Quant Quest

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1 Instructions

This is not your usual bla-bla-bla. MOVE FORWARD WITHOUT READING AT YOUR OWN RISK

- Humans will check this paper so please write legibly.
- For any discrepancy write down the discrepancy on the paper itself (asking for queries might make it unfair for other teams)
- If you notice something interesting a great solution, or some random observation, do mention separately somewhere and write above it 'INTENDED to authors of the question'.
- Try to enjoy the quiz and maximise fun and not marks (however they may be correlated for you)
- The paper is quite lengthy so please be smart about attempting it.
- The questions are tagged with the difficulty level besides them (rating from 1 to 10 out of 10) for your reference. These are relative numbers and also subject to biases and errors.
- **IF** you want, you can ask for another sheet which has "interesting" questions worth 30 points, but the catch is that opting for that sheet will cost you 5 points. :)

Good luck!

Name: _____

Entry Number: _____

Mathematics and Puzzles

1. (3 points) A standard chessboard is a grid of 8 by 8 squares. You have dominoes that cover 2 squares at a time (that is, they measure 2 by 1).
   If two opposite corners of the chessboard are removed, in how many ways can you cover the chessboard. [**Difficulty** : 4/10]

   A. < 100

   B. 100 to 1000

   C. 1000 to 10^8

   D. $> 10^8$

2. (5 points) You are a junior trader, your boss gave you 9000 INR capital. You can trade with INR, USD, EUR. The exchange rates are
   EUR/INR = 0.8
   EUR/USD = 1.2
   INR/USD = 2
   What's the maximum profit you can make in a day, assuming your boss makes you work 6 hrs a day. You are only allowed to do one transaction in one hour. [**Difficulty** : 5/10]

3. (5 points) Tejas and Pratyush have 4 and 3 coins respectively. What is the probability after they toss all their coins simultaneously Tejas has more HEADS than Pratyush ? [**Difficulty** : 5/10]

4. (6 points) What is the expected no. of tosses to get 8 consecutive heads for the first time ? [**Difficulty** : 6/10]

5. (8 points) Pravar decides to contest in the Class Representative Elections. His opponent was very strong. Total 50 votes were casted. His friend Tejas calculated that there was a 92% chance of his opponent being in a strict lead throughout the counting if the votes are counted in a random order. How many votes did Pravar get ? [**Difficulty** : 7/10]

6. (6 points) We have 2 cars parked at locations 1 and 2 of the parking lot. Spaces 3 and 4 are empty. Every minute a car is eligible to move forward if and only if the following 2 conditions are satisfied.

- The Space in front of them should be empty prior to any movement.
- Their parking label should be less than 4.

What is the expected time in which the parking spots 3 and 4 are filled in minutes ? [**Difficulty** : 6/10]

7. (8 points) Tejas decided to play a game with pratyush. At each turn he can choose 'Place' or 'Take'. There are 2 boxes. 'Place' Increments a rupee in one of the boxes randomly, 'Take' will make pratyush empty one of the boxes randomly and add the amount in Tejas's account (Tejas doesn't know the amount added to his account at any stage during the game). There are 100 turns. Tejas decided to play optimally. How many times did he say 'Place' ?

   [**Difficulty** : 7/10]

8. (6 points) How many ways to place 31 knights on the cell of a chessboard so that no two attack each other. Hint: The answer is not 64.

   [**Difficulty** : 6/10]

9. (8 points) Pravar lives on the origin of the Cartesian plane. Every second Pravar moves 1 unit up with probability 2/9 1 unit right 2/9 and 1 unit right and up simultaneously with probability 4/9. He does not move with probability with 1/9. After 2022 seconds Pravar ends up on (a, b) What is the value of $\sqrt{ab}$.

   [**Difficulty** : 7/10]

10. There is a bank in the town that goes by the name 'CONTINUUM BANK LTD.' originally built by Parth Patel: They have a scheme deposit X rupees and by the end of T days we will give you T% rate of interest. For example if you keep the money there for 100 days your money doubles (100% rate of interest)

    (a) (2 points) Suren stores the money for 110 Days. Raghav on the other hand deposits the same amount of money and takes it out on 50th Day. Then he redeposits the

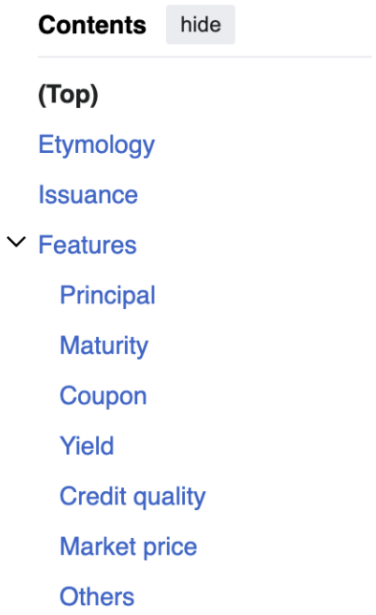
money that he took out of the bank for another 50 days. Who has more money in the bank ? [**Difficulty** : 1/10]

