Working Title

Andrew Banman

Outline

What is topology?

"...a topologist is someone who cannot tell the difference between a tea cup and a doughnut." -Crossley

- Notions of equivalence

  x = y ,   ■ = 🧩 ,   ● ≠ ●

- Study of continuous functions.

  ex) continuous integer-valued function on the real line must be constant. What matters is the topology of **R** and **Z.**

Homotopy

- Two functions (loops or paths) are *homotopic* if there is a *continuous deformation* from one to the other.

  The **homotopy** is the function that "does the deforming."

- Group all possible functions into homotopy *equivalence classes*
  → can count the number of "holes"

Simplicial Homology

- Simplicies
  - K simplex = regular ordered k+1 vertices (points)
- Simplicial complexes
  - Formal def: if sigma < Sigma, and tau < sigma, then tau < Sigma
  - Show how constructed

Point Clouds: enter the data

- Persistent homology

- Sample the torus

- Show the barcodes / persistence diagram

Mind the gap

Indicators

Children per woman (total fertility)
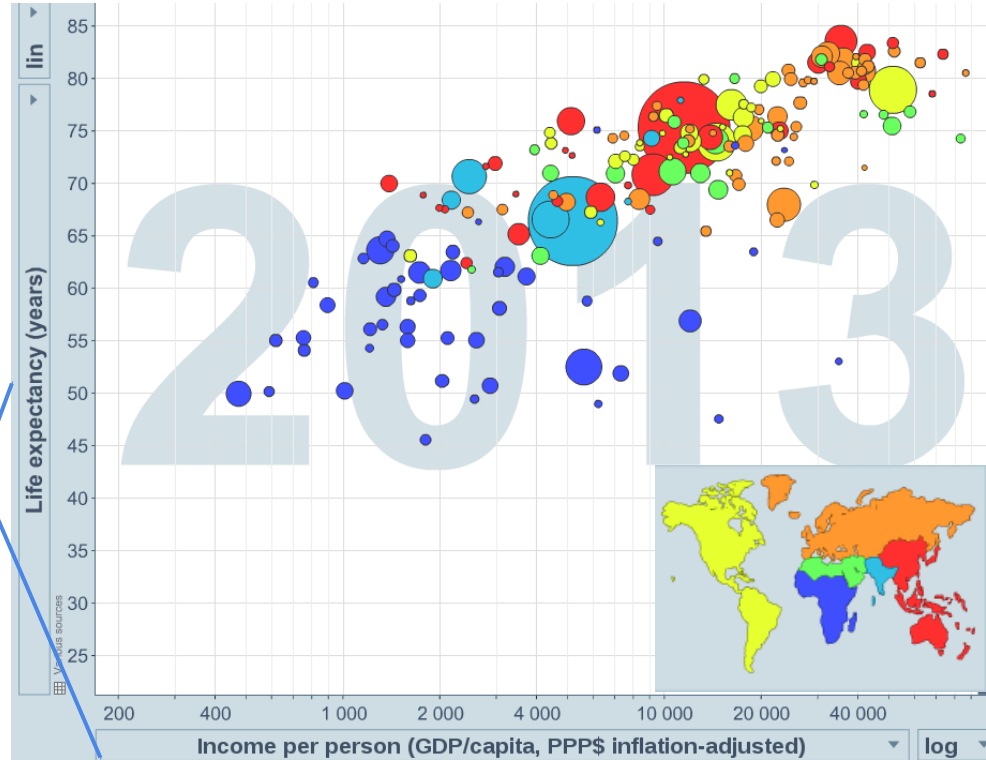
CO2 emissions (tonnes per person)

