MINIMUM EDIT DISTANCE

Definition of Minimum Edit Distance

HOW SIMILAR ARE TWO STRINGS?

- Spell correction
  - The user typed "graffe"

    Which is closest?
    - graf
    - graft
    - grail
    - giraffe

- Computational Biology
  - Align two sequences of nucleotides

    AGGCTATCACCTGACCTCCAGGCCGATGCCC
    TAGCTATCACGACCGCGGTCGATTTGCCCGAC

  - Resulting alignment:

    -AGGCTATCACCTGACCTCCAGGCCGA--TGCCC---
    TAG-CTATCAC--GACCGC--GGTCGATTTGCCCGAC

- Also for Machine Translation, Information Extraction, Speech Recognition

EDIT DISTANCE

- The minimum edit distance between two strings
- Is the minimum number of editing operations
  - Insertion
  - Deletion
  - Substitution
- Needed to transform one into the other

MINIMUM EDIT DISTANCE

- Two strings and their **alignment**:

```
I N T E * N T I O N
| | | | | | | | | |
* E X E C U T I O N
```

MINIMUM EDIT DISTANCE

```
I N T E * N T I O N
| | | | | | | | | |
* E X E C U T I O N
d s s   i s
```

- If each operation has cost of 1
  - Distance between these is 5
- If substitutions cost 2 (Levenshtein)
  - Distance between them is 8

ALIGNMENT IN COMPUTATIONAL BIOLOGY

- Given a sequence of bases

AGGCTATCACCTGACCTCCAGGCCGATGCCC

TAGCTATCACGACCGCGGTCGATTTGCCCGAC

- An alignment:

-**AG**G**CTATCAC**CT**GACC**T**C**CA**GG**C**CGA**--**TGCCC**---

T**AG**-**CTATCAC**--**GACC**G**C**--**GG**T**CGA**TT**TGCCC**GAC

- Given two sequences, align each letter to a letter or gap

OTHER USES OF EDIT DISTANCE IN NLP

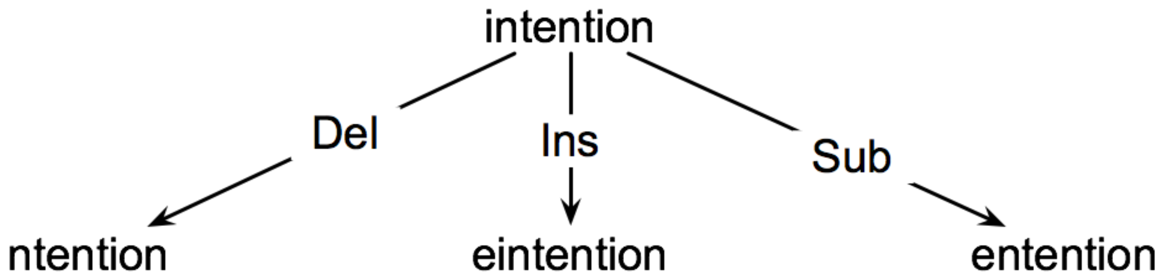
- Evaluating Machine Translation and speech recognition

```
R Spokesman confirms    senior government adviser was shot
H Spokesman said    the senior            adviser was shot dead
              S      I              D                        I
```

- Named Entity Extraction and Entity Coreference
  - IBM Inc. announced today
  - IBM profits
  - Stanford President John Hennessy announced yesterday
  - for Stanford University President John Hennessy

HOW TO FIND THE MIN EDIT DISTANCE?

- Searching for a path (sequence of edits) from the start string to the final string:
  - **Initial state**: the word we're transforming
  - **Operators**: insert, delete, substitute
  - **Goal state**: the word we're trying to get to
  - **Path cost**: what we want to minimize: the number of edits



MINIMUM EDIT AS SEARCH

- But the space of all edit sequences is huge!
  - We can’t afford to navigate naïvely
  - Lots of distinct paths wind up at the same state.
    - We don’t have to keep track of all of them
    - Just the shortest path to each of those revisted states.

DEFINING MIN EDIT DISTANCE

- For two strings
  - X of length n
  - Y of length m
- We define D(i,j)
  - the edit distance between X[1..i] and Y[1..j]
    - i.e., the first i characters of X and the first j characters of Y
  - The edit distance between X and Y is thus D(n,m)

MINIMUM EDIT DISTANCE

Definition of Minimum Edit Distance

MINIMUM EDIT DISTANCE

Computing Minimum Edit Distance

DYNAMIC PROGRAMMING FOR MINIMUM EDIT DISTANCE

- **Dynamic programming**: A tabular computation of D(n,m)
- Solving problems by combining solutions to subproblems.
- Bottom-up
  - We compute D(i,j) for small *i,j*
  - And compute larger D(i,j) based on previously computed smaller values
  - i.e., compute D(i,j) for all i ($0 < i < n$) and j ($0 < j < m$)

DEFINING MIN EDIT DISTANCE (LEVENSHTEIN)

- **Initialization**

```
D(i,0) = i
D(0,j) = j
```

- **Recurrence Relation:**

```
For each  i = 1…M
      For each  j = 1…N
```

$$D(i,j) = \min \begin{cases} D(i-1,j) + 1 \\ D(i,j-1) + 1 \\ D(i-1,j-1) + \begin{cases} 2; & \text{if } X(i) \neq Y(j) \\ 0; & \text{if } X(i) = Y(j) \end{cases} \end{cases}$$

- **Termination:**

```
D(N,M) is distance
```

THE EDIT DISTANCE TABLE

N	9									
O	8									
I	7									
T	6									
N	5									
E	4									
T	3									
N	2									
I	1									
#	**0**	1	2	3	4	5	6	7	8	9
	#	E	X	E	C	U	T	I	O	N

The Edit Distance Table

N	9									
O	8									
I	7									
T	6									
N	5									
E	4									
T	3									
N	2									
I	1									
#	0	1	2	3	4	5	6	7	8	9
	#	E	X	E	C	U	T	I	O	N

$$D(i,j) = \min \begin{cases} D(i-1,j) + 1 \\ D(i,j-1) + 1 \\ D(i-1,j-1) + \begin{cases} 2; & \text{if } S_1(i) \neq S_2(j) \\ 0; & \text{if } S_1(i) = S_2(j) \end{cases} \end{cases}$$

EDIT DISTANCE

$$D(i,j) = \min \begin{cases} D(i-1,j) + 1 \\ D(i,j-1) + 1 \\ D(i-1,j-1) + \begin{cases} 2; & \text{if } S_1(i) \neq S_2(j) \\ 0; & \text{if } S_1(i) = S_2(j) \end{cases} \end{cases}$$

N	9									
O	8									
I	7									
T	6									
N	5									
E	4									
T	3									
N	2									
I	1									
#	**0**	1	2	3	4	5	6	7	8	9
	#	E	X	E	C	U	T	I	O	N

The Edit Distance Table

N	9	8	9	10	11	12	11	10	9	**8**
O	8	7	8	9	10	11	10	9	8	9
I	7	6	7	8	9	10	9	8	9	10
T	6	5	6	7	8	9	8	9	10	11
N	5	4	5	6	7	8	9	10	11	10
E	4	3	4	5	6	7	8	9	10	9
T	3	4	5	6	7	8	7	8	9	8
N	2	3	4	5	6	7	8	7	8	7
I	1	2	3	4	5	6	7	6	7	8
#	**0**	1	2	3	4	5	6	7	8	9
	#	E	X	E	C	U	T	I	O	N

MINIMUM EDIT DISTANCE

Computing Minimum Edit Distance

MINIMUM EDIT DISTANCE

Backtrace for Computing Alignments

COMPUTING ALIGNMENTS

- Edit distance isn't sufficient
  - We often need to **align** each character of the two strings to each other
- We do this by keeping a "backtrace"
- Every time we enter a cell, remember where we came from
- When we reach the end,
  - Trace back the path from the upper right corner to read off the alignment

EDIT DISTANCE

$$D(i,j) = \min \begin{cases} D(i-1,j) + 1 \\ D(i,j-1) + 1 \\ D(i-1,j-1) + \begin{cases} 2; & \text{if } S_1(i) \neq S_2(j) \\ 0; & \text{if } S_1(i) = S_2(j) \end{cases} \end{cases}$$

N	9									
O	8									
I	7									
T	6									
N	5									
E	4									
T	3									
N	2									
I	1									
#	**0**	1	2	3	4	5	6	7	8	9
	#	E	X	E	C	U	T	I	O	N

MINEDIT WITH BACKTRACE

n	9	↓ 8	↙←↓ 9	↙←↓ 10	↙←↓ 11	↙←↓ 12	↓ 11	↓ 10	↓ 9	↙ **8**	
o	8	↓ 7	↙←↓ 8	↙←↓ 9	↙←↓ 10	↙←↓ 11	↓ 10	↓ 9	↙ **8**	← 9	
i	7	↓ 6	↙←↓ 7	↙←↓ 8	↙←↓ 9	↙←↓ 10	↓ 9	↙ **8**	← 9	← 10	
t	6	↓ 5	↙←↓ 6	↙←↓ 7	↙←↓ 8	↙←↓ 9	↙ **8**	← 9	← 10	←↓ 11	
n	5	↓ 4	↙←↓ 5	↙←↓ 6	↙←↓ 7	↙←↓ **8**	↙←↓ 9	↙←↓ 10	↙←↓ 11	↙↓ 10	
e	4	↙ 3	← 4	↙← **5**	← **6**	← 7	←↓ 8	↙←↓ 9	↙←↓ 10	↓ 9	
t	3	↙←↓ 4	↙←↓ **5**	↙←↓ 6	↙←↓ 7	↙←↓ 8	↙ 7	←↓ 8	↙←↓ 9	↓ 8	
n	2	↙←↓ **3**	↙←↓ 4	↙←↓ 5	↙←↓ 6	↙←↓ 7	↙←↓ 8	↓ 7	↙←↓ 8	↙ 7	
i	**1**	↙←↓ 2	↙←↓ 3	↙←↓ 4	↙←↓ 5	↙←↓ 6	↙←↓ 7	↙ 6	← 7	← 8	
#	**0**	1	2	3	4	5	6	7	8	9	
	#	**e**	**x**	**e**	**c**	**u**	**t**	**i**	**o**	**n**	

ADDING BACKTRACE TO MINIMUM EDIT DISTANCE

- **Base conditions:**

  D(i,0) = i   D(0,j) = j

Termination:

