Formalization of TSP problem with multiple drones as a mixed integer linear problem

Anda Buinoschi MOC2, Bogdan Luncașu MOC2

Abstract

This paper is a formalization for the problem of delivering parcels to customers with the help of a truck and multiple drones. It is an extension from classical TSP problem, but it can save more time and also it enables customers unequally distributed to receive their packages by drone. Our experiments that we ran prove that employing more drones to be launched, we quantify potential time savings.

1 Introduction

This paper is an introduction for the TSP-D problem (sometimes called Multiple Flying Sidekicks Traveling Salesman Problem or mFSTSP for short) and we did a formalization for it as a constraint linear problem. The problem states that a truck and a number of drones (or unmanned aerial vehicles, UAVs) deliver together a set of parcels for a set of customers geographically distributed in different places. The drone has the limit to deliver only one package and then it should return to the truck to take another and to make a new delivery. The final objective of the problem is to minimize the total time of the delivery process after successfully landed parcels to the customers.

2 Problem definition

Let C represent the set of customer parcels, such that $C = \{1, 2, ..., c\}$. Each customer must receive exactly one delivery by either the single truck or one of the drones available denoted by the set V. A particular customer $i \in C$ is "droneable" by UAV $v \in V$, belonging to the set $\hat{C}_v$ if the customer's parcel is eligible for drone delivery (it is not too heavy or any other restriction).

Each of the drone can only deliver one parcel at a time. It may be launched from any point on the map, either customer location or depot point. While it can be launched multiple times during the delivery process, it cannot leave the same location more than once. It can land at depot point, be retrieved by the truck at a customer's location, but it cannot land at the same customer location from which it was deployed. When a drone arrives to the truck, it may be loaded onto the truck and follow it for several locations or it can be launched with a new package. While the UAVs are airborne, the truck may deliver packages continuing its path.

The set of nodes are defined to be $N = \{0, 1, ..., c + 1\}$ where 0 and $c + 1$ represent the depot from which they start traveling and in which they have to return. The set of nodes from which the truck can depart are represented by $N_0 = \{0, 1, ..., c\}$, while $N_+ = \{1, 2, ..., c+1\}$ are the set of nodes which the truck may visit.

The truck's travel time from a node $i \in N_0$ to $j \in N_+$ is given by τ_{ij}. Also, τ'_{vij} represents the time required for a UAV $v \in V$ to fly from node $i \in N_0$ to node $j \in N_+$.

When UAV $v \in V$ is launched from node $i \in N_0$, it requires $s^L_{v,i}$ units of time. The launch is indexed on v to keep track of differences in UAVs. The launch time is also indexed on the launch location; launching from the depot may require a different amount of time. The recovery time, $s^R_{v,k}$ is similarly defined for UAV $v \in V$ retrieved at node $k \in N_+$.

Truck deliveries require σ_k units of time for service at node $k \in N_+$, where $\sigma_{c+1} \equiv 0$ as there is no delivery at the depot. Similarly, UAV $v \in V$ requires σ'_{vk} units of time to perform the delivery service at node $k \in N_+$, where $\sigma'_{v,c+1} \equiv 0$.

The summary of the parameter notation is below in Table 1.

V	Set of UAVs.
C	Set of customers; $C = \{1, 2, ..., c\}$.
$\hat{C}_v$	Set of customers that may be served by UAV $v \in V$; $\hat{C}_v \subseteq C$ for all $v \in V$.
N	Set of all nodes; $N = \{0, 1, ..., c + 1\}$.
N_0	Set of nodes from which a vehicle may depart; $N_0 = \{0, 1, ..., c\}$.
N_+	Set of nodes to which a vehicle may visit; $N_+ = \{1, 2, ..., c + 1\}$.
τ_{ij}	Truck's travel time from node $i \in N_0$ to node $j \in N_+$.
τ'_{vij}	Travel time for UAV $v \in V$ from node $i \in N_0$ to node $j \in N_+$.
$s^L_{v,i}$	Launch time for UAV $v \in V$ from node $i \in N_0$.
$s^R_{v,k}$	Recovery time for UAV $v \in V$ at node $k \in N_+$.
σ_k	Service time by truck at node $k \in N_+$, where $\sigma_{c+1} \equiv 0$.
σ'_{vk}	Service time by UAV $v \in V$ at node $k \in N_+$, where $\sigma'_{v,c+1} \equiv 0$.
P	A set of tuples of the form $\langle v, i, j, k\rangle$, specifying all possible three-node sorties that may be flown by UAV $v \in V$.
e_{vijk}	Endurance, in units of time, for UAV $v \in V$ traveling from nodes $i \in N_0$ to $j \in \hat{C}_v$ to $k \in N_+$.

