Modeling predator-prey relationships in the presence of a multi-species parasite

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What is mathematical biology?

- Studying biological systems using mathematical tools
- Tools include: differential equations models, computer simulations and more
- Models can show whether real-life experiments are viable before spending resources

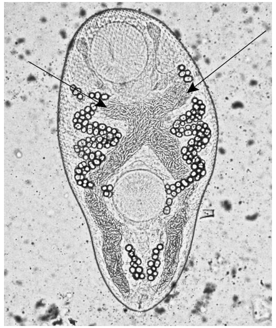
Examples:

- Modeling disease spread of COVID-19 to inform response efforts
- Understanding tumor behavior for chemotherapy

Project background

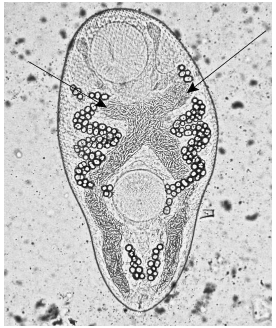
- Part of the 2022 SCMB Modeling Accelerator
- Learned to create mathematical models using ODEs (ordinary differential equations) and Netlogo (computer modeling software)
- Agent-based model - a simulation exploring the interactions of individual units, or “agents”, following a set of programmed rules
- Used ODE and agent-based models in tandem

Bio-system context



Predatory birds, prey frogs, and inter-species *Riberoia ondatrae* (frog-mutating flatworm)

Bio-system context



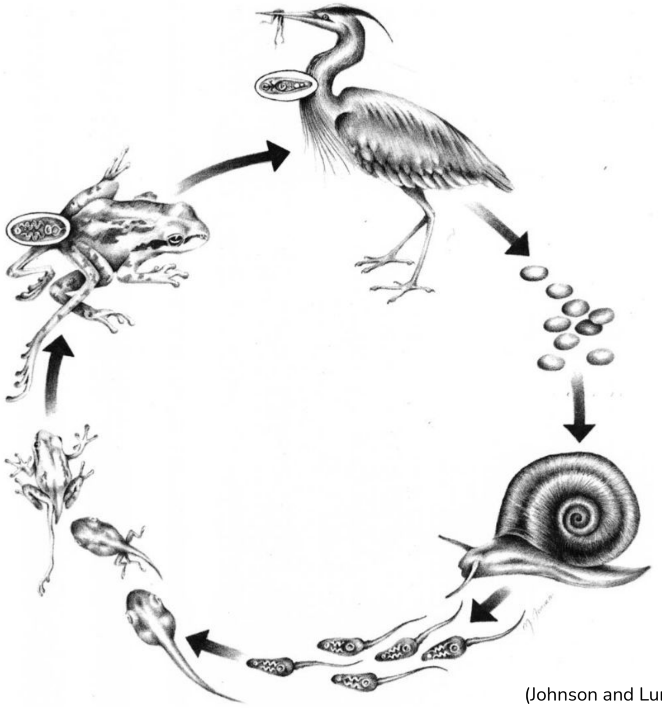
Predatory birds, prey frogs, and inter-species *Riberoia ondatrae* (frog-mutating flatworm)

Honorable mention: Ram’s horn snail

Bio-system context

Birds eat infected frogs, and parasites fully mature in birds' GI tract

Adult *Ribeiroia ondatrae* eggs spread through bird feces



(Johnson and Lunde 2005)

Parasites infect frogs and become encysted in limb buds, causing deformities

After developing in eggs, parasites grow inside snails

Bio-system context

Deformities can be severe, and research links them to increased predation rates (Sessions and Ruth 1990)

Why do we care?

Besides feeling bad for cute frogs…

- Research shows that human activity, such as polluting aquatic environments, can increase snail proliferation (Dell'Amore 2011)
- More snails means more parasites hatch to infect frogs
- More deformed frogs will increase the overall predation rate
- How does this increase in frog predation affect the biosystem as a whole?

Question: What are the effects of increased frog predation rates due to parasite-induced deformity?