D(N,M) is distance

- **Recurrence Relation:**

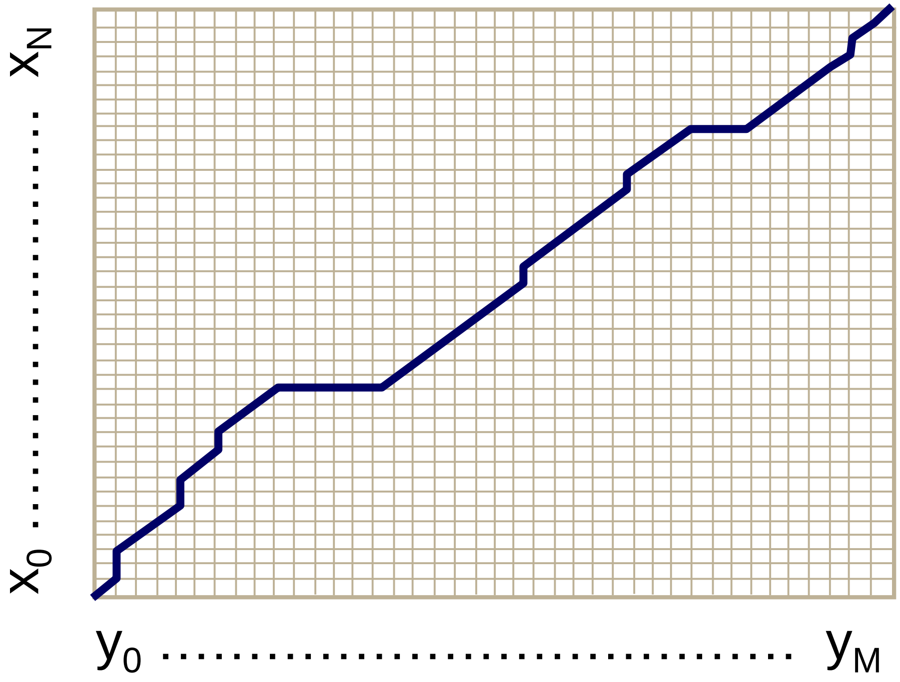
  For each i = 1…M

  For each j = 1…N

$$D(i,j) = \min \begin{cases} D(i-1,j) + 1 & \text{deletion} \\ D(i,j-1) + 1 & \text{insertion} \\ D(i-1,j-1) + \begin{cases} 2; & \text{if } X(i) \neq Y(j) \\ 0; & \text{if } X(i) = Y(j) \end{cases} & \text{substitution} \end{cases}$$

$$ptr(i,j) = \begin{cases} \text{LEFT} & \text{insertion} \\ \text{DOWN} & \text{deletion} \\ \text{DIAG} & \text{substitution} \end{cases}$$

THE DISTANCE MATRIX



Every non-decreasing path

from (0,0) to (M, N)

corresponds to
an alignment
of the two sequences

An optimal alignment is composed
of optimal subalignments

Slide adapted from Serafim Batzoglou

RESULT OF BACKTRACE

- Two strings and their **alignment**:

```
I N T E * N T I O N
| | | | | | | | | |
* E X E C U T I O N
```

PERFORMANCE

- Time:

  O(nm)

- Space:

  O(nm)

- Backtrace

  O(n+m)

MINIMUM EDIT DISTANCE

Backtrace for Computing Alignments

MINIMUM EDIT DISTANCE

Weighted Minimum Edit Distance

WEIGHTED EDIT DISTANCE

- Why would we add weights to the computation?
  - Spell Correction: some letters are more likely to be mistyped than others
  - Biology: certain kinds of deletions or insertions are more likely than others

CONFUSION MATRIX FOR SPELLING ERRORS

sub[X, Y] = Substitution of X (incorrect) for Y (correct)

Y (correct)

X	a	b	c	d	e	f	g	h	i	j	k	l	m	n	o	p	q	r	s	t	u	v	w	x	y	z
a	0	0	7	1	342	0	0	2	118	0	1	0	0	3	76	0	0	1	35	9	9	0	1	0	5	0
b	0	0	9	9	2	2	3	1	0	0	0	5	11	5	0	10	0	0	2	1	0	0	8	0	0	0
c	6	5	0	16	0	9	5	0	0	0	1	0	7	9	1	10	2	5	39	40	1	3	7	1	1	0
d	1	10	13	0	12	0	5	5	0	0	2	3	7	3	0	1	0	43	30	22	0	0	4	0	2	0
e	388	0	3	11	0	2	2	0	89	0	0	3	0	5	93	0	0	14	12	6	15	0	1	0	18	0
f	0	15	0	3	1	0	5	2	0	0	0	3	4	1	0	0	0	6	4	12	0	0	2	0	0	0
g	4	1	11	11	9	2	0	0	0	1	1	3	0	0	2	1	3	5	13	21	0	0	1	0	3	0
h	1	8	0	3	0	0	0	0	0	0	2	0	12	14	2	3	0	3	1	11	0	0	2	0	0	0
i	103	0	0	0	146	0	1	0	0	0	0	6	0	0	49	0	0	0	2	1	47	0	2	1	15	0
j	0	1	1	9	0	0	1	0	0	0	0	2	1	0	0	0	0	0	5	0	0	0	0	0	0	0
k	1	2	8	4	1	1	2	5	0	0	0	0	5	0	2	0	0	0	6	0	0	0	4	0	0	3
l	2	10	1	4	0	4	5	6	13	0	1	0	0	14	2	5	0	11	10	2	0	0	0	0	0	0
m	1	3	7	8	0	2	0	6	0	0	4	4	0	180	0	6	0	0	9	15	13	3	2	2	3	0
n	2	7	6	5	3	0	1	19	1	0	4	35	78	0	0	7	0	28	5	7	0	0	1	2	0	2
o	91	1	1	3	116	0	0	0	25	0	2	0	0	0	0	14	0	2	4	14	39	0	0	0	18	0
p	0	11	1	2	0	6	5	0	2	9	0	2	7	6	15	0	0	1	3	6	0	4	1	0	0	0
q	0	0	1	0	0	0	27	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
r	0	14	0	30	12	2	2	8	2	0	5	8	4	20	1	14	0	0	12	22	4	0	0	1	0	0
s	11	8	27	33	35	4	0	1	0	1	0	27	0	6	1	7	0	14	0	15	0	0	5	3	20	1
t	3	4	9	42	7	5	19	5	0	1	0	14	9	5	5	6	0	11	37	0	0	2	19	0	7	6
u	20	0	0	0	44	0	0	0	64	0	0	0	0	2	43	0	0	4	0	0	0	0	2	0	8	0
v	0	0	7	0	0	3	0	0	0	0	0	1	0	0	1	0	0	0	8	3	0	0	0	0	0	0
w	2	2	1	0	1	0	0	2	0	0	1	0	0	0	0	7	0	6	3	3	1	0	0	0	0	0
x	0	0	0	2	0	0	0	0	0	0	0	0	0	0	0	0	0	0	9	0	0	0	0	0	0	0
y	0	0	2	0	15	0	1	7	15	0	0	0	2	0	6	1	0	7	36	8	5	0	0	1	0	0
z	0	0	0	7	0	0	0	0	0	0	0	7	5	0	0	0	0	2	21	3	0	0	0	0	3	0


~ ` ! 1 @ 2 # 3 $ 4 % 5 ^ 6 & 7 * 8 (9) 0 — - + =
Tab Q W E R T Y U I O P { [}]
Caps Lock A S D F G H J K L : ; " ' Enter
Shift Z X C V B N M < , > . ? / Shift
Ctrl Alt Alt

WEIGHTED MIN EDIT DISTANCE

- **Initialization:**

```
D(0,0) = 0
D(i,0) = D(i-1,0) + del[x(i)];    1 < i ≤ N
D(0,j) = D(0,j-1) + ins[y(j)];    1 < j ≤ M
```

- **Recurrence Relation:**

$$D(i,j) = \min \begin{cases} D(i-1,j) + del[x(i)] \\ D(i,j-1) + ins[y(j)] \\ D(i-1,j-1) + sub[x(i),y(j)] \end{cases}$$

- **Termination:**

```
D(N,M) is distance
```

WHERE DID THE NAME, DYNAMIC PROGRAMMING, COME FROM?

…The 1950s were not good years for mathematical research. [the] Secretary of Defense …had a pathological fear and hatred of the word, research…

I decided therefore to use the word, "**programming**".

I wanted to get across the idea that this was dynamic, this was multistage… I thought, let's … take a word that has an absolutely precise meaning, namely **dynamic**… it's impossible to use the word, **dynamic**, in a pejorative sense. Try thinking of some combination that will possibly give it a pejorative meaning. It's impossible.

Thus, I thought dynamic programming was a good name. It was something not even a Congressman could object to."

Richard Bellman, "Eye of the Hurricane: an autobiography" 1984.

MINIMUM EDIT DISTANCE

Weighted Minimum Edit Distance

MINIMUM EDIT DISTANCE

Minimum Edit Distance in Computational Biology

SEQUENCE ALIGNMENT

AGGCTATCACCTGACCTCCAGGCCGATGCCC

TAGCTATCACGACCGCGGTCGATTTGCCCGAC

-**AG**G**CTATCAC**CT**GACC**T**C**CA**GG**C**CGA**--**TGCCC**---

T**AG**-**CTATCAC**--**GACC**G**C**--**GG**T**CGA**TT**TGCCC**GAC

WHY SEQUENCE ALIGNMENT?

- Comparing genes or regions from different species
  - to find important regions
  - determine function
  - uncover evolutionary forces
- Assembling fragments to sequence DNA
- Compare individuals to looking for mutations

ALIGNMENTS IN TWO FIELDS

- In Natural Language Processing
  - We generally talk about distance (minimized)
    - And weights
- In Computational Biology
  - We generally talk about similarity (maximized)
    - And scores

THE NEEDLEMAN-WUNSCH ALGORITHM

- Initialization:

  D(i,0) = -i * d

  D(0,j) = -j * d

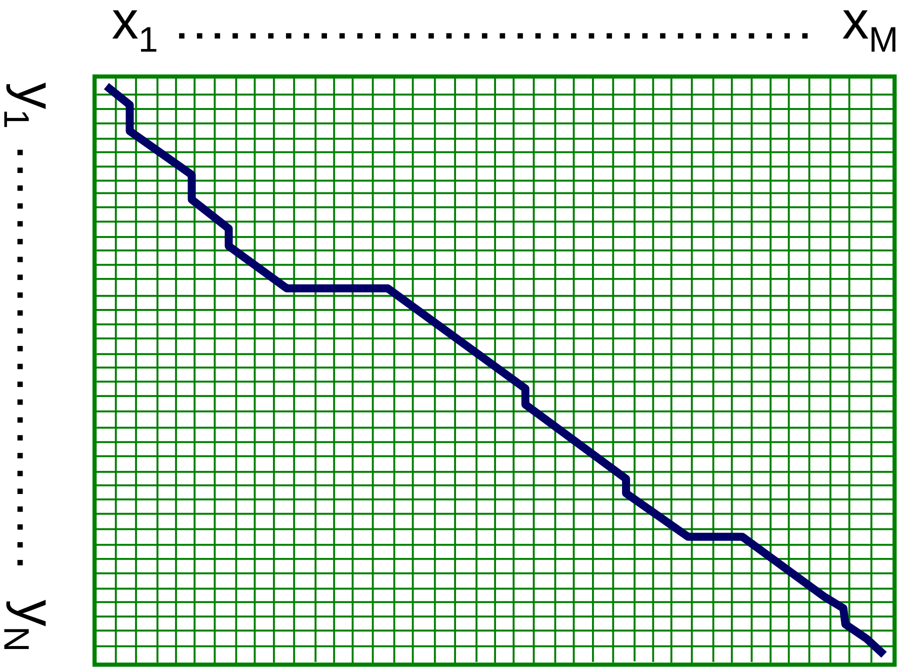
- Recurrence Relation*:*

$$
D(i,j) = \min \begin{cases} D(i-1,j) - d \\ D(i,j-1) - d \\ D(i-1,j-1) + s[x(i),y(j)] \end{cases}
$$

- Termination*:*

  D(N,M) is distance

THE NEEDLEMAN-WUNSCH MATRIX



(Note that the origin is at the upper left.)