Table 1: Parameters notation

3 Formalization as a MILP

3.1 Drone endurance

Each of the drones $v \in V$ has a unique endurance, represented as e_{vijk} and measured in units of time, for which it can be operated and launch from node $i \in N_0$, deliver to $j \in \hat{C}_v$ and then meet the truck to $k \in N_+$. These constraints are critical, as UAV operations can be affected by their limited battery capacity.

To identify potential valid UAV sorties (the sequence of a launch, customer delivery, and rendezvous), P is defined to be a set of four-tuples of the form $\langle v, i, j, k \rangle$ for $v \in V$, $i \in N_0$, $j \in \hat{C}_v$, and $k \in N_+$. This set has the properties:

- The launch node, i, must not be the ending depot node (i is restricted to N_0);
- The delivery node, j, must be an eligible customer for UAV v ($j \in \{\hat{C}_v : j \neq i\}$);
- The rendezvous point, k, may be either a customer or the ending depot (but it must not be either node i or j); and
- The drone's travel time from $i \to j \to k$ must not exceed the endurance of the UAV ($\tau'_{vij} + \sigma'_{vj} + \tau'_{vjk} \leqslant e_{vijk}$ for $k \in \{N_+ : k \neq j, k \neq i\}$).

3.2 Objective and decision variables

The objective of the TSP-D is to minimize the time required to deliver all parcels and return to the depot. This is done by determining the decision variables values across six main classes, which are shown in Table 2. First, binary decision variable $x_{ij} = 1$ if the truck travels from node $i \in N_0$ right to node $j \in \{N_+ : j \neq i\}$. This decision variable determines the route of the delivery truck. Similarly, in the second class, binary decision variable $p_{ij} = 1$ if the truck visits node $i \in N_0$ at some time prior to visiting node $j \in \{C : j \neq i\}$. We define $p_{0j} \equiv 1$ for all $j \in C$ to indicate that the truck must leave the depot (node 0).

In the third class, binary decision variable $y_{vijk} = 1$ if UAV $v \in V$ travels from node $i \in N_0$ to customer $j \in \{\hat{C}_v : j \neq i\}$, re-joining the truck at node $k \in \{N_+ : \langle v, i, j, k \rangle \in P\}$. This decision variable identifies UAV sorties.

The fourth class involves five continuous decision variables to determine the time at which key events for truck and drones occur. Specifically, $\check{t}_i \geqslant 0$ captures the truck's arrival time to node $i \in N$, where $\check{t}_0 \equiv 0$ to indicate that the truck is available to begin operations at time zero. The truck's service time completion at node $i \in N_+$ is given by $\bar{t}_i \geqslant 0$, where $\bar{t}_0 \equiv 0$ to reflect the fact that there is no customer associated with depot. This decision variable indicates the time at which customer i's parcel has been delivered. Next, $\hat{t}_i \geqslant 0$ identifies the truck's completion time at node $i \in N$ (the earliest departure time from this node if $i \in N_0$). Similarly, timing for the UAVs is determine by $\check{t'}_{vi} \geqslant 0$, which denotes the arrival time for UAV $v \in V$ to node $i \in N$, and $\hat{t'}_{vi} \geqslant 0$, which identifies the completion time for UAV $v \in V$ at node $i \in N$.

Next, numerous binary decision variables (all identified by the letter z either sub- and -super scripts) are used to determine the coordination between the driver of the truck and each drone, and to establish the sequencing of UAV launches and retrievals for each node.

Finally, $1 \leqslant u_i \leqslant c+2$ are standard truck subtour elimination variables, defined for all $i \in N_+$, that indicate the relative ordering of visits to node i. In the following subsections, we are formulating the constraints based on the belonging class for simplicity.

