Homework 3.1

9 points. End term: 12-th week (17-23 december)

1. Devise an implemetation (C, C++, C#, Java) for the Branch-and-Bound algorithm using the following pseudo-code:

$\mathscr{S} \leftarrow \varnothing$**;** *// $\mathscr{S}$ is a stack containing LP problems of the form {objective, constraints}.*
$\mathcal{P} \leftarrow \{\max \mathbf{c}^T\mathbf{x}, \mathbf{A}\mathbf{x} \leqslant b, \mathbf{x} \geqslant \mathbf{0}\}$**;**
push$(\mathscr{S}, \mathcal{P})$**;**
$(\overline{\mathrm{x}}, \overline{z}) \leftarrow (\varnothing, -\infty)$ **;** *// $\overline{\mathrm{x}}$ is the best current solution and $\overline{z}$ is its corresponding objective value.*
while $(\mathscr{S} \neq \varnothing)$ **{**
  $\mathcal{P} \leftarrow$ **pop**$(\mathscr{S})$**;**
  if ($\mathcal{P}$ is unbounded)
    return "unbounded problem"**;**
  if ($\mathcal{P}$ **is** feasible) **{** *// otherwise the current problem is fathomed by infeasibility.*
    (x^0, z_0) **= TwoPhaseMethod**$(\mathcal{P})$**;** *// x^0 is the solution of the LP relaxation and z^0 is its objective values.*
    if$(z_0 > \overline{z})$ *// otherwise the current problem is fathomed by bound.*
      if ($x_i^0 \in \mathbb{Z}$**,** $\forall\, i \in \mathfrak{I}$) **{** *// the current problem is fathomed by integrality.*
        $\overline{\mathrm{x}} \leftarrow \mathrm{x}^0$**;**
        $\overline{z} \leftarrow z_0$**;**
      }
      else {
        fie $j \in \mathfrak{I}$ **cu** $x_j^0 - [x_j^0] = \max\left\{x_i^0 - [x_i^0] \,:\, i \in \mathfrak{I}\right\}$**;**
        push$(\mathscr{S}, \mathcal{P} \cup \left\{x_j \leqslant [x_j^0]\right\})$**;**
        push$(\mathscr{S}, \mathcal{P} \cup \left\{x_j \geqslant [x_j^0] + 1\right\})$**;**
      }
  }
}
if $((\overline{\mathrm{x}}, \overline{z}) = (\varnothing, -\infty))$
  return "infeasible problem"**;**
return $(\overline{\mathrm{x}}, \overline{z})$**;**

2. Devise an implementation (C, C++, C#, Java) for a version of the Cutting-Plane algorithm using the following pseudo-code:

for ($i = 0$ **to** i_{max}) **{**
  if ($\mathcal{P}$ **is** feasible) **{**
    (x^0, f^0) **= TwoPhaseMethod**$(\mathcal{P})$**;**
    if ($\mathbf{x}^0 \in \mathbb{Z}^n$)
      return $\mathbf{x}^0$**;**
    let $x_i^0 \notin \mathbb{Z}$**;**
    let $x_i + \sum_{j \in N} \lfloor \tilde{a}_{ij} \rfloor x_j \leqslant \lfloor \tilde{b}_i \rfloor$ **be a Gomory fractional cut;**
    $\mathcal{P} \leftarrow \mathcal{P} \cup \{x_i + \sum_{j \in N} \lfloor \tilde{a}_{ij} \rfloor x_j \leqslant \lfloor \tilde{b}_i \rfloor\}$**;**
  }
  else
    return "infeasible problem"**;**
  $i++$**;**
}

3. Run the Branch-and-Bound and Cutting-Plane implemented algorithms on the following problems:

$$
\textbf{(a)} \left\{ \begin{array}{lll} \max & x_1 - x_2 + 2x_3 & \\ & x_1 - 2x_2 + 2x_3 & \leqslant 6 \\ & x_1 + x_2 + 2x_3 & \leqslant 8 \\ & x_1, x_2, x_3 \geqslant 0 & x_1, x_2, x_3 \in \mathbb{Z} \end{array} \right.
\qquad
\textbf{(b)} \left\{ \begin{array}{lll} \max & 2x_1 + x_2 & \\ & -x_1 + x_2 & \leqslant 0 \\ & 6x_1 + 2x_2 & \leqslant 21 \\ & x_1, x_2 \geqslant 0 & x_1, x_2 \in \mathbb{Z} \end{array} \right.
$$

$$
\textbf{(c)} \left\{ \begin{array}{lll} \max & x_1 + x_2 + x_3 + 2x_4 & \\ & x_1 + 2x_2 + x_3 - x_4 & \leqslant 8 \\ & x_1 + 2x_2 + 2x_4 & \leqslant 7 \\ & 4x_1 + x_2 + 3x_3 & \leqslant 8 \\ & x_1, x_2, x_3, x_4 \geqslant 0 & x_1, x_2, x_3, x_4 \in \mathbb{Z} \end{array} \right.
$$