Slide adapted from Serafim Batzoglou

A VARIANT OF THE BASIC ALGORITHM:

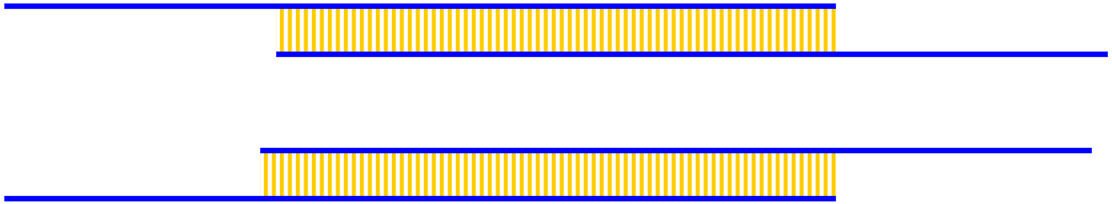
- Maybe it is OK to have an unlimited # of gaps in the beginning and end:

```
----------CTATCACCTGACCTCCAGGCCGATGCCCCTTCCGGC
GCGAGTTCATCTATCAC--GACCGC--GGTCG--------------
```

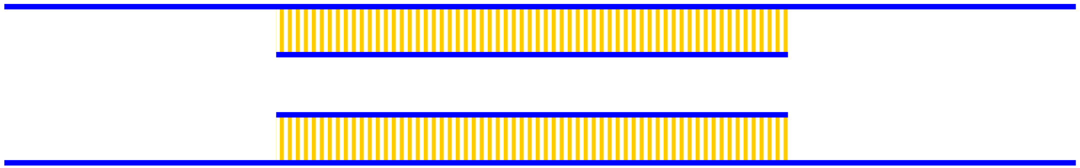
- If so, we don't want to penalize gaps at the ends

Slide from Serafim Batzoglou

DIFFERENT TYPES OF OVERLAPS



Example:
2 overlapping"*reads*" from a sequencing project



Example:
Search for a mouse gene within a human chromosome

Slide from Serafim Batzoglou

THE OVERLAP DETECTION VARIANT

Changes:

1. Initialization

   For all i, j,

   $F(i, 0) = 0$

   $F(0, j) = 0$

2. Termination

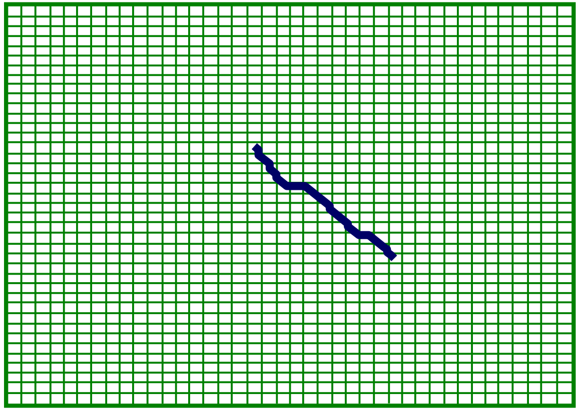
$$F_{OPT} = \max \begin{cases} \max_i F(i, N) \\ \max_j F(M, j) \end{cases}$$

Slide from Serafim Batzoglou

THE LOCAL ALIGNMENT PROBLEM

Given two strings $x = x_1 \ldots\ldots x_M$,

$y = y_1 \ldots\ldots y_N$



Find substrings x', y' whose similarity
(optimal global alignment value)
is maximum

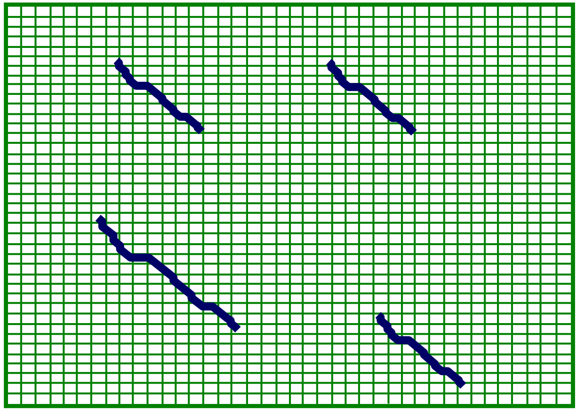
x = aaaa$\boxed{\text{cccccgggg}}$ta

y = $\boxed{\text{tcccggg}}$aaccaacc

Slide from Serafim Batzoglou

THE SMITH-WATERMAN ALGORITHM

Idea: Ignore badly aligning regions



Modifications to Needleman-Wunsch:

Initialization: $F(0, j) = 0$

$F(i, 0) = 0$

Iteration:

$$F(i, j) = \max \begin{cases} 0 \\ F(i-1, j) - d \\ F(i, j-1) - d \\ F(i-1, j-1) + s(x_i, y_j) \end{cases}$$

Slide from Serafim Batzoglou

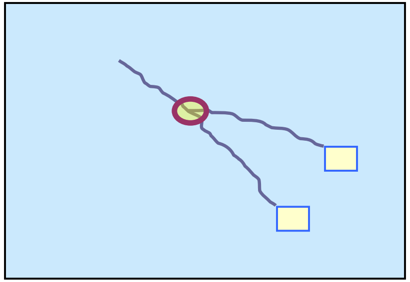
THE SMITH-WATERMAN ALGORITHM

Termination:

1. If we want the best local alignment…

$$F_{OPT} = \max_{i,j} F(i, j)$$

Find F_{OPT} and trace back



2. If we want all local alignments scoring > t

?? For all i, j find F(i, j) > t, and trace back?

Complicated by overlapping local alignments

Slide from Serafim Batzoglou

LOCAL ALIGNMENT EXAMPLE

X = ATCAT

Y = ATTATC

Let:

m = 1 (1 point for match)

d = 1 (-1 point for del/ins/sub)

		A	T	T	A	T	C
	0	0	0	0	0	0	0
A	0						
T	0						
C	0						
A	0						
T	0						

LOCAL ALIGNMENT EXAMPLE

X = ATCAT

Y = ATTATC

		A	T	T	A	T	C
	0	0	0	0	0	0	0
A	0	1	0	0	1	0	0
T	0	0	2	1	0	2	0
C	0	0	1	1	0	1	3
A	0	1	0	0	2	1	2
T	0	0	2	0	1	3	2

LOCAL ALIGNMENT EXAMPLE

X = **ATCAT**

Y = **ATTAT**C

		A	T	T	A	T	C
	0	0	0	0	0	0	0
A	0	1	0	0	1	0	0
T	0	0	2	1	0	2	0
C	0	0	1	1	0	1	3
A	0	1	0	0	2	1	2
T	0	0	2	0	1	3	2

LOCAL ALIGNMENT EXAMPLE

X = **ATC**AT

Y = ATT**ATC**

		A	T	T	A	T	C
	0	0	0	0	0	0	0
A	0	1	0	0	1	0	0
T	0	0	2	1	0	2	0
C	0	0	1	1	0	1	3
A	0	1	0	0	2	1	2
T	0	0	2	0	1	3	2

MINIMUM EDIT DISTANCE

Minimum Edit Distance in Computational Biology

LANGUAGE MODELING

Introduction to N-grams

PROBABILISTIC LANGUAGE MODELS

- Today's goal: assign a probability to a sentence
  - Machine Translation:
    - P(**high** winds tonite) > P(**large** winds tonite)
  - Spell Correction
    - The office is about fifteen **minuets** from my house
      - P(about fifteen **minutes** from) > P(about fifteen **minuets** from)
  - Speech Recognition
    - P(I saw a van) >> P(eyes awe of an)
  - + Summarization, question-answering, etc., etc.!!

Why?

PROBABILISTIC LANGUAGE MODELING

- Goal: compute the probability of a sentence or sequence of words:

  $P(W) = P(w_1,w_2,w_3,w_4,w_5\ldots w_n)$

- Related task: probability of an upcoming word:

  $P(w_5|w_1,w_2,w_3,w_4)$

- A model that computes either of these:

  $P(W)$ or $P(w_n|w_1,w_2\ldots w_{n-1})$ is called a **language model**.

- Better: **the grammar** But **language model** or **LM** is standard

HOW TO COMPUTE P(W)

- How to compute this joint probability:
  - P(its, water, is, so, transparent, that)
- Intuition: let’s rely on the Chain Rule of Probability

REMINDER: THE CHAIN RULE

- Recall the definition of conditional probabilities

Rewriting:

- More variables:

$$P(A,B,C,D) = P(A)P(B|A)P(C|A,B)P(D|A,B,C)$$

- The Chain Rule in General

$$P(x_1,x_2,x_3,\ldots,x_n) = P(x_1)P(x_2|x_1)P(x_3|x_1,x_2)\ldots P(x_n|x_1,\ldots,x_{n-1})$$

THE CHAIN RULE APPLIED TO COMPUTE JOINT PROBABILITY OF WORDS IN SENTENCE

$$P(w_1 w_2 \ldots w_n) = \prod_i P(w_i \mid w_1 w_2 \ldots w_{i-1})$$

P("its water is so transparent") =

P(its) × P(water|its) × P(is|its water)

× P(so|its water is) × P(transparent|its water is so)

How to Estimate These Probabilities

- Could we just count and divide?

$$P(\text{the} \mid \text{its water is so transparent that}) = \frac{Count(\text{its water is so transparent that the})}{Count(\text{its water is so transparent that})}$$

- No! Too many possible sentences!
- We'll never see enough data for estimating these

MARKOV ASSUMPTION

Andrei Markov

- Simplifying assumption:

$$P(\text{the} \mid \text{its water is so transparent that}) \approx P(\text{the} \mid \text{that})$$

- Or maybe

$$P(\text{the} \mid \text{its water is so transparent that}) \approx P(\text{the} \mid \text{transparent that})$$

MARKOV ASSUMPTION

$$P(w_1 w_2 \ldots w_n) \approx \prod_i P(w_i \mid w_{i-k} \ldots w_{i-1})$$

- In other words, we approximate each component in the product

$$P(w_i \mid w_1 w_2 \ldots w_{i-1}) \approx P(w_i \mid w_{i-k} \ldots w_{i-1})$$

SIMPLEST CASE: UNIGRAM MODEL

$$P(w_1 w_2 \ldots w_n) \approx \prod_i P(w_i)$$

Some automatically generated sentences from a unigram model

```
fifth, an, of, futures, the, an, incorporated, a,
a, the, inflation, most, dollars, quarter, in, is,
mass

thrift, did, eighty, said, hard, 'm, july, bullish

that, or, limited, the
```

BIGRAM MODEL

- Condition on the previous word:

$$P(w_i \mid w_1 w_2 \ldots w_{i-1}) \approx P(w_i \mid w_{i-1})$$

texaco, rose, one, in, this, issue, is, pursuing, growth, in, a, boiler, house, said, mr., gurria, mexico, 's, motion, control, proposal, without, permission, from, five, hundred, fifty, five, yen

outside, new, car, parking, lot, of, the, agreement, reached

this, would, be, a, record, november

N-GRAM MODELS

- We can extend to trigrams, 4-grams, 5-grams
- In general this is an insufficient model of language
  - because language has **long-distance dependencies**:

“The computer which I had just put into the machine room on the fifth floor crashed.”