3.3 Core constraints for TSP-D

$$
\begin{aligned}
\min \quad & \check{t}_{c+1} \\
\text{s.t.} \quad & \sum_{\substack{i \in N_0 \\ i \neq j}} x_{ij} + \sum_{v \in V} \sum_{\substack{i \in N_0 \\ i \neq j}} \sum_{\substack{k \in N_+ \\ \langle v,i,j,k \rangle \in P}} y_{vijk} = 1, \forall j \in C, \\
& \sum_{j \in N_+} x_{0j} = 1, \\
& \sum_{i \in N_0} x_{i,c+1} = 1, \\
& \sum_{\substack{i \in N_0 \\ i \neq j}} x_{ij} = \sum_{\substack{k \in N_+ \\ k \neq j}} x_{jk}, \forall j \in C, \\
& \sum_{\substack{j \in C \\ j \neq i}} \sum_{\substack{k \in N_+ \\ \langle v,i,j,k \rangle \in P}} y_{vijk} \leqslant 1, \forall i \in N_0, \forall v \in V, \\
& \sum_{\substack{i \in N_0 \\ i \neq k}} \sum_{\substack{j \in C \\ \langle v,i,j,k \rangle \in P}} y_{vijk} \leqslant 1, \forall k \in N_+, \forall v \in V, \\
& 2y_{vijk} \leqslant \sum_{\substack{h \in N_0 \\ h \neq i}} x_{hi} + \sum_{\substack{l \in C \\ l \neq k}} x_{lk}, \forall v \in V, i \in C, j \in \{C : j \neq i\}, k \in \{N_+ : \langle v,i,j,k \rangle \in P\}, \\
& y_{v0jk} \leqslant \sum_{\substack{h \in N_0 \\ h \neq k}} x_{hk}, \forall v \in V, j \in C, k \in \{N_+ : \langle v,0,j,k \rangle \in P\}, \\
& u_k - u_i \geqslant 1 - (c+2)\left(1 - \sum_{\substack{j \in C \\ \langle v,i,j,k \rangle \in P}} y_{vijk}\right), \forall i \in C, k \in \{N_+ : k \neq i\}, \forall v \in V, \\
& u_i - u_j + 1 \leqslant (c+2)(1 - x_{ij}), \forall i \in C, j \in \{N_+ : j \neq i\}, \\
& u_i - u_j \geqslant 1 - (c+2)p_{ij}, \forall i \in C, j \in \{C : j \neq i\}, \\
& u_i - u_j \leqslant -1 + (c+2)(1 - p_{ij}), \forall i \in C, j \in \{C : j \neq i\}, \\
& p_{ij} + p_{ji} = 1, \forall i \in C, j \in \{C : j \neq i\}.
\end{aligned}
$$