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Hypothesis: A higher predation rate will increase parasite spread. However, if the rate increase is too high it will destabilize the biosystem and detriment parasite spread.

Modelling the bio-system with ODEs

Finding the Goldilocks zone

The Lotka-Volterra equations

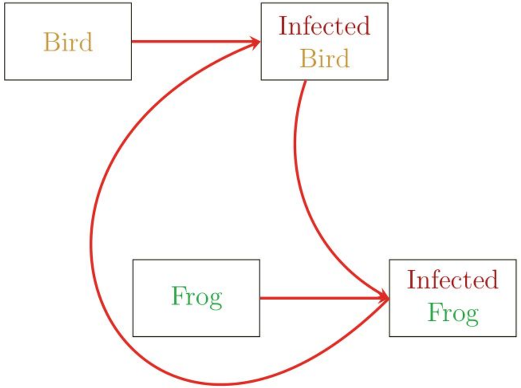
- A pair of differential equations often used to describe predator-prey relationships in nature
- $\boldsymbol{\alpha}$ - prey birth rate; assumed to reproduce exponentially in the absence of predation
- $\boldsymbol{\beta}$ - prey predation rate by predators
- $\boldsymbol{\delta}$ - predator birth rate; depends on ability to eat prey, but not necessarily equal to predation rate
- $\boldsymbol{\gamma}$ - predator death rate; can die of natural causes, or simply leave the biosystem

$$\frac{dx}{dt} = \alpha x - \beta xy,$$
$$\frac{dy}{dt} = \delta xy - \gamma y,$$

x: prey
y: predator

Simplifying the biosystem

- Simplify the system to just bird-frog predation, and model parasites as interactions between them
  - Frog infection requires a distinct coefficient
  - Bird infection rate is equal to predation rate of infected frogs



A modified Lotka-Volterra model including parasite impact

$$\frac{dB}{dt} = -e_2 F_i B + \varepsilon(FB + FB_i) + \varepsilon(1 + \frac{e_2}{e_1})(F_i B + F_i B_i) - xB$$
$$\frac{dB_i}{dt} = e_2 F_i B - xB_i$$
$$\frac{dF}{dt} = -\sigma B_i F - e_1(BF + B_i F) + r(F + F_i)$$
$$\frac{dF_i}{dt} = \sigma B_i F - e_2(BF_i + B_i F_i)$$

B: Healthy birds
B_i: Infected birds

F: Healthy frogs
F_i: Infected frogs

σ: Infection rate from birds to frogs (by parasite)

e_1: Predation rate of healthy frogs
e_2: Predation rate of infected frogs

ε: Reproduction rate of birds (proportional to frogs)
r: Reproduction rate of frogs

