Modeling and attitude control of Bi-copter

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Modeling and Attitude Control of Bi-copter

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Abstract—**Multi rotor unmanned aerial vehicle is a hot topic in the field of UAV research. This paper presents a twin rotor copter whose dynamic system is able to tilt. We have a research on the principle of motion and the design of attitude stabilizing controller. Firstly, the dynamics model of the twin rotor aircraft is established. Then the prototype is designed and built to obtain the relevant physical parameters. Finally, the real flight attitude tracking curve was measured by flight experiment which verifies the correctness and effectiveness of the proposed PID attitude controller of the Bi-copter.**

I. Introduction

Multi rotor aircraft has attracted extensive attention with its novel structure and layout, and has rapidly become a hotspot in the world. The common multi rotor aircrafts at present are four, six and eight rotor aircraft. They can only be done by changing the speed of the propellers to complete a variety of actions. Although the above multi rotor aircrafts have been widely used in aerial surveying and mapping, remote sensing, agricultural plant protection and many other fields, in the practical application, it has been unable to avoid the trouble of the short time.

Aiming at the problem of weak endurance, we put forward a Bi-copter whose dynamic system is able to tilt which is based on the easiest layout and refers to tilt rotor layout. The significant advantage of this aircraft is the reduction in the number of rotors which reduce the unit time power demand effectively. To a certain extent, the aerodynamic interference between the propellers is also reduced. At the same time, the Bi-copter also has the advantages of vertical take-off and landing, small dependence on space, etc.

The design of attitude controller is the key technology for the Bi-copter to achieve stable flight. As the accuracy of the model is highly dependent on the control of nonlinear systems. In the case of multi rotor itself with complex force, this paper finally choose PID control which is stable, reliable and highly inclusive. In this paper, the PID attitude controller is designed on the basis of the dynamic modeling of the Bi-copter. Finally, the effectiveness of the proposed Bi-copter control algorithm is verified by the successful test flight.

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II. Dynamic model of Bi-copter

A. Movement Form

Dual rotor aircraft has 6 degrees of freedom but only 4 input control. It is an under actuated system. The control quantity is coupled, which belongs to the nonlinear system. Its flight principle is as follows. The arrow direction and size represent propellers turning direction and rotating speed. The dotted line indicates the horizontal position of the motor seat under the body coordinate. The flight mechanism is summarized as follows: Vertical movement is generated by two propellers whose speed increases or decreases the same time. The pitch motion is produced by a co rotating propellers with translation at the same time. The yaw motion is produced by two propellers in the opposite direction. The roll motion is provided by two differential propellers.

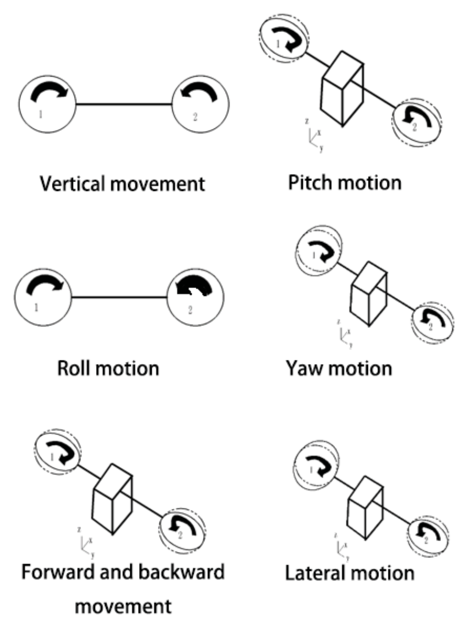

Figure 1. Motion mechanism of dual rotor aircraft

B. Dynamic Model

Then we mainly analyze the dynamic model. First of all, we establish two coordinate systems as Figure 2 shown: Body coordinate system $OX_bY_bZ_b$ and the Ground coordinate system $OX_gY_gZ_g$.