$x_{ij} \in \{0,1\}$	$x_{ij} = 1$ if the truck travels from node $i \in N_0$ immediately to node $j \in \{N_+ : j \neq i\}$.
$p_{ij} \in \{0,1\}$	$p_{ij} = 1$ if node $i \in N_0$ appears in the truck's route before node $j \in \{C : j \neq i\}$.$p_{0j} \equiv 1$ for all $j \in C$.
$y_{vijk} \in \{0,1\}$	$y_{vijk} = 1$ if UAV $v \in V$ travels from node $i \in N_0$ to customer $j \in \{\check{C}_v : j \neq i\}$, re-joining the truck at node $k \in \{N_+ : \langle v,i,j,k \rangle \in P\}$.
$\check{t}_i \geqslant 0$	Truck's arrival time to node $i \in N$, where $\check{t}_0 \equiv 0$.
$\bar{t}_i \geqslant 0$	Truck's service time completion at node $i \in N_+$, where $\bar{t}_0 \equiv 0$.
$\hat{t}_i \geqslant 0$	Truck's completion time at node $i \in N$ (the earliest departure time from this node if $i \in N_0$). If the truck is not required to be at the depot when UAVs launch, then $\check{t}_0 \equiv 0$.
$\check{t'}_{vi} \geqslant 0$	Arrival time for UAV $v \in V$ to node $i \in N$.
$\hat{t'}_{vi} \geqslant 0$	Completion time for UAV $v \in V$ at node $i \in N$.
$z^R_{v1,v2,k} \in \{0,1\}$	$z^R_{v1,v2,k} = 1$ if $v1 \in V$ and $v2 \in \{V : v2 \neq v1\}$ are recovered at node $k \in N_+$, such that $v1$ is recovered before $v2$.
$z^R_{0,v,k} \in \{0,1\}$	$z^R_{0,v,k} = 1$ if the truck completes its service activities at node $k \in N_+$ before UAV $v \in V$ is retrieved at node k.
$z^R_{v,0,k} \in \{0,1\}$	$z^R_{v,0,k} = 1$ if UAV $v \in V$ is retrieved at node $k \in N_+$ before the truck completes its service activities at node k. We define $z^R_{v,0,c+1} \equiv 0$ for all $v \in V$ (since the truck has no service activities at the depot node, the order does not matter).
$z^L_{v1,v2,i} \in \{0,1\}$	$z^L_{v1,v2,i} = 1$ if UAV $v1 \in V$ is launched from node $i \in N_0$ before $v2 \in \{V : v2 \neq v1\}$ is launched from i.
$z^L_{0,v,i} \in \{0,1\}$	$z^L_{0,v,i} = 1$ if the truck completes its service activities at node $i \in N_0$ before UAV $v \in V$ is launched from i.
$z^L_{v,0,i} \in \{0,1\}$	$z^L_{v,0,i} = 1$ if UAV $v \in V$ is launched from node $i \in N_0$ before the truck completes its service activities at node i. If the truck is not required to be present when UAVs launch from the depot, we may define $z^L_{v,0,0} = 0$ for all $v \in V$ (since the truck has no service activities at the depot node, the order does not matter).
$z'_{v1,v2,i} \in \{0,1\}$	$z'_{v1,v2,i} = 1$ if UAV $v1 \in V$ launches from node $i \in C$ before UAV $v2 \in \{V : v2 \neq v1\}$ lands at i.
$z''_{v1,v2,i} \in \{0,1\}$	$z''_{v1,v2,i} = 1$ if UAV $v1 \in V$ lands at node $i \in C$ before UAV $v2 \in \{V : v2 \neq v1\}$ launches from i.
$1 \leqslant u_i \leqslant c+2$	Truck subtour elimination variables, defined for all $i \in N_+$, which indicate the relative ordering of visits to node i.

Table 2: Decision variables

3.4 UAV timing constraints

The following constraints establish the times at which each UAV launches from either the depot or the truck, arrives at a customer location, and returns to either the truck or the depot.

$$\hat{t'}_{vl} \geqslant \check{t'}_{vk} - M\left(3 - \sum_{\substack{j\in C\\ \langle v,i,j,k\rangle\in P\\ j\neq l}} y_{vijk} - \sum_{\substack{m\in C\\ m\neq i\\ m\neq k\\ m\neq l}} \sum_{\substack{n\in N_+\\ \langle v,l,m,n\rangle\in P\\ n\neq i\\ n\neq k}} y_{vlmn} - p_{il}\right)$$

$$\forall i \in N_0, k \in \{N_+ : k \neq i\}, l \in \{C : l \neq i, l \neq k\}, \forall v \in V,$$

$$\hat{t'}_{vi} \geqslant \check{t'}_{vi} + s^L_{v,i} - M\left(1 - \sum_{\substack{j\in C\\ j\neq i}} \sum_{\substack{k\in N_+\\ \langle v,i,j,k\rangle\in P}} y_{vijk}\right) \forall v \in V, i \in N_0,$$

$$\hat{t'}_{vi} \geqslant \check{t}_i + s^L_{v,i} - M(1 - z^L_{v0i}) \forall v \in V, i \in N_0,$$

$$\hat{t'}_{vi} \geqslant \bar{t}_i + s^L_{v,i} - M(1 - z^L_{0vi}) \forall v \in V, i \in N_0,$$

$$\hat{t'}_{vi} \geqslant \hat{t'}_{v2,i} + s^L_{v,i} - M(1 - z^L_{v2,v,i}) \forall v \in V, v2 \in \{V : v2 \neq v\}, i \in N_0,$$

$$\hat{t'}_{v2,i} \geqslant \check{t'}_{vi} + s^L_{v2,i} - M(1 - z''_{v,v2,i}) \forall v \in V, v2 \in \{V : v2 \neq v\}, i \in C,$$

