Homework 1.1

5 points. End term: 4-th week (october 22 - october 28, 2020)

Model as an LP problem two of the following problems.

1. Find the smallest disk (its radix and the center coordinates) which contains a given set of n points in the plane.

2. An investor has a portfolio of n different stocks. He has bought s_i shares of stock i at price p_i, $i = \overline{1,n}$. The current price of one share of stock is q_i. The investor expects that the price of one share of stock i in one year will be r_i. If he sells shares, the investor pays transaction costs at a rate of 1% of the amount transacted. In addition, the investor pays taxes at the rate of 30% on capital gains.

(For example, suppose that the investor sales $1,000$ shares of a stock at $q = \$50$ per share. He has bought these shares at $p = \$30$ per share. He receives $\$50,000$; however, he owes $0.30 \times 50,000 = \$15,000$ on capital gains taxes and $0.01 \times (50,000) = \$500$ on transaction costs. So, by selling $1,000$ shares of this stock he nets $50,000 - 15,000 - 500 = \$34,500$.)

The investor wants to know how many shares from each stock needs to sell in order to raise an amount of money K, net of capital gains and transaction costs, while maximizing the expected value of his portfolio next year. (Ignore the fact that the number of shares must be integer.)

3. Consider a road divided into n segments that is illuminated by m lamps. Let p_j be the power of the jth lamp. The illumination I_k of the kth segment is assumed to be $\sum_{j=1}^{m} a_{kj} p_j$, where a_{kj} are known coefficients. Let I_k^* be the desired illumination of segemnt road k. We are interested in choosingthe lamp powers so that the illuminations I_k are close to the desired illuminations I_k^*. Provide a reasonable linear programming formulation of this problem. (Note that there is more than one possible formulation.)

4. A small computer manufacturing company forecasts the demand over the next n months to be d_i, $i = \overline{1,n}$. In every month it can produce r units, using regular production, at a cost b dollars per unit. By using overtime, it can produce additional units at c dollars per unit ($c > b$). The company can store units from month to month at a cost of s dollars per unit per month. We would like to minimize the total cost by respecting the schedule constraints.

Model as an LP problem two of the following problems.

5. A manager has to determine the production plan for the next year. There are four products to produce (a, b, c, and d) which use three different processes (p_1, p_2, and p_3). Each process can be run 2000 hours per year. A product needs each process a certain amount of time (given below in hours):

Process:	Product a	Product b	Product c	Product d
p_1	12	10	15	10
p_2	10	9	12	12
p_3	12	14	12	11
Total time	34	33	39	33

The unit revenues, the costs of materials, and the maximum demand for the year to come are given below:

	Product a	Product b	Product c	Product d
revenue per unit (\$)	100	110	90	105
material cost per unit (\$)	40	45	45	40
maximum demand (units)	3000	4000	5000	3500

Find the amount of each product in oredr to maximize the profit for the next year.

6. A manufacturer of concrete has two plants, P_1 and P_2. There are three distributing warehouses, W_1, W_2, and W_3. P_1 plant can supply up to 110 tons of the product per week, whereas P_2 plant can supply up to 120 tons per week. W_1 warehouse needs at least 70 tons weekly to meet its demand, W_2 needs at least 60 tons weekly, and W_3 needs at least 80 tons weekly. Each warehouse have an upper limit capacity of 85 tons weekly.

The following table gives the shipping cost (in dollars) per ton of the product:

	W_1	W_2	W_3
P_1	10	9	12
P_2	9	11	10

a) How many tons of concrete should be shipped from each plant to each warehouse in order to minimize the total shipping cost while meeting the demand?

b) Rework the problem by dismissing the upper limit capacity of warehouses.

7. The FastCargoService company needs different numbers of drivers on different days of the week. Each employess must work five consecutive days and then will have two days off. The number of employee and the pay per employee required on each day of the week follows:

Day	No. of employees	Pay/employee
Monday	20	\$150
Tuesday	17	\$140
Wednesday	19	\$140
Thursday	15	\$130
Friday	22	\$150
Saturday	18	\$190
Sunday	19	\$210

a) Find the minimum number of full-employees that FastCargoService needs in order to satisfy the above constraints.

b) Find the minimum cost of FastCargoService in order to satisfy the required constraints.

Solve the following linear programs graphically.

8.
$$\left\{\begin{array}{lrcl} \max & z = x_1 + 2x_2 & & \\ \text{s. t.} & 2x_1 + x_2 & \geqslant & 12 \\ & x_1 + x_2 & \geqslant & 5 \\ & -x_1 + 3x_2 & \leqslant & 3 \\ & 6x_1 - x_2 & \geqslant & 12 \\ & x_1, x_2 & \geqslant & 0 \end{array}\right.$$

9.
$$\left\{\begin{array}{lrcl} \max & z = -x_1 - x_2 & & \\ \text{s. t.} & x_1 - x_2 & \geqslant & 1 \\ & x_1 - 2x_2 & \geqslant & 2 \\ & x_1, x_2 & \geqslant & 0 \end{array}\right.$$

10.
$$\left\{\begin{array}{lrcl} \max & z = x_1 - 2x_2 & & \\ \text{s. t.} & x_1 - 2x_2 & \geqslant & 4 \\ & x_1 + x_2 & \leqslant & 8 \\ & x_1, x_2 & \geqslant & 0 \end{array}\right.$$

11.
$$\left\{\begin{array}{lrcl} \max & z = 3x_1 + x_2 & & \\ \text{s. t.} & x_1 - x_2 & \leqslant & 1 \\ & 3x_1 + 2x_2 & \leqslant & 12 \\ & 2x_1 + 3x_2 & \leqslant & 3 \\ & -2x_1 + 3x_2 & \geqslant & 9 \\ & x_1, x_2 & \geqslant & 0 \end{array}\right.$$