- But we can often get away with N-gram models

LANGUAGE MODELING

Introduction to N-grams

LANGUAGE MODELING

Estimating N-gram Probabilities

ESTIMATING BIGRAM PROBABILITIES

- The Maximum Likelihood Estimate

$$P(w_i \mid w_{i-1}) = \frac{count(w_{i-1}, w_i)}{count(w_{i-1})}$$

$$P(w_i \mid w_{i-1}) = \frac{c(w_{i-1}, w_i)}{c(w_{i-1})}$$

AN EXAMPLE

$$P(w_i \mid w_{i-1}) = \frac{c(w_{i-1}, w_i)}{c(w_{i-1})}$$

<s> I am Sam </s>

<s> Sam I am </s>

<s> I do not like green eggs and ham </s>

$P(\texttt{I} \mid \texttt{<s>}) = \frac{2}{3} = .67$ $\quad P(\texttt{Sam} \mid \texttt{<s>}) = \frac{1}{3} = .33$ $\quad P(\texttt{am} \mid \texttt{I}) = \frac{2}{3} = .67$

$P(\texttt{</s>} \mid \texttt{Sam}) = \frac{1}{2} = 0.5$ $\quad P(\texttt{Sam} \mid \texttt{am}) = \frac{1}{2} = .5$ $\quad P(\texttt{do} \mid \texttt{I}) = \frac{1}{3} = .33$

MORE EXAMPLES: BERKELEY RESTAURANT PROJECT SENTENCES

- can you tell me about any good cantonese restaurants close by
- mid priced thai food is what i’m looking for
- tell me about chez panisse
- can you give me a listing of the kinds of food that are available
- i’m looking for a good place to eat breakfast
- when is caffe venezia open during the day

RAW BIGRAM COUNTS

- Out of 9222 sentences

	i	want	to	eat	chinese	food	lunch	spend
i	5	827	0	9	0	0	0	2
want	2	0	608	1	6	6	5	1
to	2	0	4	686	2	0	6	211
eat	0	0	2	0	16	2	42	0
chinese	1	0	0	0	0	82	1	0
food	15	0	15	0	1	4	0	0
lunch	2	0	0	0	0	1	0	0
spend	1	0	1	0	0	0	0	0

RAW BIGRAM PROBABILITIES

- Normalize by unigrams:

i	want	to	eat	chinese	food	lunch	spend
2533	927	2417	746	158	1093	341	278

- Result

	i	want	to	eat	chinese	food	lunch	spend
i	0.002	0.33	0	0.0036	0	0	0	0.00079
want	0.0022	0	0.66	0.0011	0.0065	0.0065	0.0054	0.0011
to	0.00083	0	0.0017	0.28	0.00083	0	0.0025	0.087
eat	0	0	0.0027	0	0.021	0.0027	0.056	0
chinese	0.0063	0	0	0	0	0.52	0.0063	0
food	0.014	0	0.014	0	0.00092	0.0037	0	0
lunch	0.0059	0	0	0	0	0.0029	0	0
spend	0.0036	0	0.0036	0	0	0	0	0

BIGRAM ESTIMATES OF SENTENCE PROBABILITIES

P(<s> I want english food </s>) =

P(I|<s>)

× P(want|I)

× P(english|want)

× P(food|english)

× P(</s>|food)

= .000031

WHAT KINDS OF KNOWLEDGE?

- P(english|want) = .0011
- P(chinese|want) = .0065
- P(to|want) = .66
- P(eat | to) = .28
- P(food | to) = 0
- P(want | spend) = 0
- P (i | <s>) = .25

PRACTICAL ISSUES

- We do everything in log space
  - Avoid underflow
  - (also adding is faster than multiplying)

$$\log(p_1 \times p_2 \times p_3 \times p_4) = \log p_1 + \log p_2 + \log p_3 + \log p_4$$

LANGUAGE MODELING TOOLKITS

- SRILM
  - http://www.speech.sri.com/projects/srilm/

GOOGLE N-GRAM RELEASE, AUGUST 2006

AUG
3

All Our N-gram are Belong to You

Posted by Alex Franz and Thorsten Brants, Google Machine Translation Team

Here at Google Research we have been using word n-gram models for a variety of R&D projects,

...

That's why we decided to share this enormous dataset with everyone. We processed 1,024,908,267,229 words of running text and are publishing the counts for all 1,176,470,663 five-word sequences that appear at least 40 times. There are 13,588,391 unique words, after discarding words that appear less than 200 times.

GOOGLE N-GRAM RELEASE

- serve as the incoming 92
- serve as the incubator 99
- serve as the independent 794
- serve as the index 223
- serve as the indication 72
- serve as the indicator 120
- serve as the indicators 45
- serve as the indispensable 111
- serve as the indispensible 40
- serve as the individual 234

http://googleresearch.blogspot.com/2006/08/all-our-n-gram-are-belong-to-you.html

GOOGLE BOOK N-GRAMS

- http://ngrams.googlelabs.com/

LANGUAGE MODELING

Estimating N-gram Probabilities

LANGUAGE MODELING

Evaluating Perplexity

EVALUATION: HOW GOOD IS OUR MODEL?

- Does our language model prefer good sentences to bad ones?
  - Assign higher probability to "real" or "frequently observed" sentences
    - Than "ungrammatical" or "rarely observed" sentences?
- We train parameters of our model on a **training set**.
- We test the model's performance on data we haven't seen.
  - A **test set** is an unseen dataset that is different from our training set, totally unused.
  - An **evaluation metric** tells us how well our model does on the test set.

EXTRINSIC EVALUATION OF N-GRAM MODELS

- Best evaluation for comparing models A and B
  - Put each model in a task
    - spelling corrector, speech recognizer, MT system
  - Run the task, get an accuracy for A and for B
    - How many misspelled words corrected properly
    - How many words translated correctly
  - Compare accuracy for A and B

DIFFICULTY OF EXTRINSIC (IN-VIVO) EVALUATION OF N-GRAM MODELS

- Extrinsic evaluation
  - Time-consuming; can take days or weeks
- So
  - Sometimes use **intrinsic** evaluation: **perplexity**
  - Bad approximation
    - unless the test data looks **just** like the training data
    - So **generally only useful in pilot experiments**
  - But is helpful to think about.

INTUITION OF PERPLEXITY

- The Shannon Game:
  - How well can we predict the next word?

    I always order pizza with cheese and ____

    The 33rd President of the US was ____

    I saw a ____

  - Unigrams are terrible at this game. (Why?)
- A better model of a text
  - is one which assigns a higher probability to the word that actually occurs

mushrooms 0.1

pepperoni 0.1

anchovies 0.01

….

fried rice 0.0001

….

and 1e-100

PERPLEXITY

The best language model is one that best predicts an unseen test set

- Gives the highest P(sentence)

Perplexity is the inverse probability of the test set, normalized by the number of words:

$$PP(W) = P(w_1 w_2 \ldots w_N)^{-\frac{1}{N}}$$

$$= \sqrt[N]{\frac{1}{P(w_1 w_2 \ldots w_N)}}$$

Chain rule:

$$\mathrm{PP}(W) = \sqrt[N]{\prod_{i=1}^{N} \frac{1}{P(w_i | w_1 \ldots w_{i-1})}}$$

For bigrams:

$$\mathrm{PP}(W) = \sqrt[N]{\prod_{i=1}^{N} \frac{1}{P(w_i | w_{i-1})}}$$

Minimizing perplexity is the same as maximizing probability

THE SHANNON GAME INTUITION FOR PERPLEXITY

- From Josh Goodman
- How hard is the task of recognizing digits ‘0,1,2,3,4,5,6,7,8,9’
  - Perplexity 10
- How hard is recognizing (30,000) names at Microsoft.
  - Perplexity = 30,000
- If a system has to recognize
  - Operator (1 in 4)
  - Sales (1 in 4)
  - Technical Support (1 in 4)
  - 30,000 names (1 in 120,000 each)
  - Perplexity is 53
- Perplexity is weighted equivalent branching factor

PERPLEXITY AS BRANCHING FACTOR

- Let's suppose a sentence consisting of random digits
- What is the perplexity of this sentence according to a model that assign P=1/10 to each digit?

$$
\begin{aligned}
\mathrm{PP}(W) &= P(w_1 w_2 \ldots w_N)^{-\frac{1}{N}} \\
&= (\frac{1}{10}^N)^{-\frac{1}{N}} \\
&= \frac{1}{10}^{-1} \\
&= 10
\end{aligned}
$$

LOWER PERPLEXITY = BETTER MODEL

- Training 38 million words, test 1.5 million words, WSJ

N-gram Order	Unigram	Bigram	Trigram
Perplexity	962	170	109

LANGUAGE MODELING

Evaluation and Perplexity

LANGUAGE MODELING

Generalization and Zeros

THE SHANNON VISUALIZATION METHOD

- Choose a random bigram

  (<s>, w) according to its probability
- Now choose a random bigram (w, x) according to its probability
- And so on until we choose </s>
- Then string the words together

```
<s> I
    I want
      want to
           to eat
              eat Chinese
                  Chinese food
                          food </s>
I want to eat Chinese food
```

APPROXIMATING SHAKESPEARE

Unigram

To him swallowed confess hear both. Which. Of save on trail for are ay device and rote life have

Every enter now severally so, let

Hill he late speaks; or! a more to leg less first you enter

Are where exeunt and sighs have rise excellency took of.. Sleep knave we. near; vile like

Bigram

What means, sir. I confess she? then all sorts, he is trim, captain.

Why dost stand forth thy canopy, forsooth; he is this palpable hit the King Henry. Live king. Follow.

What we, hath got so she that I rest and sent to scold and nature bankrupt, nor the first gentleman?

Trigram

Sweet prince, Falstaff shall die. Harry of Monmouth's grave.

This shall forbid it should be branded, if renown made it empty.