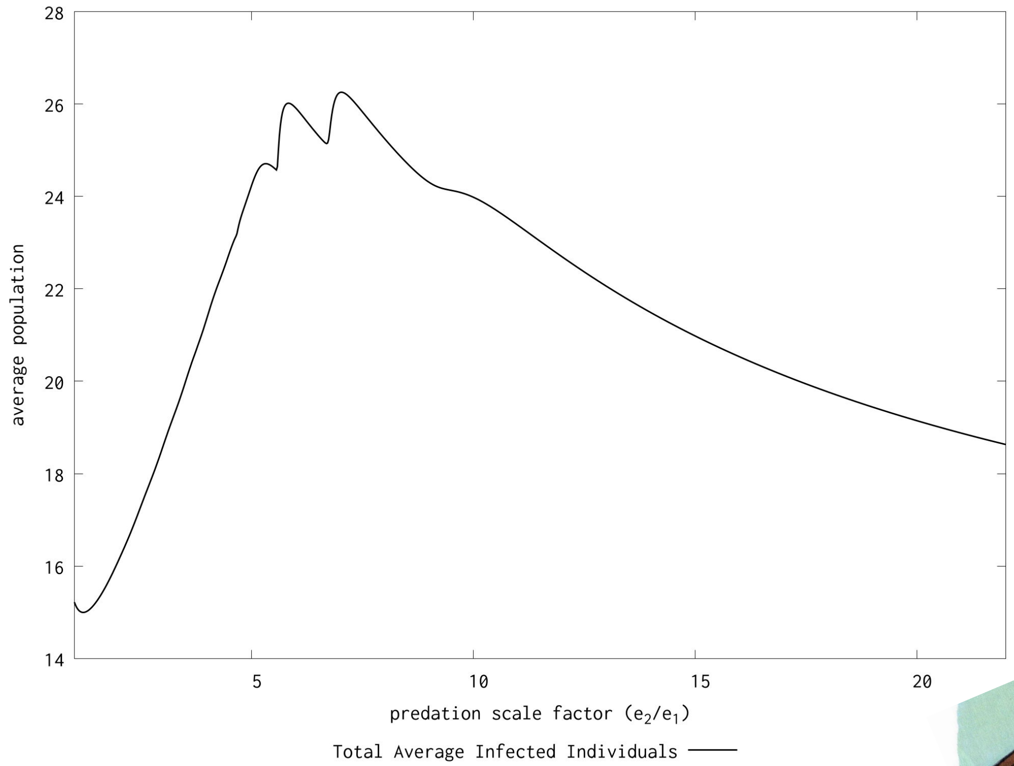
x: Death rate of birds

A modified Lotka-Volterra model including parasite impact

$$\frac{dB}{dt} = -e_2 F_i B + \varepsilon(FB + FB_i) + \varepsilon\left(1 + \frac{e_2}{e_1}\right)(F_i B + F_i B_i) - xB$$
$$\frac{dB_i}{dt} = e_2 F_i B - xB_i$$
$$\frac{dF}{dt} = -\sigma B_i F - e_1(BF + B_i F) + r(F + F_i)$$
$$\frac{dF_i}{dt} = \sigma B_i F - e_2(BF_i + B_i F_i)$$

- Parasite infection is modeled as interactions between frogs and birds (**σ**, e_2)
- Infected individuals exit the population of uninfected ones; therefore "birth" rate of infected individuals is the rate at which they leave the healthy population, AKA become infected (e_2F_iB, σB_iF)
- Bird infection rate is the predation rate of infected frogs (e_2)

Average infected population as relative predation increases



An agent-based model

Analyzing biosystem stability

Model background

- Used Netlogo to program model and run experiments
- Initial conditions scaled to approx. real-life population density of great blue herons and Pacific tree frogs
- Agents have an energy meter which is used up by moving, and refilled by feeding
- Birds attempt to “hunt” frogs near them, and frogs have a chance to escape

Model background

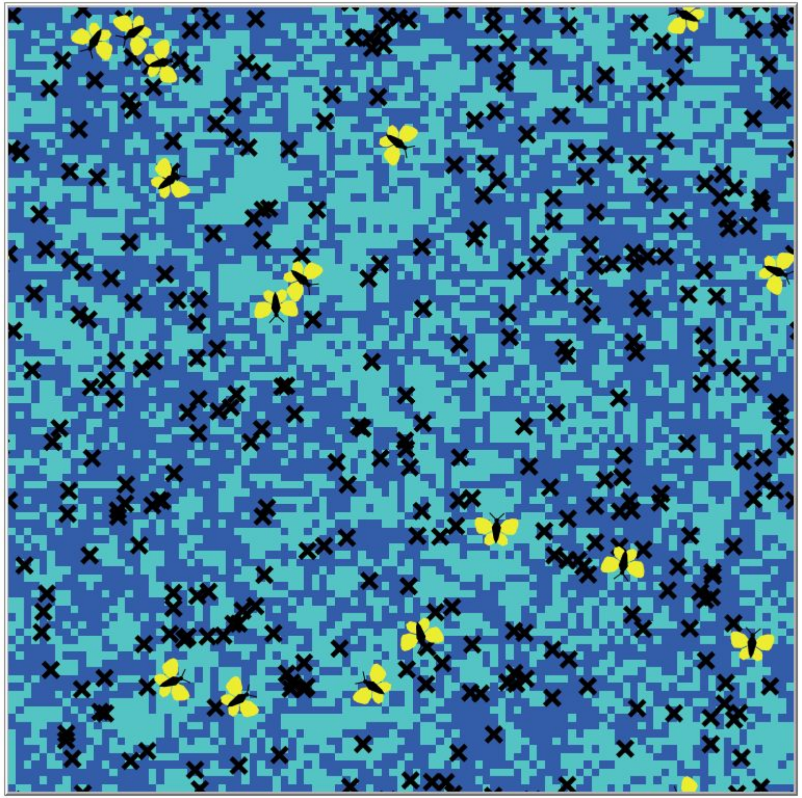
- A bird that eats an infected frog becomes infected
- For each infected bird near a healthy frog, the frog has a fixed chance to become infected
- We reduced infected frogs’ chance of escape to represent increased predation rate