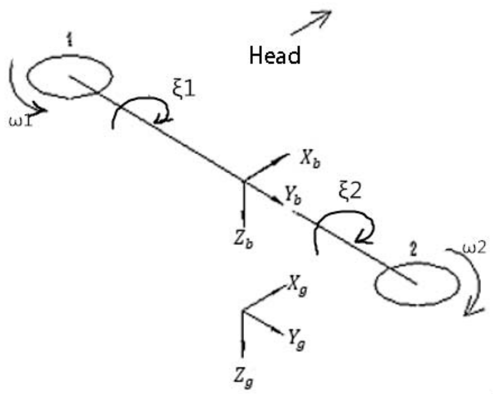

Figure 2. Body coordinate system and inertial coordinate system

As is shown in Figure 3, we define the Euler angle:

Yaw angle ψ: The angle between the axis of the body in the horizontal plane and the ground axis, the right side is positive.

Pitch angle θ: The angle between the body axis and the horizontal plane, to rise is positive.

Roll angle ϕ: The angle that symmetry plane rotate around the body axis, right roll is positive.

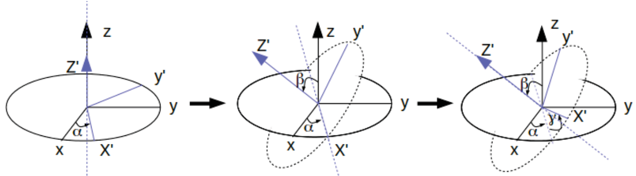

Figure 3. Definition of Euler angles

When using Euler angles to describe the attitude of an aircraft, we can obtain the transformation matrix from the ground coordinate system to the body coordinate system.

$$\mathbf{L_{bg}} = \begin{bmatrix} cos\theta cos\psi & cos\theta sin\psi & -sin\theta \\ -cos\phi sin\psi + sin\phi sin\theta cos\psi & cos\phi cos\psi + sin\phi sin\theta sin\psi & sin\phi cos\theta \\ sin\phi sin\psi + cos\phi sin\theta cos\psi & -sin\phi cos\psi + cos\phi sin\theta sin\psi & cos\phi cos\theta \end{bmatrix} \tag{1}$$

The relationship matrix between the angular velocity of the body and the first derivative of the Euler angles from the ground system is:

$$\begin{bmatrix} \dot{\phi} \\ \dot{\theta} \\ \dot{\psi} \end{bmatrix} = \begin{bmatrix} 1 & tan\theta sin\phi & tan\theta cos\phi \\ 0 & cos\phi & -sin\phi \\ 0 & sin\phi / cos\theta & cos\phi / cos\theta \end{bmatrix} \begin{bmatrix} p \\ q \\ r \end{bmatrix} \tag{2}$$

The physical model of the aircraft is assumed to simplify the model:

(1) The origin of the coordinate system is always coincident with the center of gravity of the dual rotor aircraft. The body mass center is on the OZ_b axis.

(2) Its own structure (including the external load) is symmetric with the OX_bZ_b plane. Its inertia product is $I_{x_by_b} = 0,\ I_{z_by_b} = 0,\ I_{x_bz_b} \neq 0$.

(3) The resistance and gravity of the dual rotor aircraft during flight are not affected by external conditions, such as flight height and other factors.

(4) The force produced by the propellers is proportional to the square of the speed of the motor.

We define the force on the three axis F_x, F_y, F_z. The moment of roll, pitch and yaw are L, M, N. Three axis angular velocities around the body coordinate system are p、q、r.

According to Newton's second law, the matrix L_{bg} and I, we can know that under the circumstance of no wind, the linear motion and angular motion equations of aircraft in the ground coordinate system are as follows:

$$\begin{cases} \ddot{x} = -\dfrac{\sin\phi\sin\psi + \cos\phi\sin\theta\cos\psi}{m} k_T \left(\omega_1^2 \cos\xi_1 + \omega_2^2 \cos\xi_2\right) \\ \quad -\dfrac{\cos\theta\cos\psi}{m} k_T (\omega_1^2 \sin\xi_1 + \omega_2^2 \sin\xi_2) \\ \ddot{y} = \dfrac{\sin\phi\cos\psi - \cos\phi\sin\theta\sin\psi}{m} k_T \left(\omega_1^2 \cos\xi_1 + \omega_2^2 \cos\xi_2\right) \\ \quad +\dfrac{\cos\theta\sin\psi}{m} k_T (\omega_1^2 \sin\xi_1 + \omega_2^2 \sin\xi_2) \\ \ddot{z} = \dfrac{-\cos\phi\cos\theta}{m} k_T (\omega_1^2 \cos\xi_1 + \omega_2^2 \cos\xi_2) \\ \quad -\dfrac{\sin\theta}{m} k_T (\omega_1^2 \sin\xi_1 + \omega_2^2 \sin\xi_2) + g \\ \ddot{\phi} = \dfrac{1}{I_x\left[\left(-I_z + I_y\right)qr + lk_T\left(\omega_1^2 \cos\xi_1 - \omega_2^2 \cos\xi_2\right)\right]} \\ \ddot{\theta} = \dfrac{1}{I_y\left[\left(I_z - I_x\right)pr + hk_T\left(\omega_1^2 \sin\xi_1 + \omega_2^2 \sin\xi_2\right)\right]} \\ \ddot{\psi} = \dfrac{1}{I_z\left[\left(I_x - I_y\right)pq + lk_T\left(\omega_1^2 \sin\xi_1 - \omega_2^2 \sin\xi_2\right)\right]} \end{cases} \tag{3}$$