Indeed the duke; and had a very good friend.

Fly, and will rid me these news of price. Therefore the sadness of parting, as they say, 'tis done.

Quadrigram

King Henry.What! I will go seek the traitor Gloucester. Exeunt some of the watch. A great banquet serv'd in;

Will you not tell me who I am?

It cannot be but so.

Indeed the short and the long. Marry, 'tis a noble Lepidus.

SHAKESPEARE AS CORPUS

- N=884,647 tokens, V=29,066
- Shakespeare produced 300,000 bigram types out of V^2= 844 million possible bigrams.
  - So 99.96% of the possible bigrams were never seen (have zero entries in the table)
- Quadrigrams worse: What's coming out looks like Shakespeare because it *is* Shakespeare

THE WALL STREET JOURNAL IS NOT SHAKESPEARE (NO OFFENSE)

Unigram

Months the my and issue of year foreign new exchange's september were recession exchange new endorsed a acquire to six executives

Bigram

Last December through the way to preserve the Hudson corporation N. B. E. C. Taylor would seem to complete the major central planners one point five percent of U. S. E. has already old M. X. corporation of living on information such as more frequently fishing to keep her

Trigram

They also point to ninety nine point six billion dollars from two hundred four oh six three percent of the rates of interest stores as Mexico and Brazil on market conditions

THE PERILS OF OVERFITTING

- N-grams only work well for word prediction if the test corpus looks like the training corpus
  - In real life, it often doesn’t
  - We need to train robust models that generalize!
  - One kind of generalization: Zeros!
    - Things that don’t ever occur in the training set
      - But occur in the test set

ZEROS

- Training set:
  … denied the allegations
  … denied the reports
  … denied the claims
  … denied the request

  P("offer" | denied the) = 0

- Test set
  … denied the offer
  … denied the loan

ZERO PROBABILITY BIGRAMS

- Bigrams with zero probability
  - mean that we will assign 0 probability to the test set!
- And hence we cannot compute perplexity (can't divide by 0)!

LANGUAGE MODELING

Generalization and Zeros

LANGUAGE MODELING

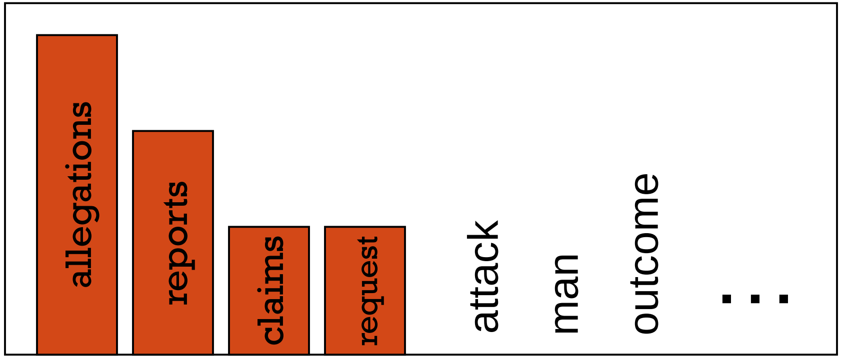
Smoothing: Add-one (Laplace) smoothing

THE INTUITION OF SMOOTHING (FROM DAN KLEIN)

- When we have sparse statistics:

P(w | denied the)
3 allegations
2 reports
1 claims
1 request

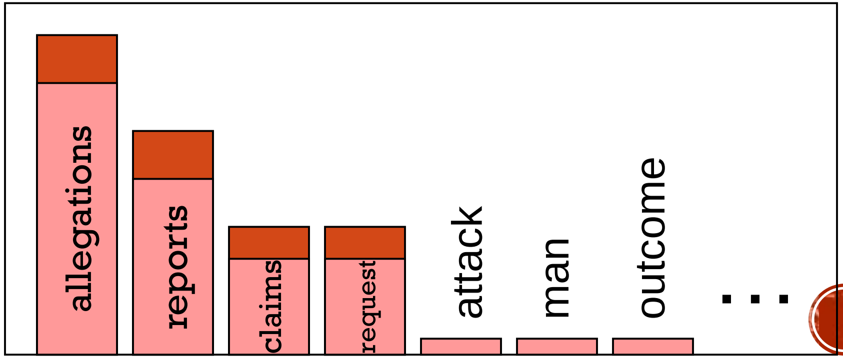
7 total



Steal probability mass to generalize better

P(w | denied the)
2.5 allegations
1.5 reports
0.5 claims
0.5 request
2 other

7 total



ADD-ONE ESTIMATION

- Also called Laplace smoothing
- Pretend we saw each word one more time than we did
- Just add one to all the counts!
- MLE estimate:

$$P_{MLE}(w_i \mid w_{i-1}) = \frac{c(w_{i-1}, w_i)}{c(w_{i-1})}$$

- Add-1 estimate:

$$P_{Add-1}(w_i \mid w_{i-1}) = \frac{c(w_{i-1}, w_i) + 1}{c(w_{i-1}) + V}$$

MAXIMUM LIKELIHOOD ESTIMATES

- The maximum likelihood estimate
  - of some parameter of a model M from a training set T
  - maximizes the likelihood of the training set T given the model M
- Suppose the word "bagel" occurs 400 times in a corpus of a million words
- What is the probability that a random word from some other text will be "bagel"?
- MLE estimate is 400/1,000,000 = .0004
- This may be a bad estimate for some other corpus
  - But it is the **estimate** that makes it **most likely** that "bagel" will occur 400 times in a million word corpus.

BERKELEY RESTAURANT CORPUS: LAPLACE SMOOTHED BIGRAM COUNTS

	i	want	to	eat	chinese	food	lunch	spend
i	6	828	1	10	1	1	1	3
want	3	1	609	2	7	7	6	2
to	3	1	5	687	3	1	7	212
eat	1	1	3	1	17	3	43	1
chinese	2	1	1	1	1	83	2	1
food	16	1	16	1	2	5	1	1
lunch	3	1	1	1	1	2	1	1
spend	2	1	2	1	1	1	1	1

LAPLACE-SMOOTHED BIGRAMS

$$P^*(w_n|w_{n-1}) = \frac{C(w_{n-1}w_n)+1}{C(w_{n-1})+V}$$

	i	want	to	eat	chinese	food	lunch	spend
i	0.0015	0.21	0.00025	0.0025	0.00025	0.00025	0.00025	0.00075
want	0.0013	0.00042	0.26	0.00084	0.0029	0.0029	0.0025	0.00084
to	0.00078	0.00026	0.0013	0.18	0.00078	0.00026	0.0018	0.055
eat	0.00046	0.00046	0.0014	0.00046	0.0078	0.0014	0.02	0.00046
chinese	0.0012	0.00062	0.00062	0.00062	0.00062	0.052	0.0012	0.00062
food	0.0063	0.00039	0.0063	0.00039	0.00079	0.002	0.00039	0.00039
lunch	0.0017	0.00056	0.00056	0.00056	0.00056	0.0011	0.00056	0.00056
spend	0.0012	0.00058	0.0012	0.00058	0.00058	0.00058	0.00058	0.00058

RECONSTITUTED COUNTS

$$c^*(w_{n-1}w_n) = \frac{[C(w_{n-1}w_n)+1] \times C(w_{n-1})}{C(w_{n-1})+V}$$

	i	want	to	eat	chinese	food	lunch	spend
i	3.8	527	0.64	6.4	0.64	0.64	0.64	1.9
want	1.2	0.39	238	0.78	2.7	2.7	2.3	0.78
to	1.9	0.63	3.1	430	1.9	0.63	4.4	133
eat	0.34	0.34	1	0.34	5.8	1	15	0.34
chinese	0.2	0.098	0.098	0.098	0.098	8.2	0.2	0.098
food	6.9	0.43	6.9	0.43	0.86	2.2	0.43	0.43
lunch	0.57	0.19	0.19	0.19	0.19	0.38	0.19	0.19
spend	0.32	0.16	0.32	0.16	0.16	0.16	0.16	0.16

COMPARE WITH RAW BIGRAM COUNTS

	i	want	to	eat	chinese	food	lunch	spend
i	5	827	0	9	0	0	0	2
want	2	0	608	1	6	6	5	1
to	2	0	4	686	2	0	6	211
eat	0	0	2	0	16	2	42	0
chinese	1	0	0	0	0	82	1	0
food	15	0	15	0	1	4	0	0
lunch	2	0	0	0	0	1	0	0
spend	1	0	1	0	0	0	0	0

	i	want	to	eat	chinese	food	lunch	spend
i	3.8	527	0.64	6.4	0.64	0.64	0.64	1.9
want	1.2	0.39	238	0.78	2.7	2.7	2.3	0.78
to	1.9	0.63	3.1	430	1.9	0.63	4.4	133
eat	0.34	0.34	1	0.34	5.8	1	15	0.34
chinese	0.2	0.098	0.098	0.098	0.098	8.2	0.2	0.098
food	6.9	0.43	6.9	0.43	0.86	2.2	0.43	0.43
lunch	0.57	0.19	0.19	0.19	0.19	0.38	0.19	0.19
spend	0.32	0.16	0.32	0.16	0.16	0.16	0.16	0.16

ADD-1 ESTIMATION IS A BLUNT INSTRUMENT

- So add-1 isn't used for N-grams:
  - We'll see better methods
- But add-1 is used to smooth other NLP models
  - For text classification
  - In domains where the number of zeros isn't so huge.

LANGUAGE MODELING

Smoothing: Add-one (Laplace) smoothing

LANGUAGE MODELING

Interpolation, Backoff, and Web-Scale LM's

BACKOFF AND INTERPOLATION

- Sometimes it helps to use **less** context
  - Condition on less context for contexts you haven't learned much about
- **Backoff:**
  - use trigram if you have good evidence,
  - otherwise bigram, otherwise unigram
- **Interpolation:**
  - mix unigram, bigram, trigram

- Interpolation works better

LINEAR INTERPOLATION

- Simple interpolation

$$\begin{aligned}\hat{P}(w_n|w_{n-1}w_{n-2}) &= \lambda_1 P(w_n|w_{n-1}w_{n-2}) \\ &+\lambda_2 P(w_n|w_{n-1}) \\ &+\lambda_3 P(w_n)\end{aligned}$$

$$\sum_i \lambda_i = 1$$

- Lambdas conditional on context:

$$\begin{aligned}\hat{P}(w_n|w_{n-2}w_{n-1}) &= \lambda_1(w_{n-2}^{n-1})P(w_n|w_{n-2}w_{n-1}) \\ &+\lambda_2(w_{n-2}^{n-1})P(w_n|w_{n-1}) \\ &+\lambda_3(w_{n-2}^{n-1})P(w_n)\end{aligned}$$

HOW TO SET THE LAMBDAS?