Key differences from ODE model

- Frog food respawn rate enforces a prey carrying capacity
- Models frog escape chance, rather than predation rate
- Goldilocks zone harder to ascertain due to noise
- Stability is more easily visible in fluctuations of graphs

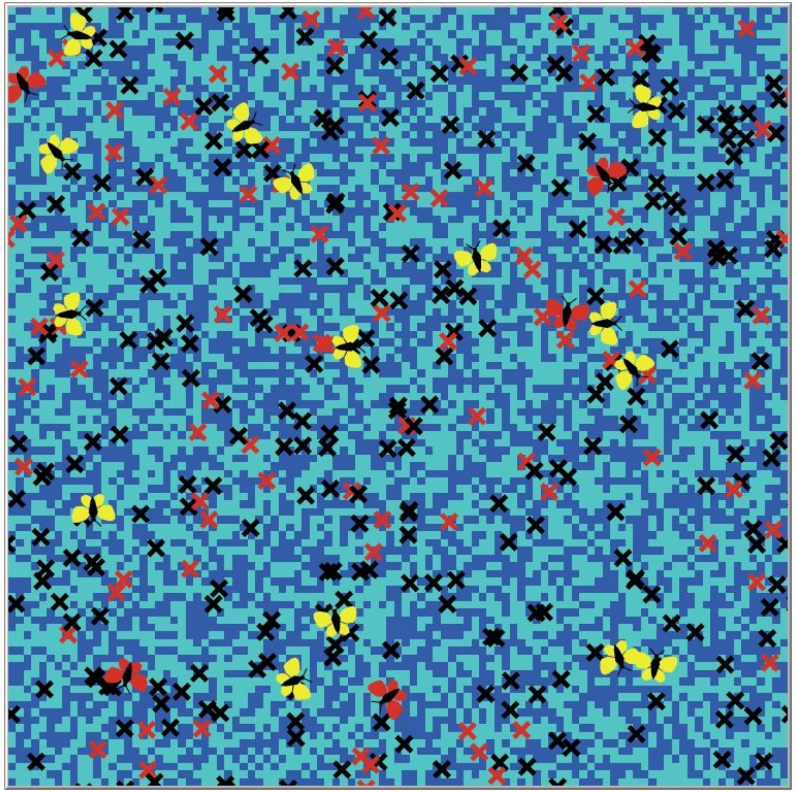
Base predator-prey simulation

- Butterflies: birds
- ‘x’: frogs
- Light blue patch: contains frog food
- Dark blue patch: waiting for frog food to respawn