(b) (3 points) You are given 100 days time period and 100 Rupees. Assume that you cannot make more than 1 deposit in a day. Suppose you can make X amount of money at the end. Report $\lfloor X/10 \rfloor$ [**Difficulty** : 3/10]

11. (2 points) For a set of five whole numbers, the mean is 4, the mode is 1, and the median is 5. What are the five numbers? [**Difficulty** : 2/10]

12. (3 points) Imagine you have 100 kilograms of food that is 99 percent water by weight. In the hot sun, the food dehydrates until it is 98 percent water. How much will the food weigh? Assume the only weight loss is from water evaporating away. [**Difficulty** : 3/10]

Theory of Assets and Markets



13. (2 points) The above screenshot is taken from the wikipedia page of which of the following financial instruments ? [**Difficulty** : 3/10]

    A. Options

    B. Bonds

    C. Futures

    D. Mutual Funds

14. (2 points) You suspect that a major news is going to be uncovered in the future about the firm X but are uncertain about the nature of this news. You want to bet on X. What should you do for the optimum reward/risk ?

    [**Difficulty** : 3/10]

    A. Buy puts of X

    B. Sell calls of X

    C. Buy puts and Buy calls of X

    D. Sell puts and Buy calls of X

15. (3 points) An investor purchases 100 shares of XYZ at 60 and also writes an XYZ 65 call @3. What is the investor's maximum potential loss ?

    [**Difficulty** : 3/10]

16. (3 points) An investor purchases 100 shares of XYZ at 60 and also writes an XYZ 65 call @3. If the call is exercised when the market price of XYZ is 70, what is the investor's profit?

    [**Difficulty** : 3/10]

17. (3 points) If one asset has standard deviation σ_1, another asset has standard deviation σ_2. Suppose that the correlation between the asset prices is $|\rho| < 1$. You buy 1 unit of asset 1 and 1 unit of asset 2. What is the standard deviation of your portfolio?

[**Difficulty** : 3/10]

18. (4 points) You buy 1 unit of asset 1, and x units of asset 2 (Negative x indicates selling). Find the value of x such that the RISK (i.e Standard Deviation) of the portfolio is 0 units. (i.e. SD is 0)

    A. 1

    B. 0

    C. -1

    D. ρ

    E. $-\rho$

    F. ρ^{-1}

    G. $-\rho^{-1}$

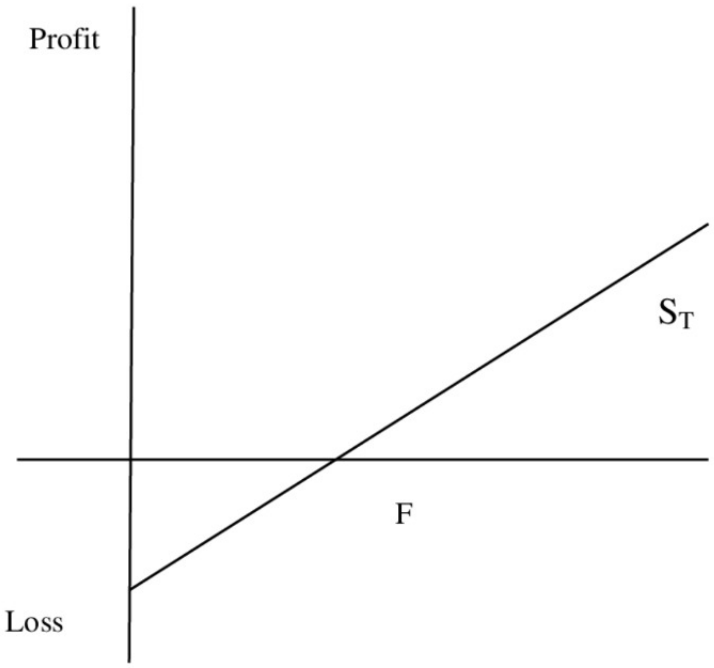
    H. There exists no such x (you cannot make risk = 0)

    [**Difficulty** : 4/10]

Payoff Graphs

Payoff graphs are graphs that describe the payoff to a person in different scenarios... The X axis represents the different scenarios and the Y axis represents the various payoffs. Here we will discuss the payoff graphs of bonds, options and futures. (Assume European options (If you don't know what this means just ignore it)). For these assets, on the X axis we plot the spot price... or say the price at expiry of the underlying. On the Y axis there is profit or payoff in all the scenarios.

For example... The Payoff graph for a future is :



Here F is the strike price which means the price at which the buyers and sellers pre decided to exchange the asset. Say A and B decided that B will sell A, 1 kilo of wheat on 31st June for a price of 40 Rupees. Now if on 31st June, the market price of wheat is 40 Rupees then both A and B are indifferent about the future. It is like selling or buying it from the market.

So , if finally the actual price in the market is also F the 0 profit. Now if the Final price is 30 Rupees then A is happy (i.e. profit) because he buys 1 Kg of wheat at 30 from B which would have cost him 40 from the market. The opposite is also correct.

Assume bonds are risk free and stop thinking about arbitrage. Assume that for a fixed asset there is an infinite buffet of bonds, futures and options.

