L_1 under scenario 1.

TABLE III
PN METHODS FOR ANALYSIS, IMPLEMENTATION, OPTIMIZATION, AND INTELLIGENT TRAFFIC CONTROL

Research group	**Type of PN**	**Key Features**
Li and Li [41]	DSFPN and Free-Choice PN	Analyze reliability of the traffic signal control by analyzing liveness with siphons and traps and safeness using state machines.
Tzes *et. al* [42]	Ordinary PN	Analyze reliability of the traffic signal control for simulations in a microprosessor. Liveness is analyzed using marked graph while reachability is determined using reachability tree.
Huang *et. al* [43][44]	TCPN	Model complex phase transition and evaluate reliability of the traffic signal control based on liveness of the TCPN verified with occurrence graph [43] and invariant analysis [44].
Mladnovic and Abbas [45]	TCPN with stochastic times	Model traffic signal control at ring barrier and determine a deadlock-free signal control with reachability analysis using coverability tree.
List and Cetin [46]	Ordinary PN	Evaluate safety of traffic signal control using P-invariant analysis and reachability analysis via the coverability tree.
Soares & Vrancken [47]	p-timed Petri Net	Applied Linear Logic to analyze reachability and reinitiability for safe implementation of traffic signal.
Mladnovic and Abbas [49]	CPN	Model preemption features of a traffic controller and conduct transition-based simulation to assess critical behaviors in firmware.
Barzegar and Motanemi [50] Barzegar [51]	CPN	Models fuzzy rules implemented in a fuzzy logic controller for traffic signal control and evaluates its performance based on all possible inputs.
Ganiyu *et. al* [52] Wang *et. al* [53]	TCPN TPN	Both works model traffic signal control and validate the efficacy of the signal control plan with real traffic data [52] and traffic parameters calculated from analytical approaches [53].
Lin *et. al* [60], Jbira [61] List *et. al* [46] and Guia [62]	Ordinary PN [46][60][61], HPN [62]	Translation of PN into SFC [60][61] and C-code for [46][62] implementation into PLC and microprocessor-based controller.
Soares and Vrancken [47][69]	p-timed PN	Implement traffic-responsive control by extending green time of main road based on traffic data measured in arterials.
Di Febbraro and Giglio [13][14]	DTPN	Minimize number of vehicles in network through a program that solve the underlying mathematics in the PN.
Basile *et. al* [19][71]	HPN	Applied model predictive control to predict number of vehicles in the network and develop optimizing algorithm to minimize the average number of vehicles.
Huang *et. al* [72] Zhang *et. al* [74] Di Febbraro *et. al* [75]	FSPN [72] HPN [74] STPN [75]	Models vehicle flow rate [72][75] and queue sizes [74] and optimize traffic flow through offset adjustment.
Di Febbraro *et. al* [15] Jbira and Ahmed [70] Jbira *et. al* [73]	DSTPN [15] HPN [70] [73]	Predict queue sizes and optimize traffic flow rate through optimizing stage duration [15], adjusting green splits [70] and extending green time [73].

III. PERFORMANCE ANALYSIS, IMPLEMENTATION, OPTIMIZATION, AND INTELLIGENT TRAFFIC SIGNAL CONTROL

Traffic signal control strategies have significant implications on the traffic network. These strategies can be categorized as fixed-timed and traffic-responsive strategies [7] and predictive control [40]. PNs have been instrumental in traffic control as it provides a platform for describing the underlying control algorithm and provides a simple and clear means for analysis. The structural description of the control logic supports implementation into real-time logic controllers or embedded processors. On the other hand, the analytical method aims to evaluate the properties of the net for safeness and a reliable control signal. PNs can also be integrated with various artificial intelligent methods to perform traffic optimization and intelligent traffic-responsive and predictive control. Our literature survey presents works done on optimization, intelligent control, analysis, synthesis, and implementation using PNs, which are listed in Table III.

A. *Performance Analysis of Traffic Signal Control*

Analysis of the good properties of PNs, such as liveness, safety, reachability, and reversibility, gives an evaluation of the system performance. This is of great importance with regard to signal control as these good properties can assess whether the control signal could reach all desirable states, is able to recover from error, provides a deadlock-free regulation of vehicles at an intersection, and ensures that infrastructure usage is within capacity. In [41], the liveness of the PN model for the traffic control is determined by analyzing siphon and traps. The safeness of the traffic control is ascertained by generating a strongly connected state machine (SM) component from the PN. Tzes *et al.* [42] analyzed the PN-based signal timings on the properties of liveness, safeness, and reachability to prove that the PN is safe and able to synchronize the different components. However, liveness and safeness are analyzed using the marked graph, whereas reachability is evaluated using a reachability tree. These methods are conducted to ensure safe, consistent, and deadlock-free control so that simulations on a multiprocessor computer can be implemented as the program execution safety is secured.