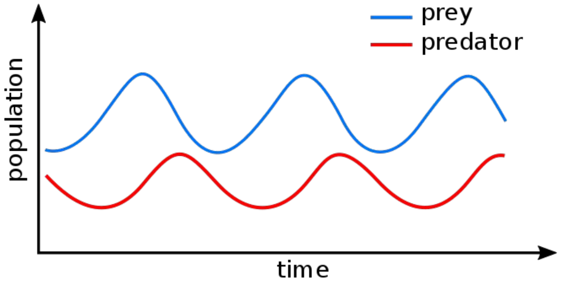


Simulation with parasite infection

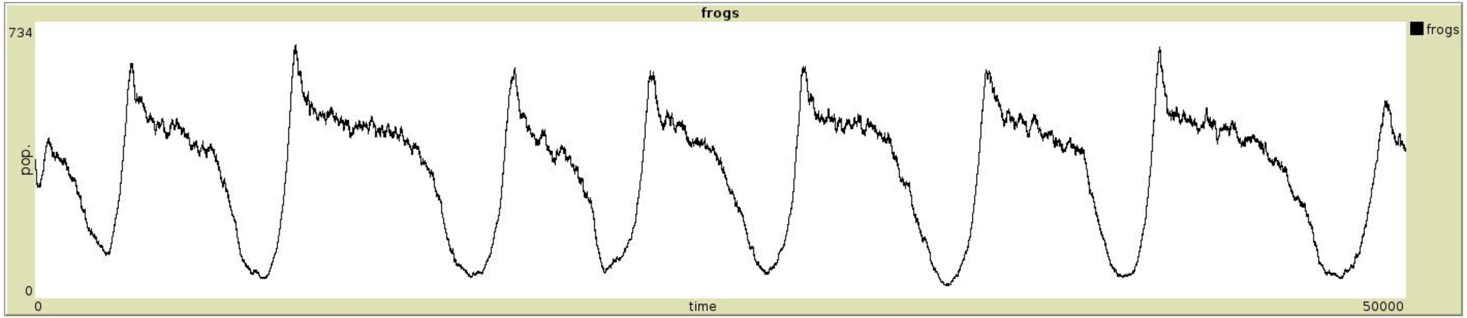
- Red agents are infected
- Extremes of changing populations are more pronounced