$$\check{t'}_{vj} \geqslant \hat{t'}_{vi} + \tau'_{vij} - M\left(1 - \sum_{\substack{k\in N_+\\ \langle v,i,j,k\rangle\in P}} y_{vijk}\right) \forall v \in V, j \in C, i \in \{N_0 : i \neq j\},$$

$$\check{t'}_{vj} \leqslant \hat{t'}_{vi} + \tau'_{vij} + M\left(1 - \sum_{\substack{k\in N_+\\ \langle v,i,j,k\rangle inP}} y_{vijk}\right) \forall v \in V, j \in C, i \in \{N_0 : i \neq j\},$$

$$\hat{t'}_{vj} \geqslant \check{t'}_{vj} + \sigma'_{vj} \sum_{\substack{i\in N_0\\ i\neq j}} \sum_{\substack{k\in N_+\\ \langle v,i,j,k\rangle\in P}} y_{vijk} \forall v \in V, j \in C,$$

$$\hat{t'}_{vj} \leqslant \check{t'}_{vj} + \sigma'_{vj} + M\left(1 - \sum_{\substack{i\in N_0\\ i\neq j}} \sum_{\substack{k\in N_+\\ \langle v,i,j,k\rangle\in P}} y_{vijk}\right) \forall v \in V, j \in C,$$

$$\check{t'}_{vk} \geqslant \check{t}_k + s^R_{v,k} - M(1 - z^R_{v0k}) \forall v \in V, k \in N_+,$$

$$\check{t'}_{vk} \geqslant \bar{t}_k + s^R_{v,k} - M(1 - z^R_{0vk}) \forall v \in V, k \in N_+,$$

$$\check{t'}_{vk} \geqslant \check{t'}_{v2,k} + s^R_{v,k} - M(1 - z^R_{v2,v,k}) \forall v \in V, v2 \in \{V : v2 \neq v\}, k \in N_+,$$

$$\check{t'}_{vk} \geqslant \hat{t'}_{v2,k} + s^R_{v,k} - M(1 - z'_{v2,v,k}) \forall v \in V, v2 \in \{V : v2 \neq v\}, k \in C,$$

$$\check{t'}_{vk} \geqslant \hat{t'}_{vj} + \tau'_{vjk} + s^R_{v,k} - M\left(1 - \sum_{\substack{i\in N_0\\ \langle v,i,j,k\rangle\in P}} y_{vijk}\right) \forall v \in V, k \in N_+, j \in \{C : j \neq k\},$$

$$(\check{t'}_{vk} - s^R_{v,k}) - \hat{t'}_{vi} \leqslant e_{vijk} + M(1 - y_{vijk}) \forall v \in V, i \in N_0, j \in \{\hat{C}_v : j \neq i\}, k \in \{N_+ : \langle v,i,j,k\rangle \in P\}$$

3.5 Truck timing constraints

The following constraints govern the arrival, service, and departure activities for the truck.

$$\check{t_j} \geqslant \hat{t}_i + \tau_{ij} - M(1 - x_{ij}) \forall i \in N_0, j \in N_+ : j \neq i\},$$

$$\bar{t_k} \geqslant \check{t_k} + \sigma_k \sum_{\substack{j \in N_0 \\ j \neq k}} x_j k \forall k \in N_+,$$

$$\bar{t_k} \geqslant \check{t}'_{vk} + \sigma_k - M(1 - z^R_{v0k}) \forall kinN_+, v \in V,$$

$$\bar{t_k} \geqslant \hat{t}'_{vk} + \sigma_k - M(1 - z^L_{v0k}) \forall k \in C, v \in V,$$

$$\hat{t_k} \geqslant \bar{t_k} \forall k \in N_+,$$

$$\hat{t_k} \geqslant \check{t}'_{vk} - M\left(1 - \sum_{\substack{i \in N_0 \\ i \neq k}} \sum_{\substack{j \in C \\ \langle v,i,j,k \rangle \in P}} y_{vijk}\right) \forall k \in N_+, v \in V,$$

$$\hat{t_k} \geqslant \hat{t}'_{vk} - M\left(1 - \sum_{\substack{l \in C \\ l \neq k}} \sum_{\substack{m \in N_+ \\ \langle v,k,l,m \rangle \in P}} y_{vklm}\right) \forall k \in N_0, v \in V.$$

3.6 Sequencing of retrievals, launches, and truck service

In this section, constraints are provided to establish proper values of the binary decision variables used to sequence the activities at each node.