Income per person (GDP/capita, PPP$ inflation-adjusted)
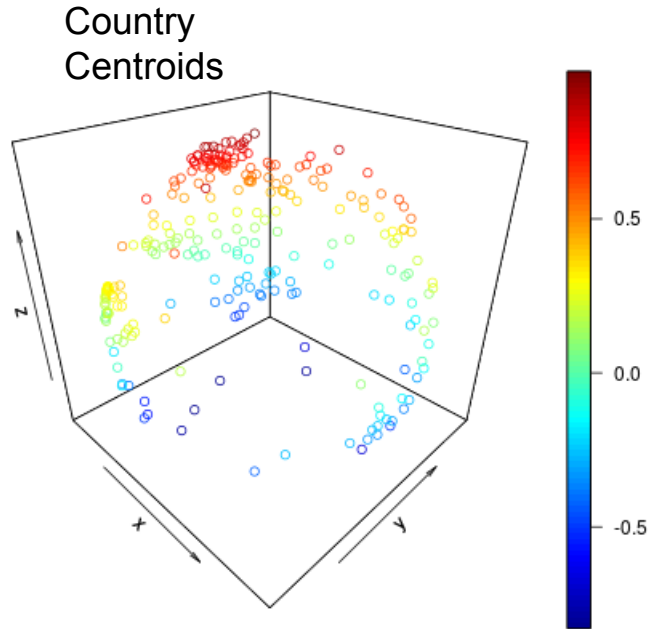
Child mortality (0-5 year-olds dying per 1,000 born)
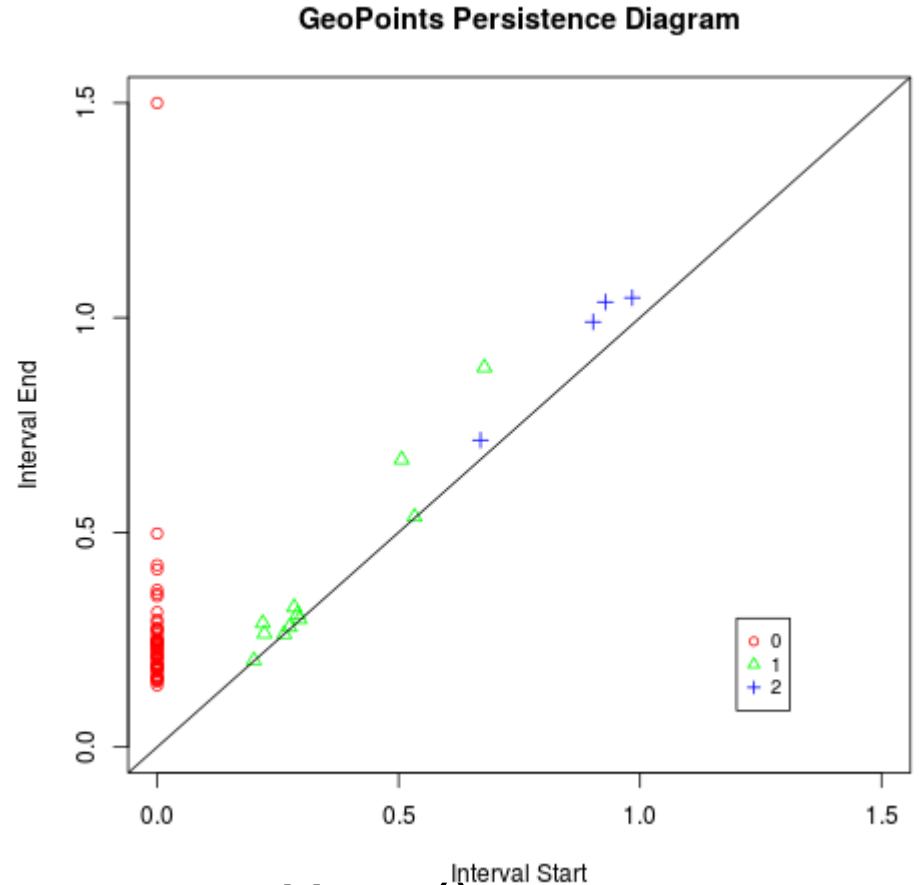
Life expectancy (years)