Huang *et al.* [43], [44] models traffic signal alternation that comprised of two-phase, six-phase, and eight-phase transitions using a timed CPN (TCPN). Their work aims to model complex phase transitions in traffic lights. The TCPN operates based on a global clock, and the movement of tokens that represent signal alternation is based on a timestamp associated with them. The dynamic information on the TCPN is analyzed using the occurrence graph method. With the occurrence graph, liveness and reversibility can be analyzed to ascertain reliability of the signal control [43]. The authors also assessed signal control using invariant analysis [44] that creates equations to prove all reachable states. Mladnovic and Abbas [45] add stochastic times to the TCPN to describe signal control logic at a ring barrier. The control logic is ascertained of its reliability and safety using the coverability tree method to ensure deadlock freedom and reachability of all states.

Another research using invariant analysis is found in [46]. List and Cetin [46] developed, using a PN, a signal control model that describes an eight-phase traffic light. The model is capable of facilitating phase transitioning when interfaced with an "optimization layer" for adaptive traffic control system. The P-invariant analysis conducted confirms that the control logic implemented in the PN meet up to safety rules. In addition, the PN is confirmed deadlock free using the coverability tree method. Soares and Vrancken [47] modeled traffic signal control with an ordinary PN. To ensure that all signal cycle could be reached and an unsafe state does not occur, sequent calculus of linear logic is applied to analyze reachability and reinitiability of the net to evaluate a set of possible markings from a given marking. This approach, which has been conducted by Girault *et al.* [48], has an advantage over the classic coverability tree method, which has the problem of state-space explosion due to the high number of states that could be reached in the model.

The PN has also found its use in traffic-controller firmware capability assessment. Mladnovic and Abbas [49] applied a CPN to model preemption features of a type of a traffic controller. A CPN tool is employed for model development, and the network structure is analyzed using a discrete transition-based simulation and incidence matrix. The analysis conducted is able to identify critical firmware features. As a CPN does not provide means to further verify and test these critical behaviors, software-in-the-loop simulation is conducted to investigate questionable firmware capabilities. This paper contributes in enhancing user understanding on unknown firmware capabilities to reach user optimum, as well as in evaluating firmware capabilities in meeting Advanced Transporter Controller standards.

A CPN tool has also been used to analyze traffic signal control based on fuzzy logic. The fuzzy logic of the fuzzy controller that receives queue and density of passing cars is capable of regulating timing operation to minimize stop time according to the traffic situation. The CPN tool is used to describe the fuzzy rules and to provide simulated results to evaluate its performance based on all possible inputs from the traffic stream [50], [51].

Adding to the given analytical and simulative methods is the practice to validate the PN model with real traffic or proven analytical data. Ganiyu *et al.* [52] modeled a multiphase traffic-light-controlled cross-type intersection with fixed signal timing plan using a TCPN. The simulated results from the TCPN are compared with the measured traffic data to ascertain the efficacy of the TCPN to correctly describe traffic behavior. In [53], a TPN is used to model pretimed and actuated signal control. The simulated results from the models are compared with analytical methods and the Highway Capacity Manual.

B. Implementation of Traffic Signal Control

PNs have provided a formal method in implementing logic control into real-time logic controllers or embedded processors. The PN models describing systems control logic can be translated into usable computer codes [54]–[59].

PN models describing traffic signal control sequence can be translated into a sequential function chart for implementation into programmable logic controllers [60], [61]. Other works on translating PN models of traffic signal control include translating into executable C code for microprocessor-based controllers [46], [62]. Code translation had been made possible with reference to standardized methods, such as IEC1131 and IEC1499 [63], and customized tools, such as NETMAN [64] and UltraSAN [65], which facilitate code generation.