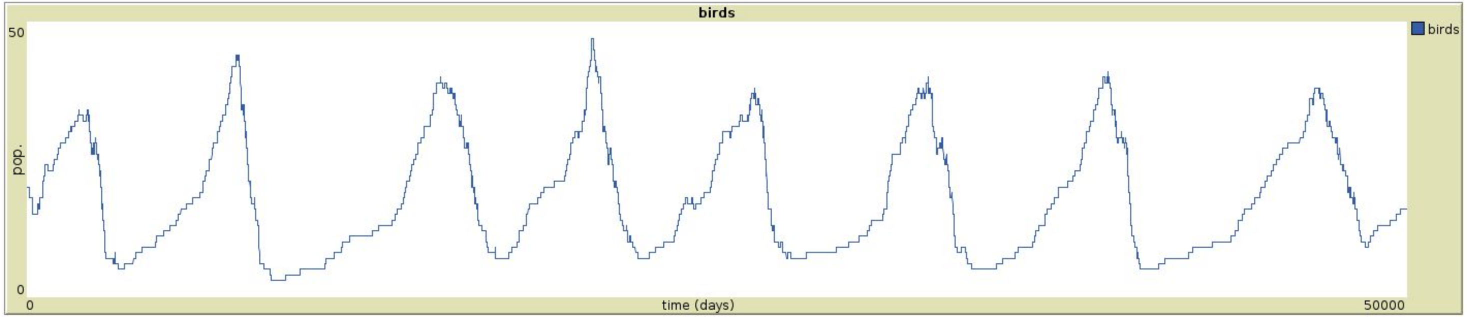
No parasite



prey
predator
population
time

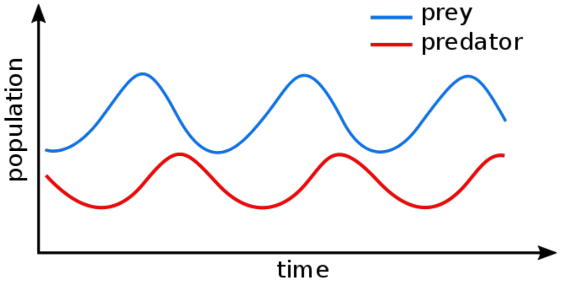


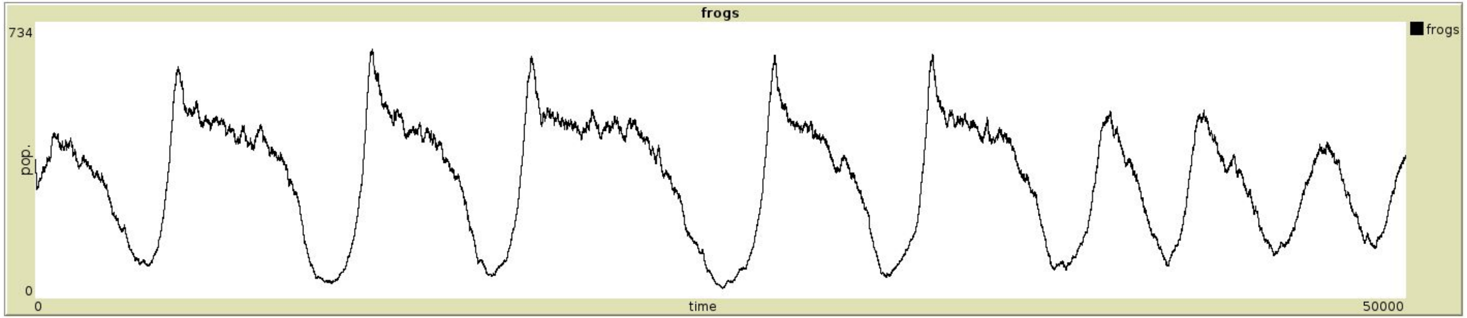
frogs
734
pop.
0
0
time
50000
frogs

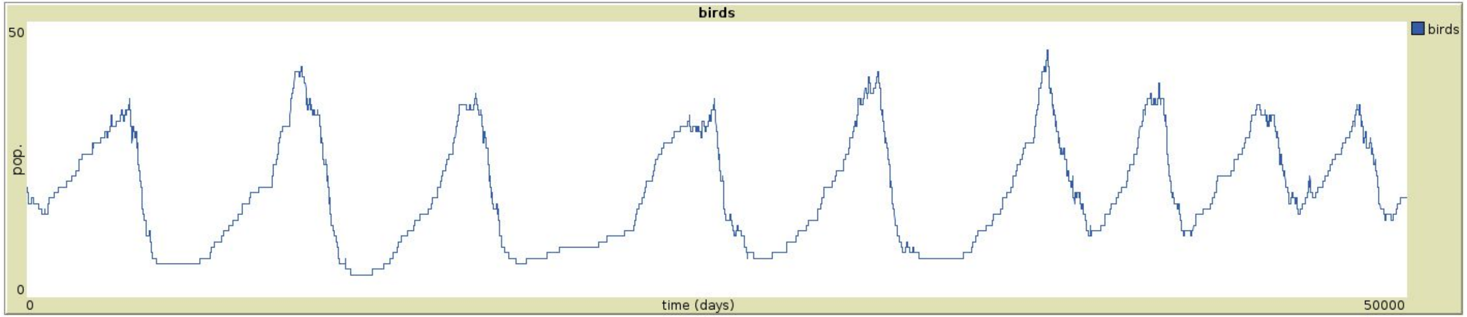


birds
50
pop.
0
0
time (days)
50000
birds

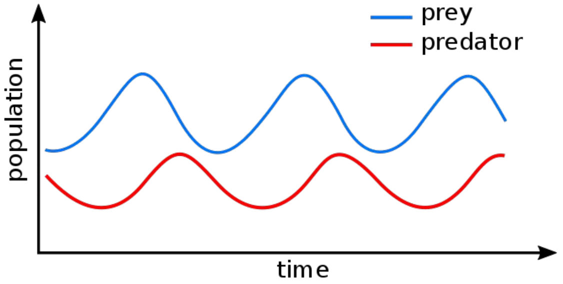
Parasite causing small increase in predation rate