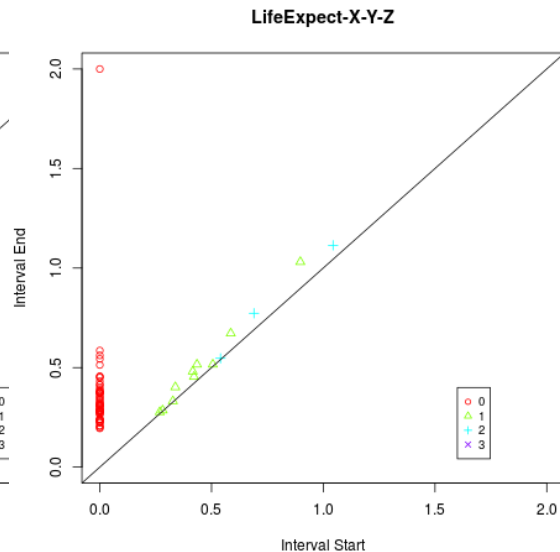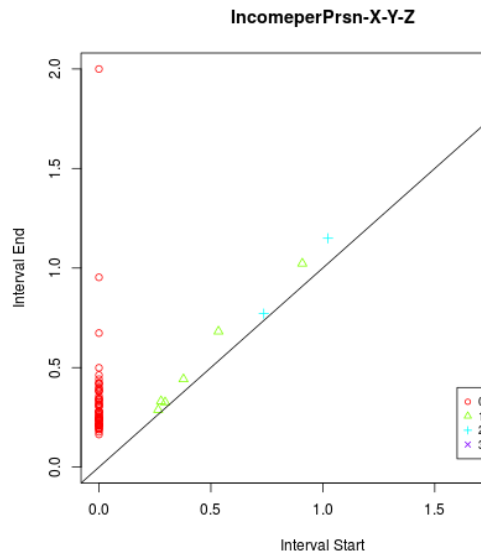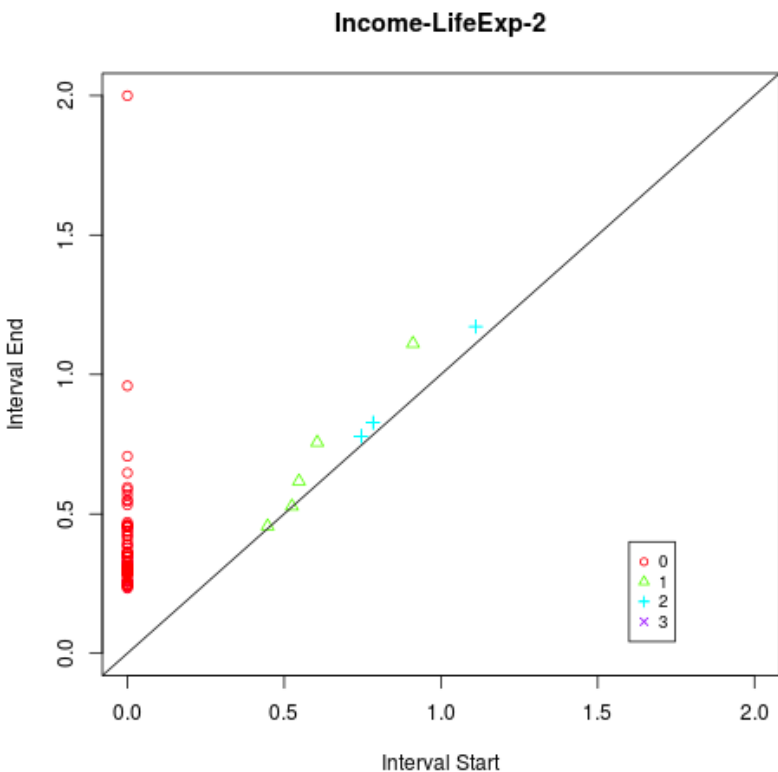Aid given (2007 US$)

Aid given per person (2007 US$)