C. Optimization and Intelligent Traffic Signal Control

PN-based methods in traffic signal can be fixed time, traffic responsive, and predictive in nature. However, most work falls into traffic-responsive and predictive control strategies. In the fixed-time strategy, a PN is used to describe and facilitate transitions between different signal timing plans for different time of the day [66]. Otherwise, the use of the PN for fixed-time control is quite "straightforward." Traffic-responsive control employs a PN-based model of controllers' logic to control intersections based on inputs from physical sensors, such as loop detectors or cameras. In predictive strategies, the PN model of the traffic network predicts future network situations and devises an "optimization plan" for the signal control via an "optimization layer."

Some existing well-known methods in traffic-responsive control schemes are SCOOT [67] and SCATS [68]. These schemes are fed with real traffic data and incrementally adjust offset, splits, and cycle time to achieve optimal performance of traffic signal control. These adjustments are also applied in PN-based models for traffic-responsive control and predictive control. A traffic-responsive model is developed by Soares and Vrancken [47], [69], using a p-timed PN to model the dynamic behavior of a group of traffic signals controlling a network of intersections comprised of a main road and arterial roads. Time is associated with places that are given a minimum and maximum duration for enabling transitions. Green time duration at places can be extended from minimum to maximum to give priority movement to vehicles on the main road, depending on the demand on the nonpriority road or arterial roads, where demands from nonpriority roads are detected by sensors. In [70], a real-time correction of green time phases is applied based on measurements of physical queue sizes at intersections.

The DTPN model in [13] has been further defined by high-level and low-level control systems [14]. This traffic-responsive method requires information from on-field devices. The high-level control system acts over the modular representation to modify some parts of the model such as signal timing plans

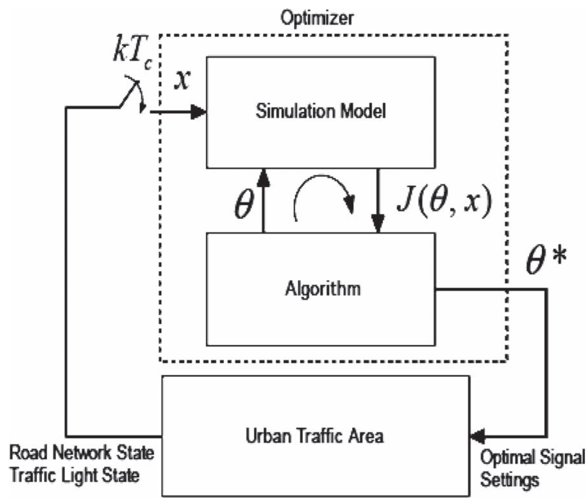

Fig. 6. Simulation-based urban traffic control scheme.

and turning rates. The low-level control system optimizes the performances of the traffic system by solving a mathematical programming problem, which minimizes the number of vehicles in the system.

While traffic-responsive control strategy is very much dependent on real traffic data that comes from sensors, predictive control provides an estimation of the traffic situation via a PN model describing the traffic system using past historical or online data. Basile *et al.* [19], [71] describe traffic intersection, on-ramps, and road links using a hybrid model with a CTPN describing the intersections, whereas road links are coarsely modeled as a discrete-event system. The model is simulation based and is best described in Fig. 6, which shows a type of model-predictive control where the model is used to predict objective function $J(\theta, x)$ that considers the average number of vehicles in the urban area in a future time horizon based on online data (traffic states, x). An optimization algorithm is needed to calculate a vector of control sequence, i.e., θ^*, that comprises of traffic light switching times to optimize J in anticipation of predicted values. Simulation conducted on the model successfully predicts the average number of vehicles in the urban network, and the optimizing algorithm aims to minimize the average vehicles in the network to avoid congestion via an optimized signal timing plan. A similar approach based on fluid stochastic PN is used to describe traffic flows in an urban traffic system. Estimated flows are used as input to a "decision-maker" that outputs control events to the traffic light that makes changes to the green time and, possibly, the cycle length [72].

To this end, it is observed that predictive control usually employs PN models to predict queue sizes [16], [71], [74] and traffic flow rate [75]. Hence, based on these predictions, adjustment can be made to the traffic signal plan to optimized traffic. Febbraro *et al.* [15] incorporates stochastic times to their DTPN model [12] to predict queues. Then, stochastic optimization is done using a simultaneous perturbation stochastic approximation approach to optimize stage duration in its aim to minimize queue lengths at intersections. On the other hand, the predicted queue sizes in [70] are handled by splitting green time proportionally to the predicted queue sizes, whereas green time is extended to cope with oversaturated input links in [73]. In anticipation of vehicle flow rates or queues at intersection, offset calculation between two intersections can be incorporated into the PN model to optimize vehicle movement through the intersections [74], [75].

