

Consider a situation when we randomly choose $(k-1)$ points in the interval $[0, 1-kc]$. So suppose these points when sorted in increasing order are y_1, y_2, \dots, y_{k-1} and let $y_k = 1-kc$.

Now consider $x_i = i \cdot c + y_i$.
We see that $x_i - x_{(i-1)} = c + (y_i - y_{i-1}) \geq c$
and $x_1 \geq c$

So randomly choosing $(k-1)$ points in the interval $[0, 1-kc]$ correspondingly generates randomly chosen $(k-1)$ points in the interval $[0, 1]$ in which any 2 consecutive points dist $\geq c$.

So the probability is

$$= \frac{\text{Volume of } (k-1) \text{ dimensional cube of size } (1-kc)}{\text{Volume of } (k-1) \text{ dimensional cube of size } 1}$$

So by scaling argument we
can say answer is

$$\begin{aligned} &= (Hkc)^{k-1} && \text{for } kc \leq 1 \\ &= 0 && \text{for } kc > 1 \end{aligned}$$