- Use a **held-out** corpus

Training Data | Held-Out Data | Test Data

- Choose λs to maximize the probability of held-out data:
  - Fix the N-gram probabilities (on the training data)
  - Then search for λs that give largest probability to held-out set:

$$\log P(w_1 \ldots w_n \mid M(\lambda_1 \ldots \lambda_k)) = \sum_i \log P_{M(\lambda_1 \ldots \lambda_k)}(w_i \mid w_{i-1})$$

UNKNOWN WORDS: OPEN VERSUS CLOSED VOCABULARY TASKS

- If we know all the words in advanced
  - Vocabulary V is fixed
  - Closed vocabulary task
- Often we don’t know this
  - **Out Of Vocabulary** = OOV words
  - Open vocabulary task
- Instead: create an unknown word token <UNK>
  - Training of <UNK> probabilities
    - Create a fixed lexicon L of size V
    - At text normalization phase, any training word not in L changed to <UNK>
    - Now we train its probabilities like a normal word
  - At decoding time
    - If text input: Use UNK probabilities for any word not in training

HUGE WEB-SCALE N-GRAMS

- How to deal with, e.g., Google N-gram corpus
- Pruning
  - Only store N-grams with count > threshold.
    - Remove singletons of higher-order n-grams
  - Entropy-based pruning
- Efficiency
  - Efficient data structures like tries
  - Bloom filters: approximate language models
  - Store words as indexes, not strings
    - Use Huffman coding to fit large numbers of words into two bytes
  - Quantize probabilities (4-8 bits instead of 8-byte float)

SMOOTHING FOR WEB-SCALE N-GRAMS

- "Stupid backoff" (Brants *et al*. 2007)
- No discounting, just use relative frequencies

$$S(w_i \mid w_{i-k+1}^{i-1}) = \begin{cases} \dfrac{\text{count}(w_{i-k+1}^{i})}{\text{count}(w_{i-k+1}^{i-1})} & \text{if } \text{count}(w_{i-k+1}^{i}) > 0 \\ 0.4 S(w_i \mid w_{i-k+2}^{i-1}) & \text{otherwise} \end{cases}$$

$$S(w_i) = \frac{\text{count}(w_i)}{N}$$

N-GRAM SMOOTHING SUMMARY

- Add-1 smoothing:
  - OK for text categorization, not for language modeling
- The most commonly used method:
  - Extended Interpolated Kneser-Ney
- For very large N-grams like the Web:
  - Stupid backoff

ADVANCED LANGUAGE MODELING

- Discriminative models:
  - choose n-gram weights to improve a task, not to fit the training set
- Parsing-based models
- Caching Models
  - Recently used words are more likely to appear
  - These perform very poorly for speech recognition (why?)

$$P_{CACHE}(w \mid history) = \lambda P(w_i \mid w_{i-2} w_{i-1}) + (1 - \lambda) \frac{c(w \in history)}{|history|}$$

LANGUAGE MODELING

Interpolation, Backoff, and Web-Scale LM's

EXERCISE

<s> I am Sam </s>

<s> Sam I am </s>

<s> I am Sam </s>

<s> I do not like green eggs and Sam </s>

Using a biagram language model with add-one smoothing, what is P(Sam | am)?

TEXT CLASSIFICATION AND NAÏVE BAYES

The Task of Text Classification

IS THIS SPAM?

Subject: **Important notice!**
From: Stanford University <newsforum@stanford.edu>
Date: October 28, 2011 12:34:16 PM PDT
To: undisclosed-recipients:;

Greats News!

You can now access the latest news by using the link below to login to Stanford University News Forum.

http://www.123contactform.com/contact-form-StanfordNew1-236335.html

Click on the above link to login for more information about this new exciting forum. You can also copy the above link to your browser bar and login for more information
about the new services.

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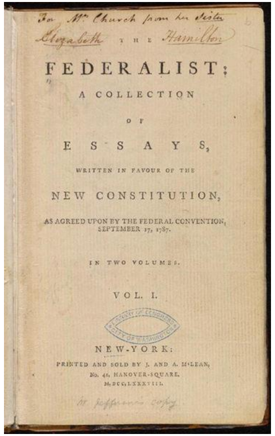
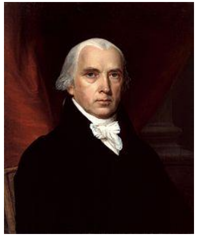
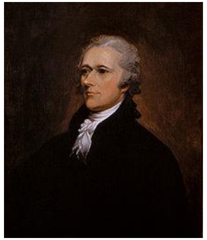
WHO WROTE WHICH FEDERALIST PAPERS?

- 1787-8: anonymous essays try to convince New York to ratify U.S Constitution: Jay, Madison, Hamilton.
- Authorship of 12 of the letters in dispute
- 1963: solved by Mosteller and Wallace using Bayesian methods


THE
FEDERALIST:
A COLLECTION
OF
ESSAYS,
WRITTEN IN FAVOUR OF THE
NEW CONSTITUTION,
AS AGREED UPON BY THE FEDERAL CONVENTION,
SEPTEMBER 17, 1787.
IN TWO VOLUMES.
VOL. I.
NEW-YORK:
PRINTED AND SOLD BY J. AND A. M'LEAN,
No. 41, HANOVER-SQUARE.
M,DCC,LXXXVIII.


James Madison


Alexander Hamilton

MALE OR FEMALE AUTHOR?

1. By 1925 present-day Vietnam was divided into three parts under French colonial rule. The southern region embracing Saigon and the Mekong delta was the colony of Cochin-China; the central area with its imperial capital at Hue was the protectorate of Annam…
2. Clara never failed to be astonished by the extraordinary felicity of her own name. She found it hard to trust herself to the mercy of fate, which had managed over the years to convert her greatest shame into one of her greatest assets…

S. Argamon, M. Koppel, J. Fine, A. R. Shimoni, 2003. "Gender, Genre, and Writing Style in Formal Written Texts," Text, volume 23, number 3, pp. 321–346

POSITIVE OR NEGATIVE MOVIE REVIEW?

- unbelievably disappointing

- Full of zany characters and richly applied satire, and some great plot twists

- this is the greatest screwball comedy ever filmed

- It was pathetic. The worst part about it was the boxing scenes.

WHAT IS THE SUBJECT OF THIS ARTICLE?

MEDLINE Article


Brain and Cognition

Syntactic frame and verb bias in aphasia: Plausibility judgments of undergoer-subject sentences

?

MeSH Subject Category Hierarchy

- Antogonists and Inhibitors
- Blood Supply
- Chemistry
- Drug Therapy
- Embryology
- Epidemiology
- …

TEXT CLASSIFICATION

- Assigning subject categories, topics, or genres
- Spam detection
- Authorship identification
- Age/gender identification
- Language Identification
- Sentiment analysis
- …

TEXT CLASSIFICATION: DEFINITION

- *Input*:
  - a document d
  - a fixed set of classes $C = \{c_1, c_2, \ldots, c_J\}$

- *Output*: a predicted class $c \in C$

CLASSIFICATION METHODS: HAND-CODED RULES

- Rules based on combinations of words or other features
  - spam: black-list-address OR (“dollars” AND“have been selected”)
- Accuracy can be high
  - If rules carefully refined by expert
- But building and maintaining these rules is expensive

CLASSIFICATION METHODS: SUPERVISED MACHINE LEARNING

- *Input:*
  - a document d
  - a fixed set of classes $C = \{c_1, c_2,\ldots, c_J\}$
  - A training set of m hand-labeled documents $(d_1,c_1),\ldots,(d_m,c_m)$
- *Output:*
  - a learned classifier $\gamma: d \rightarrow c$

CLASSIFICATION METHODS: SUPERVISED MACHINE LEARNING

- Any kind of classifier
  - Naïve Bayes
  - Logistic regression
  - Support-vector machines
  - k-Nearest Neighbors

  - …

TEXT CLASSIFICATION AND NAÏVE BAYES

The Task of Text Classification

TEXT CLASSIFICATION AND NAÏVE BAYES

Naïve Bayes

NAÏVE BAYES INTUITION

- Simple ("naïve") classification method based on Bayes rule
- Relies on very simple representation of document
  - Bag of words

THE BAG OF WORDS REPRESENTATION

$\gamma($

I love this movie! It's sweet, but with satirical humor. The dialogue is great and the adventure scenes are fun… It manages to be whimsical and romantic while laughing at the conventions of the fairy tale genre. I would recommend it to just about anyone. I've seen it several times, and I'm always happy to see it again whenever I have a friend who hasn't seen it yet.

$)=c$

THE BAG OF WORDS REPRESENTATION

$\gamma($

I **love** this movie! It's **sweet**, but with **satirical** humor. The dialogue is **great** and the adventure scenes are **fun**… It manages to be **whimsical** and **romantic** while **laughing** at the conventions of the fairy tale genre. I would **recommend** it to just about anyone. I've seen it **several** times, and I'm always **happy** to see it **again** whenever I have a friend who hasn't seen it yet.

$)=c$

THE BAG OF WORDS REPRESENTATION: USING A SUBSET OF WORDS

$\gamma($

x **love** xxxxxxxxxxxxxxxx **sweet**
xxxxxxx **satirical** xxxxxxxxxx
xxxxxxxxxxx **great** xxxxxxx
xxxxxxxxxxxxxxxxxxx **fun** xxxx
xxxxxxxxxxxxx **whimsical** xxxx
romantic xxxx **laughing**
xxxxxxxxxxxxxxxxxxxxxxxxxxxxxx
xxxxxxxxxxxxxx **recommend** xxxxx
xxxxxxxxxxxxxxxxxxxxxxxxxxxxxx
xx **several** xxxxxxxxxxxxxxxxx
xxxxx **happy** xxxxxxxxx **again**
xxxxxxxxxxxxxxxxxxxxxxxxxxxxxx
xxxxxxxxxxxxxxxxx

$)=c$

THE BAG OF WORDS REPRESENTATION

$\gamma($

great	2
love	2
recommend	1
laugh	1
happy	1
...	...

$)=c$

BAG OF WORDS FOR DOCUMENT CLASSIFICATION

?

Test document

parser
language
label
translation
…

Machine Learning	NLP	Garbage Collection	Planning	GUI
learning	parser	garbage	planning	…
training	tag	collection	temporal	
algorithm	training	memory	reasoning	
shrinkage	translation	optimization	plan	
network...	language...	region...	language...	

BAYES' RULE APPLIED TO DOCUMENTS AND CLASSES

- For a document *d* and a class *c*

$$P(c \mid d) = \frac{P(d \mid c)P(c)}{P(d)}$$

NAÏVE BAYES CLASSIFIER (I)

$$c_{MAP} = \underset{c \in C}{\operatorname{argmax}}\, P(c \mid d)$$

MAP is "maximum a posteriori" = most likely class

$$= \underset{c \in C}{\operatorname{argmax}}\, \frac{P(d \mid c)P(c)}{P(d)}$$

Bayes Rule

$$= \underset{c \in C}{\operatorname{argmax}}\, P(d \mid c)P(c)$$

Dropping the denominator

Naïve Bayes Classifier (II)

$$c_{MAP} = \underset{c \in C}{\operatorname{argmax}}\, P(d \mid c) P(c)$$

$$= \underset{c \in C}{\operatorname{argmax}}\, P(x_1, x_2, \ldots, x_n \mid c) P(c)$$

Document d represented as features x1..xn

NAÏVE BAYES CLASSIFIER (IV)

$$c_{MAP} = \underset{c \in C}{\operatorname{argmax}} P(x_1, x_2, \ldots, x_n \mid c) P(c)$$

O(|X|^n•|C|) parameters

Could only be estimated if a very, very large number of training examples was available.