$$z^R_{0vk} + z^R_{v0k} = \sum_{\substack{i \in N_0 \\ i \neq k}} \sum_{\substack{j \in C \\ \langle v,i,j,k \rangle \in P}} y_{vijk} \forall v \in V, k \in N_+,$$

$$z^R_{v,v2,k} \leqslant \sum_{\substack{i \in N_0 \\ i \neq k}} \sum_{\substack{j \in C \\ \langle v,i,j,k \rangle \in P}} y_{vijk} \forall v \in V, v2 \in \{V : v2 \neq v\}, k \in N_+,$$

$$z^R_{v,v2,k} \leqslant \sum_{\substack{i \in N_0 \\ i \neq k}} \sum_{\substack{j \in C \\ \langle v2,i,j,k \rangle \in P}} y_{v_2ijk} \forall v \in V, v_2 \in \{V : v_2 \neq v\}, k \in N_+,$$

$$z^R_{v,v2,k} + z^R_{v2,v,k} \leqslant 1 \forall v \in V, v_2 \in \{V : v_2 \neq v\}, k \in N_+,$$

$$z^R_{v,v_2,k} + z^R_{v_2,v,k} + 1 \geqslant \sum_{\substack{i \in N_0 \\ i \neq k}} \sum_{\substack{j \in C \\ \langle v,i,j,k \rangle \in P}} y_{vijk} + \sum_{\substack{i \in N_0 \\ i \neq k}} \sum_{\substack{j \in C \\ \langle v_2,i,j,k \rangle \in P}} y_{v_2ijk} \forall v \in V, v_2 \in \{V : v_2 \neq v\}, k \in N_+.$$

$$z^L_{0vi} + z^L_{v0i} = \sum_{\substack{j \in C \\ j \neq i}} \sum {}^{k \in N_+}_{\langle v,i,j,k \rangle \in P} y_{vijk} \forall v \in V, i \in N_0,$$

$$z^L_{v,v_2,i} \leqslant \sum_{\substack{j\in C\\ j\neq i}} \sum_{k\in N_+ \langle v,i,j,k\rangle \in P} y_{vijk} \forall v \in V, v_2 \in \{V : v_2 \neq v\}, i \in N_0,$$

$$z^L_{v,v_2,i} \leqslant \sum^{j\in C}_{j\neq i} \sum_{\substack{k\in N_+\\ \langle v_2,i,j,k\rangle \in P}} y_{v_2ijk} \forall v \in V, v_2 \in \{V : v_2 \neq v\}, i \in N_0,$$

$$z^L_{v,v_2,i} + z^L_{v_2,v,i} \leqslant 1 \forall v \in V, v_2 \in \{V : v_2 \neq v\}, i \in N_0,$$

$$z^L_{v,v_2,i}+z^L_{v_2,v,i}+1 \geqslant \sum_{\substack{j\in C\\ j\neq i}} \sum_{\substack{k\in N_+\\ \langle v,i,j,k\rangle \in P}} y_{vijk}+\sum_{\substack{j\in C\\ j\neq i}} \sum_{\substack{k\in N_+\\ \langle v_2,i,j,k\rangle \in P}} y_{v_2,i,j,k} \forall v \in V, v_2 \in \{V : v_2 \neq v\}, i \in N_0.$$

$$z'_{v_2,v,k} \leqslant \sum_{\substack{l\in C\\ l\neq k}} \sum_{\substack{m\in N_+\\ \langle v_2,k,l,m\rangle \in P}} y_{v_2,k,l,m} \forall v_2 \in V, v \in \{V : v \neq v_2\}, k \in C,$$

$$z''_{v_2,v,k} \leqslant \sum_{\substack{l\in C\\ l\neq k}} \sum_{\substack{m\in N_+\\ \langle v,k,l,m\rangle \in P}} y_{v,k,l,m} \forall v \in V, v_2 \in \{V : v_2 \neq v\}, k \in C,$$

$$z'_{v_2,v,k} \leqslant \sum_{\substack{i\in N_0\\ i\neq k}} \sum_{\substack{j\in C\\ \langle v,i,j,k\rangle \in P}} y_{vijk} \forall v \in V, v_2 \in \{V : v_2 \neq v\}, k \in C,$$