K_T is the pull coefficient of the propellers, ω_1, ω_2 are the angular velocity of two motors, ξ_1, ξ_2 are the tilt angle of two motors, h is the distance from the center of the rotor to the center of the aircraft.

U_1, U_2, U_3, U_4, is introduced into the control:

$$\mathbf{U}=\begin{bmatrix}U_1\\U_2\\U_3\\U_4\end{bmatrix}=\begin{bmatrix}K_T(\omega_1^2\cos\xi_1+\omega_2^2\cos\xi_2)\\K_T(\omega_1^2\cos\xi_1-\omega_2^2\cos\xi_2)\\K_T(\omega_1^2\sin\xi_1+\omega_2^2\sin\xi_2)\\K_T(\omega_1^2\sin\xi_1-\omega_2^2\sin\xi_2)\end{bmatrix} \tag{4}$$

After the introduction of four control variables, the nonlinear coupling model of the attitude angle is decomposed into a relatively independent control channel. For the above analysis, we can conclude that the angular motion is not affected by the linear motion, and the line motion is affected by the angular motion. The following mathematical model can be obtained by ignoring the drag coefficient:

$$\begin{cases}\ddot{x}=-\dfrac{U_1}{m}(\sin\phi\sin\psi+\cos\phi\sin\theta\cos\psi)-\dfrac{\cos\theta\cos\psi}{m}U_3\\ \ddot{y}=\dfrac{U_1}{m}(\sin\phi\cos\psi-\cos\phi\sin\theta\sin\psi)+\dfrac{\cos\theta\sin\psi}{m}U_3\\ \ddot{z}=-\cos\phi\cos\theta\dfrac{U_1}{m}-\dfrac{\sin\theta}{m}U_3+g\\ \ddot{\phi}=\dfrac{lU_2}{I_x}\\ \ddot{\theta}=\dfrac{hU_3}{I_y}\\ \ddot{\psi}=\dfrac{lU_4}{I_z}\end{cases} \tag{5}$$

III. DESIGN OF THE ATTITUDE STABILIZATION CONTROL

Based on the kinetic model established above, the MATLAB/Simulink model is established. The physical parameters can be acquired by the Bi-copter.

The implementation of tilting movement is to use the servo to pull the base of motor in the vertical plane of steering gear, so that the motor generates a certain inclination angle.

Figure 4. Bi-copter

TABLE I. SELECTION OF THE SIMULATED PHYSICAL PARAMETER

Parameter	Physical meaning	Value	Company
m	Quality	0.87	Kg
g	Acceleration of gravity	9.81	m/s^2
ρ	Air density	1.225	Kg/m^3
h	The vertical distance between the center of gravity and the plane of propellers	0.085	m
l	Horizon distance between rotor center and center of mass	0.175	m
K_T	Lift coefficient	6.46	Ns^2/rad^2
Ix	Inertia on the X axis	0.0043	$kg\cdot m^2$
I_y	Inertia on the Y axis	0.0142	$kg\cdot m^2$
I_z	Inertia on the Z axis	0.0176	$kg\cdot m^2$
I_{xz}	Inertia product on XZ plane	0.0001	$kg\cdot m^2$

The above physical parameters are used as the parameters of the simulation model and the attitude control law is designed. Take pitch channel as an example, in order to control the pitch angle, feedback is usually introduced to determine whether a predetermined value is given. Meanwhile, in order to improve the performance of pitch angle, pitch angle velocity feedback is also needed to be improved. Figure 5 is the circuit diagram designation of pitch angle control using the PID controller.