IV. Discussion

Here, we aim to discuss the viability and the extendibility of PNs by assessing the contributions and critical issues with regard to this paper and potential extension with regard to future works. Contributions of PNs to the urban network are notable in terms of its ability in providing a visual representation of the urban traffic networks and a balance between modeling power and analysis capability, as compared with other mathematical models.

Based on the survey, PNs have added a new modeling paradigm in estimating traffic states, such as average speed, flow, density, queues, etc., which directly reveal the performance of the traffic system. Although the function is similar to other works, the main advantage is the capability of deriving state equations from these models for implementation in software such as Matlab. Executable computer algorithms for programs such as Java and C++ can also be obtained.

Although PN models can produce good estimates of the traffic states and produces usable algorithms in a formal and systematic way, the main concern is the complexity of such derivation when the dimension of the net grows. This can be observed in microscopic models. Traffic is modeled at a very high level of detail, and the behavior of every single vehicle is considered. Simulations of these models showed high similarity to measured real data. However, disadvantages arise from the enormous amount of data that is required for microscopic models and the resulting computational efforts. Hence, PN description of microscopic models leads to large nets with tedious notations and complex analysis. This is mainly due to the detailed description of vehicle occupancy and parameters. As a consequence, the possibility of deadlocks arises, and conflict handling is needed [12]–[14]. The analysis of PNs with large notations is very tedious, particularly for discrete systems with a large population. This may result in the state explosion problem [76]. In addition, the computational or simulation time for this type of net is longer. Hence, they have less of a computational performance compared with their CTS counterpart [25]. Therefore, microscopic models are not suitable for large networks, and they only cover a relatively small area of observation. To address this issue, a modular approach is normally adopted [13], [14]. Therefore, hierarchical representation of microscopic models using PNs should be taken into account. This will help decompose the net into smaller subnets to enable easy handling and perhaps "troubleshooting."

Due to these complexities, continuous and hybrid (discrete and continuous) PNs are preferred to model traffic macroscopically. Macroscopic models considered vehicles as an element of fluid whose development over time is described by aggregated variables. The amount of data required by the model is much lesser compared with that of microscopic models. Hence, there is less computational effort. They are suitable to simulate large networks in terms of density, speed, and flow. Hence, the PN macroscopic model is more "relaxed," as compared with a purely discrete model. The continuous model can provide a good approximation of the evolution of the large number of discrete entities [76]. The potential of the hybrid model in modeling traffic systems is increasing as the continuous behavior of the traffic stream is best described by the continuous part, whereas traffic signal control can be described by the discrete part. To ascertain the "accuracy" of such models, validation with real traffic data is normally conducted [22].

One crucial question is how accurately these models describe the traffic system with its stochastic-time and deterministic-time components. An important consideration in these models is that traffic arrivals are not deterministic (cannot be represented by deterministic time), whereas stochastic arrivals are somewhat indirect and not capable of describing oversaturated situations [77]. These time descriptions are best suited for evaluation for undersaturated conditions and for isolated intersections (e.g., in [12]–[19]). However, in most urban and suburb traffic systems where signalized intersections are part of a coordinated control system, vehicle arrivals are best described in organized platoons [78]. These platoon arrivals require an adjustment to the vehicle arrival description in some works, e.g., in [37] and [75], future models of PNs intended for coordinated traffic light system, and in the event of oversaturated conditions. In addition, the effects of queue overflow have not been substantially described and may need special consideration as residue queue aids in the calculation of time delay [79], [80].

As a tool that provide a two-edged approach, e.g., modeling and analysis, a major contribution of PNs to urban traffic studies is the mathematical facility to analyze signal timing performance, which is not provided by other types of modeling tools. Its analytical methods to analyze behavioral properties, e.g., liveness, reachability, and safeness of the net, assist in ensuring reliable traffic signal control. Methods, such as the coverability tree analysis, are useful to analyze "small" nets [8]. The matrix-equation approach that analyzes the behavior of the net by solving some equations using an incidence matrix is applicable to ordinary PNs and generalized PNs with a deterministic firing nature. In addition, the invariant properties that ascended from this approach ensure that not just any markings can be reached or that not just any transition sequence can be fired. Hence, certain allowable reachable markings and firable transitions can be characterized, thus ensuring that signal control abides to allowable states. Other analysis method listed in the literature can be categorized as graph-based analysis [41]–[43]. A structural analysis of these graphs can give deeper results about the system [81].