$$z''_{v_2,v,k} \leqslant \sum_{\substack{i\in N_0\\ i\neq k}} \sum_{\substack{j\in C\\ \langle v_2,i,j,k\rangle \in P}} y_{v_2,i,j,k} \forall v_2 \in V, v \in \{V : v \neq v_2\}, k \in C,$$

$$z'_{v_2,v,k}+z''_{v,v_2,k}+1 \geqslant \sum_{\substack{i\in N_0\\ i\neq k}} \sum_{\substack{j\in C\\ \langle v,i,j,k\rangle \in P}} y_{vijk}+\sum_{\substack{l\in C\\ l\neq k}} \sum_{\substack{m\in N_+\\ \langle v_2,k,l,m\rangle \in P}} y_{v_2klm} \forall v \in V, v_2 \in \{V : v_2 \neq v\}, k \in C,$$

$$z'_{v_2,v,k} + z''_{v,v_2,k} \leqslant 1 \forall v \in V, v_2 \in \{V : v_2 \neq v\}, k \in C,$$
$$z'_{v_2,v,k} + z'_{v,v_2,k} \leqslant 1 \forall v \in V, v_2 \in \{V : v_2 \neq v\}, k \in C,$$
$$z''_{v_2,v,k} + z''_{v,v_2,k} \leqslant 1 \forall v \in V, v_2 \in \{V : v_2 \neq v\}, k \in C.$$

3.7 Variant 1: Truck not required at depot

The model above assumes that the truck must be present at the depot when UAVs are launched, and must also be at the depot when UAVs return. This reflects the case that the driver is responsible for manually performing these activities. However, the model may be relaxed to allow the UAVs to launch from, and return to, the depot independent of the driver.

$$\check{t'}_{vk} \geqslant \check{t_k} + s^R_{v,k} - M(1 - z^R_{v0k}) \forall v \in V, k \in C,$$

$$\check{t'}_{vk} \geqslant \bar{t_k} + s^R_{v,k} - M(1 - z^R_{0vk}) \forall v \in V, k \in C,$$

$$\bar{t_k} \geqslant \check{t'}_{vk} + \sigma_k - M(1 - z^R_{v0k}) \forall v \in V, k \in C,$$

$$\hat{t_k} \geqslant \hat{t'}_{vk} - M\left(1 - \sum_{\substack{l\in C\\ l\neq k}} \sum_{\substack{m\in N_+\\ \langle v,k,l,m\rangle \in P}} y_v klm\right) \forall k \in C, v \in V.$$

3.8 Variant 2: Automated launch and recovery systems

The default mFSTSP model assumes that the truck driver must be engaged in the UAV launch and recovery process at customer locations. In order to satisfy automated lunch and recovery system of the drones, the following restrictions should be added:

$$\hat{t}'_{vi} \geqslant \check{t}_i + s^L_{v,i} - M\Big(1 - \sum_{j \in \{C: j \neq i\}} \sum_{k \in \{N_+ : \langle v,i,j,k \rangle \in P\}} y_{v,i,j,k}\Big) \forall v \in V, i \in N_0,$$

$$\check{t}'_{vk} \geqslant \check{t}_k + s^R_{v,k} - M\Big(1 - \sum_{i \in N_0} \sum_{j \in \{C: \langle v,i,j,k \rangle \in P\}} y_{v,i,j,k}\Big) \forall v \in V, k \in N_+.$$

4 Experiments

Number of nodes	Number of drones	Objective function
8	1	1123.017009
8	2	1098.860889
8	3	1098.861271
8	4	1098.861270
10	1	1251.228216
10	2	1204.949671
10	3	1177.663505
10	4	1175.899710

5 Conclusion

The TSP problem with multiple drones can be formulated as a mixed integer linear programming form. For larger instances it requires a lot of time of computation since there are so many constraints that raise the difficulty of the problem. In our experiments we did solve some instances only up to 10 customer nodes due to the limited hardware. For additional information regarding the constraints mentioned above, it is possible to consult the authors' paper [1].

References

[1] Chase C. Murray, Ritwik Raj, "The Multiple Flying Sidekicks Traveling Salesman Problem: Parcel Delivery with Multiple Drones", 2019
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[3] Răschip Mădălina, Experimental Analysis of Algorithms class course, 2020
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