Homework 2.3

7 points. End term: 8-th week (19-25 november 2020)

1. For one of the following LP problems: solve the problem using the Dual Simplex method.

$$\text{(a)}\left\{\begin{array}{ll}\min & z = 3x_1 + 4x_2 + 2x_3 \\ \text{s. t.} & x_1 + 2x_2 + 3x_3 \leqslant 4 \\ & 2x_1 + 2x_2 + x_3 \geqslant 5 \\ & x_1, x_2, x_3 \geqslant 0\end{array}\right. \qquad \text{(b)}\left\{\begin{array}{ll}\min & z = 24x_1 + 6x_2 + x_3 + x_4 \\ \text{s. t.} & 4x_1 + 2x_2 + x_3 + x_4 \geqslant 4 \\ & 5x_1 + x_2 - x_3 \geqslant 5 \\ & x_1, x_2, x_3, x_4 \geqslant 0\end{array}\right.$$

2. The following tableau corresponds to an iteration of the Dual Simplex method (using Bland's rule) when solving a minimization LP problem:

	x_1	x_2	x_3	x_4	x_5	x_6	x_7	RHS
	0	-3	1	2	-2	0	1	3
	1	1	0	d	e	0	f	g
	0	h	0	-1	-3	i	-2	3
z	k	a	0	0	1	b	c	j

Categorize the variables as basic and nonbasic and provide the current values of all the variables. Find conditions on the parameters $a, b, \ldots, i$ so that the following statements are true.

(i) The above tableau is a valid tableau for the dual simplex algorithm.

(ii) The current basis is infeasible.

(iii) The current basis is not feasible and x_4 enters the basis.

(iv) The problem is infeasible.

(v) x_7 enters the basis which is infeasible, and the resulting solution is still infeasible.

3. Devise an implementation (C, C++, C#, Java) for the Dual Simplex algorithm (with Bland rule: breaking ties by choosing the smallest index variable) using the following pseudo-code (the simplex tableau has $(m+1) \times (n+1)$ dimensions):

// if $t_{i,n+1} \geqslant 0$, $\forall 1 \leqslant i \leqslant m$, the dual simplex table is <u>optimal</u>: the basic current solution is optimal.

while $(\exists\, t_{i,n+1} < 0,\ 1 \leqslant i \leqslant m)$ **{**

  let $1 \leqslant k \leqslant m$ **such that** $t_{k,n+1} < 0$**;**

  *// k is the pivot row, i.e., on row k will be the **<u>leaving basic variable</u>**.*

  if $(t_{k,j} \geqslant 0,\ \forall\, 1 \leqslant j \leqslant n)$ *// the **<u>problem</u>** is **<u>infeasible</u>** - and its dual is unbounded.*

    return;

  let $1 \leqslant l \leqslant n$ **such that** $\left|\dfrac{t_{m+1,l}}{t_{k,l}}\right| = \min\left\{\left|\dfrac{t_{m+1,j}}{t_{k,j}}\right| \,:\, 1 \leqslant j \leqslant n, t_{k,j} < 0\right\}$

  *// l is the pivot column, on column l will be the **<u>entering basic variable</u>***

  for $i = \overline{1, m+1},\ i \neq k$

    for $j = \overline{1, n+1},\ j \neq l$

      $t_{i,j} \leftarrow \dfrac{t_{i,j}t_{k,l} - t_{i,l}t_{k,j}}{t_{k,l}}$**;** *// **<u>pivoting rule</u>**.*

  for $i = \overline{1, m+1},\ i \neq k$

    $t_{i,l} \leftarrow 0$**;**

for $j = \overline{1, n+1}$**,** $j \neq l$

$\quad t_{k,j} \leftarrow \dfrac{t_{k,j}}{t_{k,l}}$**;**

$t_{k,l} \leftarrow 1$**;**

}