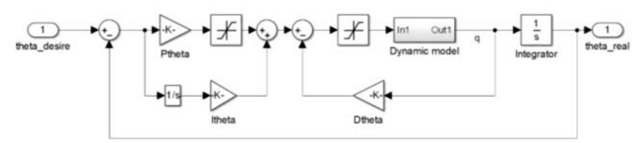

Figure 5. Control diagram on pitch channel

We can have a simulation test on the pitch, yaw and roll channel. Under the step input, the result of attitude angle is as follows:

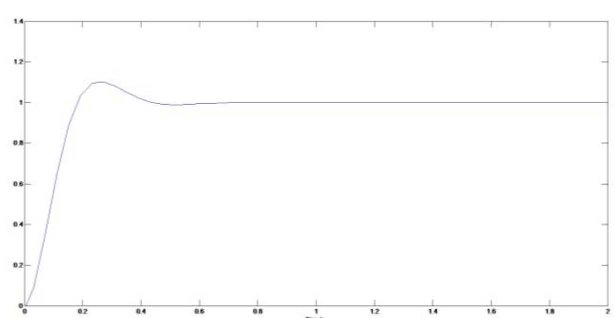

Figure 6. Step response curve of pitch angle

Due to the limitation of the structural characteristics of the dual rotor aircraft. The distance between the center of gravity of the body structure and the propellers shaft is small, which leads to the difficulty to increase the pitch moment. Therefore, the pitch angle step response of the adjustment time is longer, but the response speed is fast, the overshoot is 11%, the steady state error is close to 0. Therefore, it can be considered that the controller plays an ideal role in regulation.

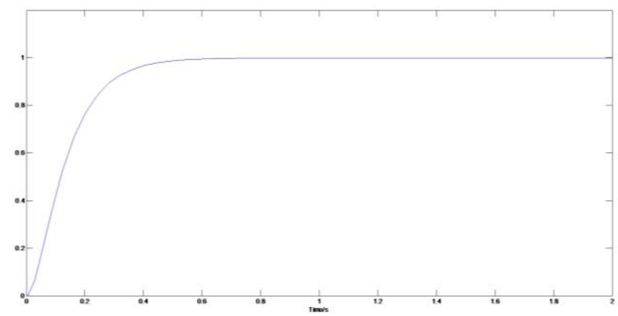

Figure 7. Step response curve of roll angle

From the above motion, we know that rolling moment is provided by two differential propellers. At the same time, the rolling movement can lead to linear movement of y axial. So the adjustment of roll will be relatively slow. From the above figure, we can see that the step response of rolling angle is longer, the overshoot and steady state error is close to 0. The control effect is ideal.

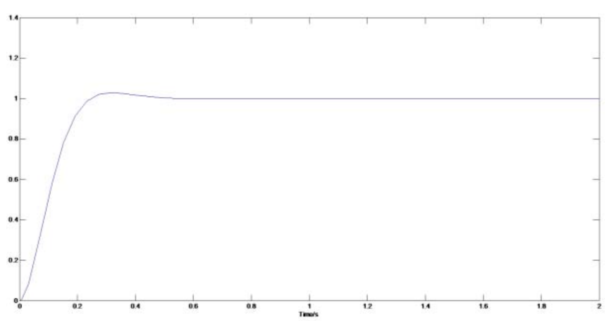

Figure 8. Step response curve of yaw angle

When doing the yaw motion, yaw moment is provided by the horizontal component of two propellers. So the aircraft around the *Z* axis movement is more flexible. The above step response simulation curve can be seen that the yaw angle response time is short, the overshoot is 3% and the steady state error is close to 0, which indicates that the PID control algorithm is effective.

IV. Dynamic Response Simulation and Verification

By comparing the actual control response and simulation model response, we test the reliability of the simulation model. The actual control of bi-copter response and simulation results are compared in the following Figure 9.