How often does this class occur?

We can just count the relative frequencies in a corpus

MULTINOMIAL NAÏVE BAYES INDEPENDENCE ASSUMPTIONS

$$P(x_1, x_2, \ldots, x_n \mid c)$$

- **Bag of Words assumption**: Assume position doesn't matter
- **Conditional Independence**: Assume the feature probabilities $P(x_i|c_j)$ are independent given the class c.

$$P(x_1, \ldots, x_n \mid c) = P(x_1 \mid c) \cdot P(x_2 \mid c) \cdot P(x_3 \mid c) \cdot \ldots \cdot P(x_n \mid c)$$

MULTINOMIAL NAÏVE BAYES CLASSIFIER

$$c_{MAP} = \operatorname*{argmax}_{c \in C} P(x_1, x_2, \ldots, x_n \mid c) P(c)$$

$$c_{NB} = \operatorname*{argmax}_{c \in C} P(c_j) \prod_{x \in X} P(x \mid c)$$

APPLYING MULTINOMIAL NAIVE BAYES CLASSIFIERS TO TEXT CLASSIFICATION

positions ← all word positions in test document

$$c_{NB} = \underset{c_j \in C}{\operatorname{argmax}} P(c_j) \prod_{i \in positions} P(x_i | c_j)$$

LEARNING THE MULTINOMIAL NAÏVE BAYES MODEL

- First attempt: maximum likelihood estimates
  - simply use the frequencies in the data

$$\hat{P}(c_j) = \frac{doccount(C = c_j)}{N_{doc}}$$

$$\hat{P}(w_i \mid c_j) = \frac{count(w_i, c_j)}{\sum_{w \in V} count(w, c_j)}$$

PARAMETER ESTIMATION

$$\hat{P}(w_i \mid c_j) = \frac{count(w_i, c_j)}{\sum_{w \in V} count(w, c_j)}$$

fraction of times word w_i appears among all words in documents of topic c_j

- Create mega-document for topic *j* by concatenating all docs in this topic
  - Use frequency of *w* in mega-document

PROBLEM WITH MAXIMUM LIKELIHOOD

- What if we have seen no training documents with the word ***fantastic*** and classified in the topic **positive** (***thumbs-up)***?

$$\hat{P}(\text{"fantastic"} \mid \text{positive}) = \frac{count(\text{"fantastic", positive})}{\sum_{w \in V} count(w, \text{positive})} = 0$$

- Zero probabilities cannot be conditioned away, no matter the other evidence!

$$c_{MAP} = \operatorname{argmax}_c \hat{P}(c) \prod_i \hat{P}(x_i \mid c)$$

LAPLACE (ADD-1) SMOOTHING FOR NAÏVE BAYES

$$\hat{P}(w_i \mid c) = \frac{count(w_i, c) + 1}{\sum_{w \in V} \left(count(w, c) + 1 \right)}$$

$$= \frac{count(w_i, c) + 1}{\left(\sum_{w \in V} count(w, c) \right) + |V|}$$

Multinomial Naïve Bayes: Learning

- From training corpus, extract *Vocabulary*

- Calculate $P(c_j)$ terms
  - For each c_j in C do

    $docs_j \leftarrow$ all docs with class $= c_j$

$$P(c_j) \leftarrow \frac{|docs_j|}{|\text{total \# documents}|}$$

- Calculate $P(w_k \mid c_j)$ terms
  - $Text_j \leftarrow$ single doc containing all $docs_j$
  - For each word w_k in *Vocabulary*

    $n_k \leftarrow$ # of occurrences of w_k in $Text_j$

$$P(w_k \mid c_j) \leftarrow \frac{n_k + \alpha}{n + \alpha|Vocabulary|}$$

LAPLACE (ADD-1) SMOOTHING: UNKNOWN WORDS

Add one extra word to the vocabulary, the "unknown word" w_u

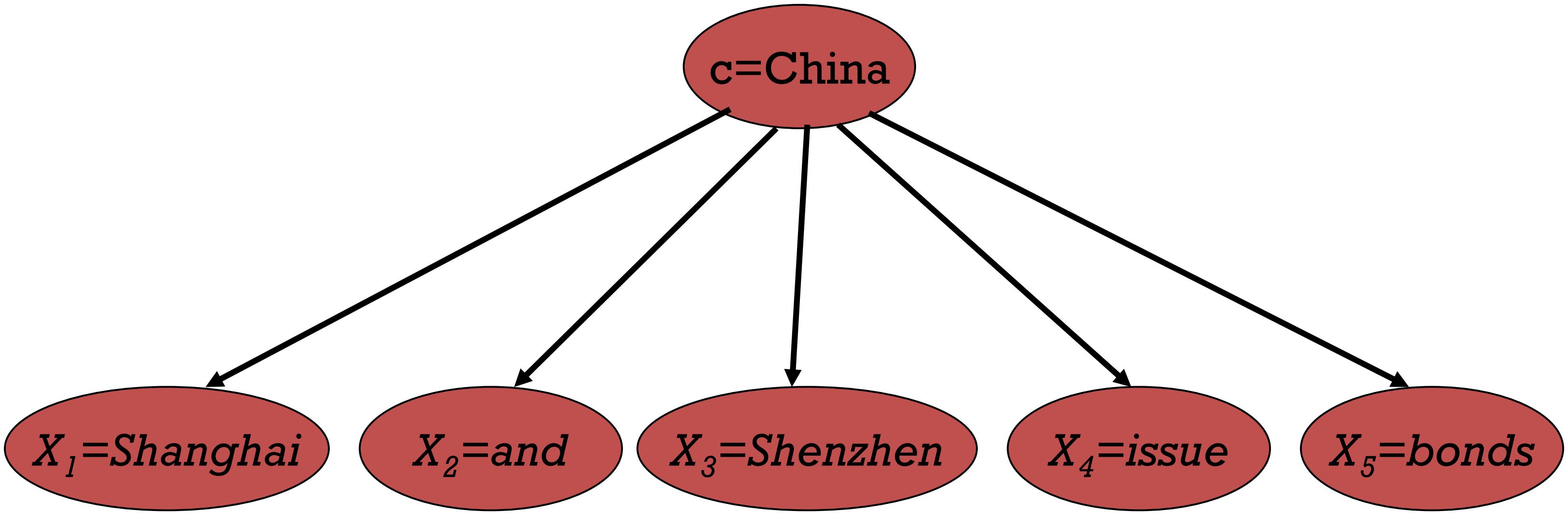
$$\hat{P}(w_u \mid c) = \frac{count(w_u, c) + 1}{\left(\sum_{w \in V} count(w, c)\right) + \left|V + 1\right|}$$

$$= \frac{1}{\left(\sum_{w \in V} count(w, c)\right) + \left|V + 1\right|}$$

TEXT CLASSIFICATION AND NAÏVE BAYES

Naïve Bayes: Relationship to Language Modeling

GENERATIVE MODEL FOR MULTINOMIAL NAÏVE BAYES



NAÏVE BAYES AND LANGUAGE MODELING

- Naïve bayes classifiers can use any sort of feature
  - URL, email address, dictionaries, network features
- But if, as in the previous slides
  - We use **only** word features
  - we use **all** of the words in the text (not a subset)
- Then
  - Naïve bayes has an important similarity to language modeling.

EACH CLASS = A UNIGRAM LANGUAGE MODEL

- Assigning each word: P(word | c)
- Assigning each sentence: P(s|c)=∠P(word|c)

Class *pos*

0.1 I

0.1 love

0.01this

0.05fun

0.1 film

I	love	this	fun	film
0.1	0.1	.05	0.01	0.1

P(s | pos) = 0.0000005

NAÏVE BAYES AS A LANGUAGE MODEL

- Which class assigns the higher probability to s?

Model pos	
0.1	I
0.1	love
0.01	this
0.05	fun
0.1	film

Model neg	
0.2	I
0.001	love
0.01	this
0.005	fun
0.1	film

I	love	this	fun	film
0.1	0.1	0.01	0.05	0.1
0.2	0.001	0.01	0.005	0.1

$P(s|pos) > P(s|neg)$

$$\hat{P}(c) = \frac{N_c}{N}$$

$$\hat{P}(w \mid c) = \frac{count(w,c)+1}{count(c)+|V|}$$

	Doc	Words	Class
Training	1	Chinese Beijing Chinese	c
	2	Chinese Chinese Shanghai	c
	3	Chinese Macao	c
	4	Tokyo Japan Chinese	j
Test	5	Chinese Chinese Chinese Tokyo Japan	?

Priors:

$P(c) = \frac{3}{4}$

$P(j) = \frac{1}{4}$

Conditional Probabilities:

P(Chinese | c) = (5+1) / (8+6) = 6/14 = 3/7

P(Tokyo | c) = (0+1) / (8+6) = 1/14

P(Japan | c) = (0+1) / (8+6) = 1/14

P(Chinese | j) = (1+1) / (3+6) = 2/9

P(Tokyo | j) = (1+1) / (3+6) = 2/9

P(Japan | j) = (1+1) / (3+6) = 2/9

Choosing a class:

P(c | d5) $\propto$ 3/4 * $(3/7)^3$ * 1/14 * 1/14 $\approx$ 0.0003

P(j | d5) $\propto$ 1/4 * $(2/9)^3$ * 2/9 * 2/9 $\approx$ 0.0001

NAÏVE BAYES IN SPAM FILTERING

- SpamAssassin Features:
  - Mentions Generic Viagra
  - Online Pharmacy
  - Mentions millions of (dollar) ((dollar) NN,NNN,NNN.NN)
  - Phrase: impress ... girl
  - From: starts with many numbers
  - Subject is all capitals
  - HTML has a low ratio of text to image area
  - One hundred percent guaranteed
  - Claims you can be removed from the list
  - 'Prestigious Non-Accredited Universities'
  - http://spamassassin.apache.org/tests_3_3_x.html

SUMMARY: NAIVE BAYES IS NOT SO NAIVE

- Very Fast, low storage requirements
- Robust to Irrelevant Features

  Irrelevant Features cancel each other without affecting results

- Very good in domains with many equally important features

  Decision Trees suffer from *fragmentation* in such cases – especially if little data

- Optimal if the independence assumptions hold: If assumed independence is correct, then it is the Bayes Optimal Classifier for problem
- A good dependable baseline for text classification
  - **But we will see other classifiers that give better accuracy**

TEXT CLASSIFICATION AND NAÏVE BAYES

The Task of Text Classification

TEXT CLASSIFICATION AND NAÏVE BAYES

Precision, Recall & F1Score

THE 2-BY-2 CONTINGENCY TABLE

	correct	not correct
selected	tp	fp
not selected	fn	tn

PRECISION AND RECALL

- **Precision**: % of selected items that are correct
  Recall: % of correct items that are selected

	correct	not correct
selected	tp	fp
not selected	fn	tn

A COMBINED MEASURE: F

- A combined measure that assesses the P/R tradeoff is F measure (weighted harmonic mean):

$$F = \frac{1}{a\frac{1}{P} + (1 - a)\frac{1}{R}} = \frac{(b^2 + 1)PR}{b^2 P + R}$$

- The harmonic mean is a very conservative average; see *IIR* § 8.3
- People usually use balanced F1 measure
  - i.e., with $\beta = 1$ (that is, $\alpha = ½$): $F = 2PR/(P+R)$

TEXT CLASSIFICATION AND NAÏVE BAYES

Precision, Recall & F1Score

TEXT CLASSIFICATION AND NAÏVE BAYES

Text Classification: Evaluation

MORE THAN TWO CLASSES: SETS OF BINARY CLASSIFIERS

- Dealing with any-of or multivalue classification
  - A document can belong to 0, 1, or >1 classes.