Analysis using the coverability tree may lead to the state explosion problem. For avoidance of the state explosion problem, linear logic provides a potential analysis platform [47], [48]. Else, if the net is too large, reduction rules may need to be applied to reduce the model to a simpler one to facilitate analysis [82]–[84]. This may be viable for future research with PN models with large notations.

In the area of intelligent traffic control and optimization, prediction of traffic states such as queues and traffic flow is useful for deciding on which suitable action is to be taken on the signal timing plan, e.g., adjustments to the green time, green split, offset, and cycle time [69]–[75]. It is observed that the PN plays a role as a predictive model of traffic states, whereas an "external" algorithm is developed to manipulate the predicted states for decision-making and regulating an optimal signal control. However, real implementation of PN models in traffic control has not been outstanding. Although PNs can be used to resemble the optimizing rules [50], [51], they are purely for performance analysis and simulations.

Future developments should focus on developing PN models or extending these models with added predictive power. In terms of extendibility, it is important to take note that a PN by itself is not a self-learning tool and is nonadaptive. Therefore, it is not possible to assess previous available information. To facilitate greater modeling power particularly in predictive control, extension to the original PN is needed. For instance, the PN has been further extended with fuzzy rules and integrated with learning automata to manage traffic signals [85]. In addition to extension with fuzzy logic, the PN can be extended with neural networks to enhance its learning capability of a particular system [86]–[93]. While extension to the PN with added learning capability may pose design and analysis complexity, a simpler practice is to integrate the PN with optimizing or assisting algorithms, such as those literature listed in this paper. PNs can also be integrated with a separate artificial neural network module for enhancement of learning capability. Thus far, this integration is seen in the manufacturing sector [94], [95] and robotics control [96]. Therefore, it is worthy to take note that there is great potential in using these extended or integrated PN to model the traffic systems particularly with fuzzy logic and neural networks.

One potential area for future research is the estimation of dynamic origin–destination trip flows [40]. As the traffic network paths can be described with this net approach, vehicle route choice and origin–destination flow may be estimated. This may facilitate iterative methods to achieve network equilibrium [97]–[99]. In addition, the path taken by the vehicles throughout the network may affect flow, density, and queuing behaviors at certain road stretches and intersections in the network.

Another potential area is the use of PNs to "schedule" the signal control of coordinated traffic lights as PNs have been proven to be a good scheduling tool, particularly for manufacturing systems [100]–[102]. In the context of urban traffic, it can serve to improve platoon progression through a group of signalized intersections using platoon-based scheduling. As this area has been approached with some existing methods [103], [104], the "maturity" of PNs in scheduling manufacturing processes can be further capitalized for realizable control implementation in real traffic systems.

Although the scope of this paper is limited to urban road traffic systems, this does not limit the scope of PNs in other areas of transportation. It is worthy for the reader to know that PNs have been used in the area of road safety to regulate movement of transit vehicles and pedestrians crossing [105], [106], modeling urban transportation [107], describing multiagent systems for traffic control [108], [109], modeling of advance driving-assistance systems [110], and even in traffic control of marine vessels [111].

V. Conclusion

For over a decade, applications of PNs in modeling, analysis, and control of urban traffic systems have been studied. Various works on such PN-based applications have been reported in the literature. The research results from these PN-based modeling and simulation show the potential of using PN models to predict traffic situations, analyze control logic and controller capabilities, and improve traffic performances. Our discussion highlighted that PNs had contributed traffic models that provide avenue for code implementation and an analysis platform for safe and reliable signal control. However, future research needs to take into consideration the modularity of the design, particularly with regard to large nets. Attention also needs to be paid to how accurate the traffic states are encapsulated into the model, particularly when describing vehicle arrivals. With regard to the choice of analytical methods, it is very much dependent on the type of PNs used. Invariant analysis is deemed useful for safety assurance of traffic control. However, if the structure is of a higher level PN, a different approach may be taken. In the area of signal control and optimization, few works based on PNs have been implemented on real-time controllers for traffic signal control. Hence, venturing into the estimation of vehicle route choice and the use of PNs in platoon-based scheduling is suggested. These optimization methods may be realized by further extension of PNs or integration with other artificial intelligent methods. The extension and integration of PNs with other artificial intelligence methods will enhance the ability of PNs to self-learn the traffic environment and to produce better predictions and regulation of traffic dynamics.

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