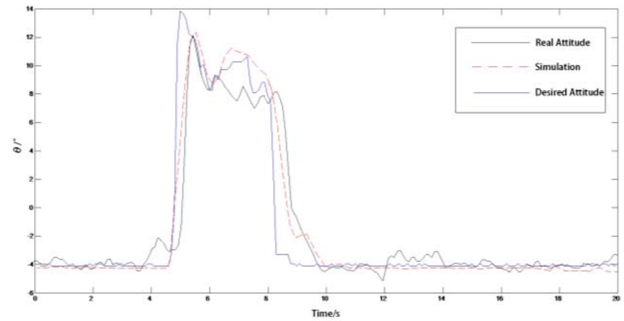

Figure 9. Comparison of pitch angle response

Take the pitch loop as example. Use the actual command as the input of Simulink model for simulation system response, we can see from the graph that the Simulink response is slightly faster and smoother than the actual response. The main reason is that the simulation model doesn't consider eddy current, ground effect, wind resistance and other interference. But in general, the error of Simulink and the actual response is small, which shows that the simulation model is reliable.

V. Flight Validation

In order to verify the effectiveness of the proposed control algorithm, an autonomous set of dual rotor aircraft is used to carry out the flight experiment. We use gyro sensor to get data. And then we transmit the information to the host computer via the digital transmission module. The contrast between the actual attitude response curve and the input command is as follows:

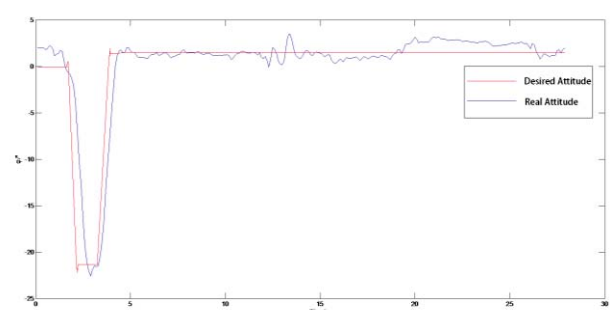

Figure 10. Real response to the roll angle

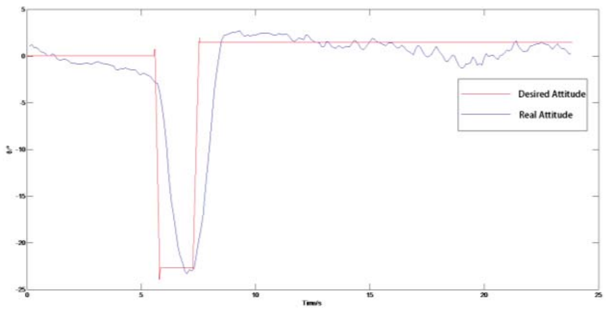

Figure 11. Real response to the pitch angle

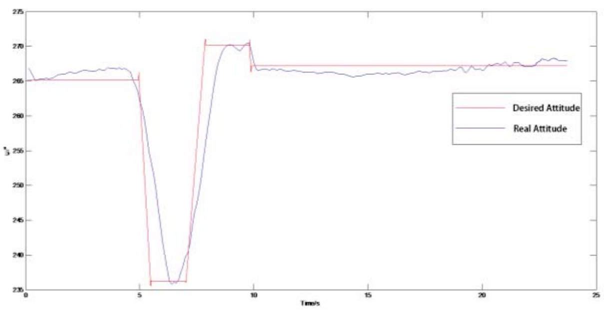

Figure 12. Real response to the roll angle

The dashed line is the input curve, the solid line is the actual attitude curve. From the figure we can see that compared to the input command, the actual response has a short lag and the maximum difference is less than 3 degrees while the average error is about 1.5 degrees which are in the acceptable range. The realization of the attitude angle quickly converge to the input command value. Small disturbances such as wind can cause disturbance to the attitude of the spacecraft during the actual flight. However, the PID controller can also realize the real-time adjustment of the attitude changes caused by small disturbances in order to keep the aircraft stable in the disturbed condition. Therefore, the flight experiment verifies the effectiveness of the PID control algorithm validly.

VI. CONCLUSION

This paper presents a new rotor aircraft whose dynamic system can tilt. And we build up a dynamic model and attitude controller based on PID according to the flight mode of dual rotor aircraft. By combining the dynamic model and physical parameters of the physical prototype, we obtain the motion characteristics of the prototype. The attitude stabilizing control law is constructed according to the PID controller parameters and the motion response. Finally, through the test of flight, the relationship between the simulation response curve and the actual response curve is obtained, which proves the accuracy of model. At the same time, the flight experiment also shows that the attitude control algorithm can effectively achieve the attitude stability control of the dual rotor aircraft.

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