- For each class $c \in C$
  - Build a classifier γ_c to distinguish c from all other classes $c' \in C$
- Given test doc d,
  - Evaluate it for membership in each class using each γ_c
  - d belongs to any class for which γ_c returns true

MORE THAN TWO CLASSES: SETS OF BINARY CLASSIFIERS

- One-of or multinomial classification
  - Classes are mutually exclusive: each document in exactly one class

- For each class c∈C
  - Build a classifier γ_c to distinguish c from all other classes c' ∈C
- Given test doc d,
  - Evaluate it for membership in each class using each γ_c
  - d belongs to the one class with maximum score

EVALUATION: CLASSIC REUTERS-21578 DATA SET

- Most (over)used data set, 21,578 docs (each 90 types, 200 toknens)
- 9603 training, 3299 test articles (ModApte/Lewis split)
- 118 categories
  - An article can be in more than one category
  - Learn 118 binary category distinctions
- Average document (with at least one category) has 1.24 classes
- Only about 10 out of 118 categories are large

Common categories (#train, #test)

- Earn (2877, 1087)
- Acquisitions (1650, 179)
- Money-fx (538, 179)
- Grain (433, 149)
- Crude (389, 189)
- Trade (369,119)
- Interest (347, 131)
- Ship (197, 89)
- Wheat (212, 71)
- Corn (182, 56)

REUTERS TEXT CATEGORIZATION DATA SET (REUTERS-21578) DOCUMENT

<REUTERS TOPICS="YES" LEWISSPLIT="TRAIN" CGISPLIT="TRAINING-SET" OLDID="12981" NEWID="798">

<DATE> 2-MAR-1987 16:51:43.42</DATE>

<TOPICS><D>livestock</D><D>hog</D></TOPICS>

<TITLE>AMERICAN PORK CONGRESS KICKS OFF TOMORROW</TITLE>

<DATELINE> CHICAGO, March 2 - </DATELINE><BODY>The American Pork Congress kicks off tomorrow, March 3, in Indianapolis with 160 of the nations pork producers from 44 member states determining industry positions on a number of issues, according to the National Pork Producers Council, NPPC.

Delegates to the three day Congress will be considering 26 resolutions concerning various issues, including the future direction of farm policy and the tax law as it applies to the agriculture sector. The delegates will also debate whether to endorse concepts of a national PRV (pseudorabies virus) control and eradication program, the NPPC said.

A large trade show, in conjunction with the congress, will feature the latest in technology in all areas of the industry, the NPPC added. Reuter

&#3;</BODY></TEXT></REUTERS>

CONFUSION MATRIX C

- For each pair of classes <c_1,c_2> how many documents from c_1 were incorrectly assigned to c_2?
  - $c_{3,2}$: 90 wheat documents incorrectly assigned to poultry

Docs in test set	Assigned UK	Assigned poultry	Assigned wheat	Assigned coffee	Assigned interest	Assigned trade
True UK	95	1	13	0	1	0
True poultry	0	1	0	0	0	0
True wheat	10	90	0	1	0	0
True coffee	0	0	0	34	3	7
True interest	-	1	2	13	26	5
True trade	0	0	2	14	5	10

PER CLASS EVALUATION MEASURES

Recall:

Fraction of docs in class *i* classified correctly: $$\frac{c_{ii}}{\sum_j c_{ij}}$$

Precision:

Fraction of docs assigned class *i* that are actually about class *i*: $$\frac{c_{ii}}{\sum_j c_{ji}}$$

Accuracy: (1 - error rate)

Fraction of docs classified correctly: $$\frac{\sum_i c_{ii}}{\sum_j \sum_i c_{ij}}$$

MICRO- VS. MACRO-AVERAGING

- If we have more than one class, how do we combine multiple performance measures into one quantity?
- **Macroaveraging**: Compute performance for each class, then average.
- **Microaveraging**: Collect decisions for all classes, compute contingency table, evaluate.

MICRO- VS. MACRO-AVERAGING: EXAMPLE

Class 1

	Truth: yes	Truth: no
Classifier: yes	10	10
Classifier: no	10	970

Class 2

	Truth: yes	Truth: no
Classifier: yes	90	10
Classifier: no	10	890

Micro Ave. Table

	Truth: yes	Truth: no
Classifier: yes	100	20
Classifier: no	20	1860

- Macroaveraged precision: (0.5 + 0.9)/2 = 0.7
- Microaveraged precision: 100/120 = .83
- Microaveraged score is dominated by score on common classes

DEVELOPMENT TEST SETS AND CROSS-VALIDATION

Training set | Development Test Set | Test Set

- Metric: P/R/F1 or Accuracy
- Unseen test set
  - avoid overfitting ('tuning to the test set')
  - more conservative estimate of performance
- Cross-validation over multiple splits
  - Handle sampling errors from different datasets
  - Pool results over each split
  - Compute pooled dev set performance

Training Set | Dev Test

Training Set | Dev Test

Dev Test | Training Set

Test Set

TEXT CLASSIFICATION AND NAÏVE BAYES

Text Classification: Evaluation

TEXT CLASSIFICATION AND NAÏVE BAYES

Text Classification: Practical Issues

THE REAL WORLD

- Gee, I'm building a text classifier for real, now!
- What should I do?

NO TRAINING DATA? MANUALLY WRITTEN RULES

If (wheat or grain) and not (whole or bread) then

Categorize as grain

- Need careful crafting
  - Human tuning on development data
  - Time-consuming: 2 days per class

VERY LITTLE DATA?

- Use Naïve Bayes
  - Naïve Bayes is a “high-bias” algorithm (Ng and Jordan 2002 NIPS)
- Get more labeled data
  - Find clever ways to get humans to label data for you
- Try semi-supervised training methods:
  - Bootstrapping, EM over unlabeled documents, …

A REASONABLE AMOUNT OF DATA?

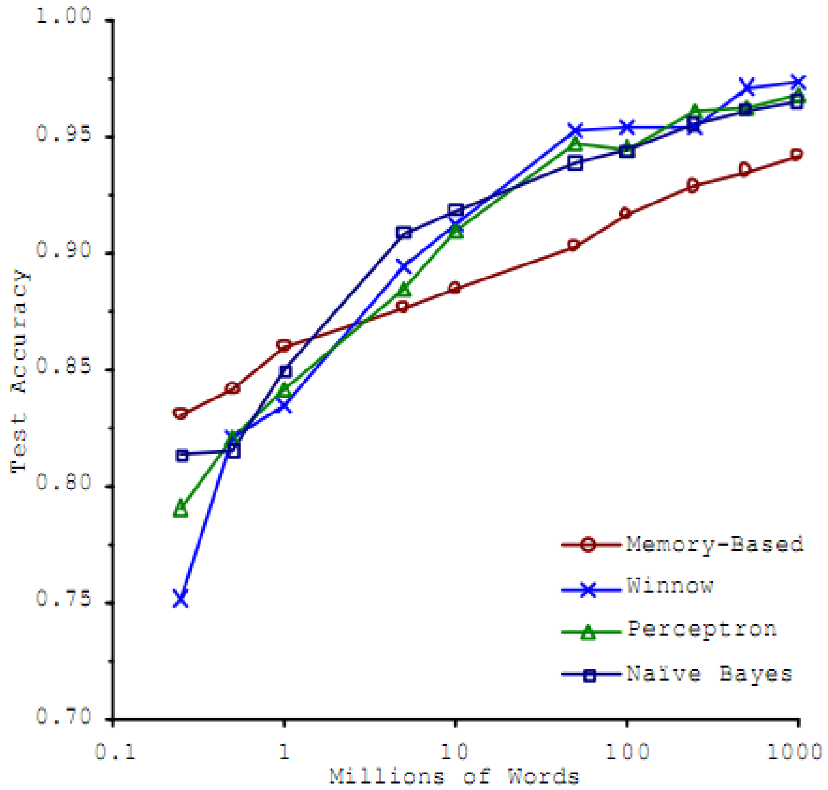
- Perfect for all the clever classifiers
  - SVM
  - Regularized Logistic Regression
- You can even use user-interpretable decision trees
  - Users like to hack
  - Management likes quick fixes

A HUGE AMOUNT OF DATA?

- Can achieve high accuracy!
- At a cost:
  - SVMs (train time) or kNN (test time) can be too slow
  - Regularized logistic regression can be somewhat better
- So Naïve Bayes can come back into its own again!

ACCURACY AS A FUNCTION OF DATA SIZE

- With enough data
  - Classifier may not matter



Brill and Banko on spelling correction

REAL-WORLD SYSTEMS GENERALLY COMBINE:

- Automatic classification
- Manual review of uncertain/difficult/"new” cases

UNDERFLOW PREVENTION: LOG SPACE

- Multiplying lots of probabilities can result in floating-point underflow.
- Since log(xy) = log(x) + log(y)
  - Better to sum logs of probabilities instead of multiplying probabilities.
- Class with highest un-normalized log probability score is still most probable.

$$c_{NB} = \operatorname*{argmax}_{c_j \in C} \log P(c_j) + \sum_{i \in positions} \log P(x_i \mid c_j)$$

- Model is now just max of sum of weights

HOW TO TWEAK PERFORMANCE

- Domain-specific features and weights: *very* important in real performance
- Sometimes need to collapse terms:
  - Part numbers, chemical formulas, …
  - But stemming generally doesn't help
- Upweighting: Counting a word as if it occurred twice:
  - title words (Cohen & Singer 1996)
  - first sentence of each paragraph (Murata, 1999)
  - In sentences that contain title words (Ko *et al,* 2002)

TEXT CLASSIFICATION AND NAÏVE BAYES

Text Classification: Practical Issues