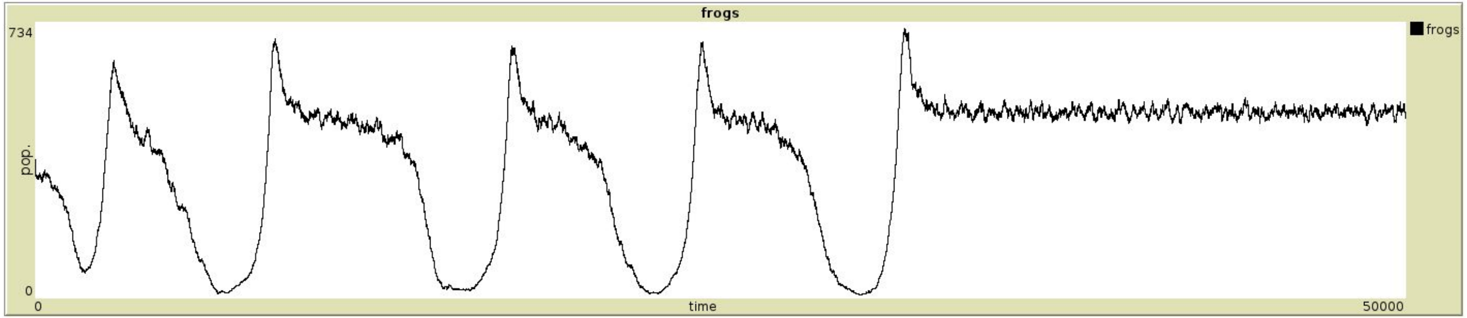


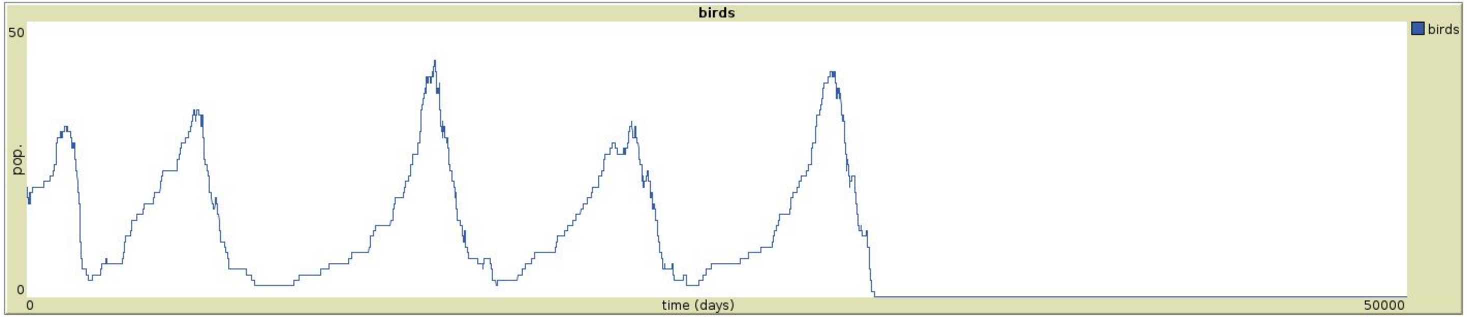




Parasite causing large increase in predation rate







Experiment

- Varied frog escape chance using `deformity-level`, kept other parameters constant
  - Escape chance = (base chance)(100 - `deformity-level`)
- Compared frequency of three events across `deformity-level` changes:
  - Parasite extinction
  - Bird extinction
  - Frog extinction
- Ran 50 trials for 50,000 time ticks each per `deformity-level`

How often does the **parasite** die out across repeated simulations?

	5%	20%	50%
	1/50	13/50	33/50

`deformity-level` →

How often do **birds** go extinct across repeated simulations?

	1/50	6/50	23/50
`deformity-level` →	5%	20%	50%

How often do **frogs** go extinct across repeated simulations?

	$\frac{0}{50}$	$\frac{0}{50}$	$\frac{3}{50}$
`deformity-level` →	5%	20%	50%

Conclusions

Conclusions

- Parasite spread is maximized within a certain range of deformity severity
- Outside this range, the parasite does not spread effectively; above it, the host populations also begin to destabilize
- Frogs are more likely than birds to survive destabilization
- Factors that increase parasite proliferation, and therefore frog predation, can have drastic effects on the host species

Sources

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