You can get one for any interest rate or strike price respectively.

19. Now answer the following questions

    (a) (1 point) Find a mistake in the paragraph above? (Please ignore grammatical mistakes) Hint : The intended mistake is not technical. [**Difficulty** : 1/10]

(b) (3 points) What does a payoff chart for a bond look like? [**Difficulty** : 4/10]

(c) (4 points) What does a payoff chart for a call option look like? [**Difficulty** : 5/10]

(d) (3 points) Can an arbitrary payoff chart be approximated using just many BONDS assuming you have the same underlying but possibly different interest rates (in other words just using bond for a same asset, can a person create any payoff chart that he thinks is suitable for him/her) True/False [**Difficulty** : 4/10]

(e) (4 points) Can an arbitrary payoff chart be approximated using just many BONDS and FUTURES assuming you have the same underlying for everything.(in other words just using bond for a same asset, can a person create any payoff chart that he thinks is suitable for him/her) True/False [**Difficulty** : 5/10]

(f) (4 points) Can an arbitrary payoff chart be approximated using just many BONDS and OPTIONS assuming you have the same underlying for everything.(in other words just using bond for a same asset, can a person create any payoff chart that he thinks is suitable for him/her) True/False [**Difficulty** : 5/10]

20. (5 points) This question is multiple choice multiple correct type. A stock moves up 10%

up or 10% down in one day and nothing in between. Both with a probability of 0.5. Given the value of the stock today is 1 dollar, What is the expected value of the stock after 1000 days ? Given you want to gain money with more than 50% probability, would you buy such a stock ? [**Difficulty** : 5/10]

A. EV = $(0.99)^{50}$ $
B. EV = $(1.01)^{50}$ $
C. EV = $(0.99)^{100}$ $
D. EV = 1 $
E. I wouldn't invest because EV < 1
F. I would invest because EV > 1
G. I would invest even though EV $= 1$
H. I wouldn't invest even though EV $= 1$

Algorithms

21. You are Thanos and you are given an array, any array or list. All numbers in the list are distinct. And because you are Thanos, you like everything balanced... in this context you want the list sorted. And because you are Thanos you can snap your fingers to make exactly half of the elements disappear... $\lfloor n/2 \rfloor$. You want to make the final array balanced, tell how many snaps do you need. For example if your list was [1,3,2] you could remove $\lfloor n/2 \rfloor = 1$ elements disappear. So you snap and in 1 snap you make '3' disappear. Now find how many operations you would need in the following lists.

    1. [9, 12 , 15 , 22 ,5 , 21]
    2. [16 , 5, 14, 13 , 25, 2, 12, 1 ,3, 11 , 6 ,15 , 24, 23 , 9 , 20 , 8 , 4 , 18 , 21 , 7 , 19]

    [2.5 mark + 3.5 marks]

    Bonus : Find the secret phrase (Hint : 'pen my black fox with drugs' has no repeating letters) [2 marks] [**Difficulty** : 4/10]

22. (5 points) You have been given a set of n integers and you have to find the min and max of the sequence using the minimum number of comparisons. A comparison tells you a larger and a smaller number of the two. Let the number of minimum comparisons be $f(n)$. Find the value of $\lfloor \frac{2f(n)}{n} \rfloor$ [**Difficulty** : 5/10]

23. (6 points) There are n heaps of sticks and Pravar and Tejas move alternately. On each move, either Tejas or Pravar chooses a non-empty heap and removes 1, 2, or 3 sticks.

The player who removes the last stick wins the game. Your task is to find out who wins if both players play optimally. The stick are "5, 7, 6, 5" in the 4 heaps. [**Difficulty** : 7/10]

24. (6 points) Sort a random array using bubble sort. What is the expected no. of swaps for $n = 13$? [**Difficulty** : 7/10]

Modelling

Assume a rational buyer who always acts on maximising his/her utility. The rational buyer has a major utility component of expected value of his asset and some negative utility for risk of his asset.

Let R denote the Risk (Standard Deviation in rough sense and E denote Expected Value in rough sense) Everything is normalised to make them unit free.

Take 2 people with utilities as such:
$U_1(E, R) = E - 0.1R$
$U_2(E, R) = E - 0.2R$

25. (3 points) Which of the above agents is more risk averse (hates risk) ? [**Difficulty** : 2/10]

26. (3 points) Now consider the following Utility equation.
$U(E, R) = E - ER - ER^2$ [**Difficulty** : 3/10]

Metaquestions (Questions about Questions)

27. What do you think is the expected score for the following quiz apart from this question and the next ? Your marks are a function of how close you are to the actual answer
Explicit : within 5% 4 points
Within 10% 3 points

Within 25% 2 points
Within 50% 1 point Else : 0 marks

28. (3 points) What do you think the deviation of average of answers to the previous question will be from actual ?

    A. -10% **to** 0%

    B. 0% **to** 10%

    C. 10% **or above**

    D. -10% **or below**

    Hints : We ourselves do not know the answer but some things that might help are : Law of Large Numbers AND Notice a little bit of asymmetry in the above question w.r.t direction.... AND What is the psychology of people giving the test.

 End of exam.