Geography

Country Centroids

scatter3D() {plot3D}

GeoPoints Persistence Diagram

Interval End

Interval Start

0
1
2

pHom()
{phom}

Adding social dimensions

Income per person "pulls" the countries into two distinct geographic groups. Life expectancy is not strong enough to pull them back together.
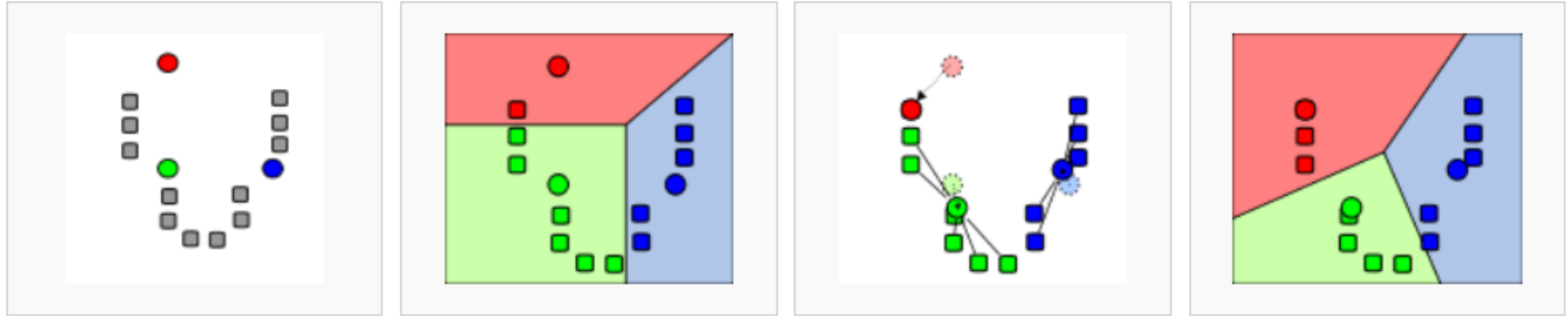
Understanding B_0 with clustering (cluster slides to be replaced w/ improved bettir 0 anlysis)

Can we use persistent homology as a clustering algorithm?

- Slow
- Sensitive to outliers
- Bridges collapse clusters
- Preprocessing algorithms required

In the meantime we'll use k-mean.
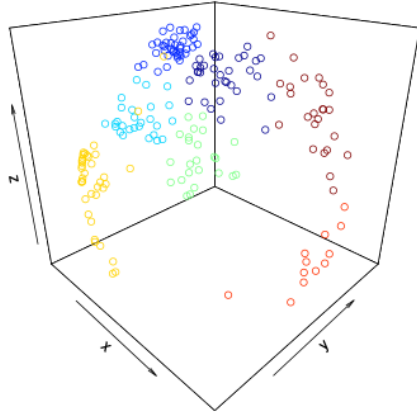
A k-means to an end

1. Initialize cluster centers.
2. Generate Voronoi Diagram for each center.
3. Let the centroid of each region be the new center.

- Requires choice of k
- Fast
- Global solution NP-hard
- Heuristic Algorithm

Voronoi Diagram *The partitioning of a plane with points into underlined convex polygons such that each underlined polygon contains underlined exactly one generating point and every point in a given polygon is closer to its generating point than to any other* -WolframMathWolrd
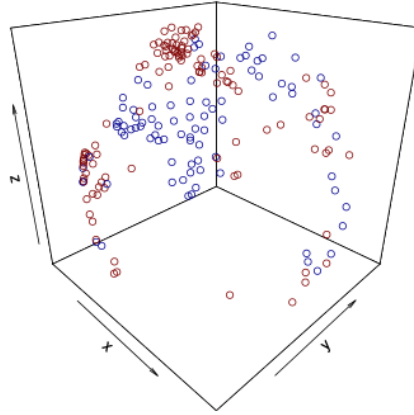
Geographic and social clusters

+

=

k = 7

k = 2

k = 4

Clusters based on geographic data only.
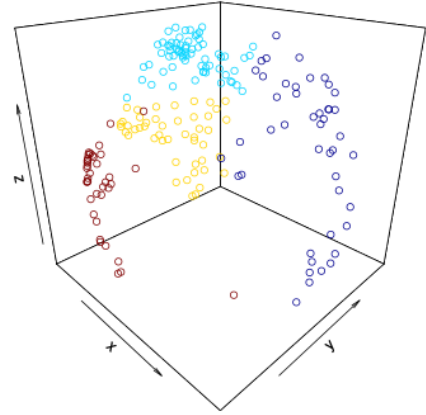
Clusters based on Income per person and Life Expectancy

Combined Geographic, Life expectancy, Income
 0.45, -0.74 Europe/Eurasia
 0.60. -0.58 Asia/South Pacific
-0.38, -0.94 Africa
 0.53, -0.78 Americas

pamk() {fpc package}

Limitations

- Coordinate space not theoretically justified
- Statistical significance (examine difference in means)
- Slow as Canadian molasses
- Ask a sociologist

Acknowledgment & References

- Gapminder
- http://www.statmethods.net/advstats/cluster.html (clustering)
- http://earthobservatory.nasa.gov/IOTD/view.php?id=885
  (Earth Image)
- Wikipedia
- WolframMathWorld
- Ghrist
- Carlson
- Topology textbook (Crossley)

Thank you...

Lori Zeigelmeir & topology class

MSCS

y'all