

# Problem Set 2 - Andreas Bloch

## Data Preprocessing

This function preprocesses the case data and creates various training (and test) sets used in the exercises 1-5. I suggest to skip this step as it will take a long time. To avoid having to do these preprocessing steps every time I already provide the preprocessed datasets in the repository. This allowed me to train all models with the full dataset (5000 cases).

In [ ]:

```
if False: # set this to True if you want to preprocess the datasets
    from data_creation_and_loader import create_datasets
    create_datasets()
```

## Exercise 1 & 5

**Q1:** Take your best hyperparameters for the logistic regression model from Problem Set 1 (or find them with a new model). Use `cross_val_predict()` to form model predictions of reversed for each observation. Construct a confusion matrix for the predictions. Report the `precision_recall_curve()` and `roc_curve()`. Make a calibration plot (as in Bansak et al 2018, appendix page 24).

**Q5:** Replicate your sklearn logistic regression model (no hidden layers, L2 regularization, and sigmoid output layer) in Keras or some other deep learning library. Compare performance. Now add two hidden layers, dropout, and batch normalization. Compare performance.

**A:** I've decided to solve exercise 1 and 5 together, as the models run on the same dataset and we were also asked to compare the models (to the one from exercise 1). I'm using the full dataset of ~5'700 cases with each 10'000 features (the number of occurrences of the most popular 3-grams). The evaluation scores for the logistic regression model are reported through cross-validation. The logistic regression model is trained via grid-search. For performance reasons, the evaluation scores for the neural nets are only reported through one validation via a dedicated train and test set and the models aren't trained via grid-search. Unfortunately, all models are as good as a random classifier. Hence, a comparison is rather useless. I've tried a lot of things: from 1'000 to 10'000 features, increasing regularization, ... But all models had an f-score of about 0.5.

## Imports

In [2]:

```
%matplotlib inline
import os
import numpy as np
import matplotlib.pyplot as plt
from sklearn.linear_model import LogisticRegression
from sklearn.metrics import roc_curve
from sklearn.metrics import auc
from sklearn.metrics import precision_recall_curve
from sklearn.model_selection import train_test_split
from sklearn.model_selection import GridSearchCV
from sklearn.model_selection import cross_val_predict
from sklearn.utils import shuffle
from sklearn.metrics import average_precision_score
from sklearn.calibration import calibration_curve
from sklearn.metrics import confusion_matrix
from keras.wrappers.scikit_learn import KerasClassifier
from keras.models import Sequential
from keras.layers import Dense
from keras.layers import BatchNormalization
from keras.layers import Dropout
from keras.regularizers import l2
from data_creation_and_loader import get_exercise_1_and_5_dataset
```

Using TensorFlow backend.

## Force CPU to be used

In [3]:

```
# force keras to use CPU (GPU on shared server is mostly busy)
os.environ["CUDA_DEVICE_ORDER"] = "PCI_BUS_ID"
os.environ["CUDA_VISIBLE_DEVICES"] = ""
```

## Loading of dataset

In [4]:

```
# get datasets
X, y = get_exercise_1_and_5_dataset()
y = y.astype(float)

# shuffle the data (again just to make sure)
X, y = shuffle(X, y, random_state=71)

# create training and test set
X_train, X_test, y_train, y_test = train_test_split(X, y, test_size=0.2)
```

## Model definitions

Here's the definition of the logistic regression model. Training with the f1-score (as it's the harmonic mean of precision and recall) greatly improved the performance of the model (compared to using accuracy as score).

In [5]:

```
# use logistic regression model
# (specify solver to avoid warnings)
# (increase max_iter to ensure convergence)
log_reg = LogisticRegression(
    solver='liblinear',
    max_iter=10000
)

# specify parameter grid_search
param_grid = {
    'penalty': ['l1', 'l2'],
    'C': [0.0001, 0.001, 0.01, 0.1, 0.2, 0.3, 0.5, 0.7, 1.0, 1.5, 2.0, 3.0, 5.0,
          7.0, 10.0]
}

# specify grid_search search
grid = GridSearchCV(
    estimator=log_reg,          # estimator to use
    param_grid=param_grid,     # parameters to do grid_search search over
    scoring='f1',              # use F1 score to evaluate models
    n_jobs=4,                  # use 4 cores
    iid=True,                  # assume data was i.i.d. (to avoid warning)
    cv=10,                     # use stratified 10-fold CV
    refit=True,                # re-fit best model
    verbose=1,                 # do not print training progress
    return_train_score=True    # save training scores
)
```

Here's the definition of the shallow NN model that replicates logistic regression.

In [6]:

```
def build_shallow_model():
    shallow_alpha = 0.01
    model = Sequential()
    model.add(Dense(1, input_dim=X.shape[1],
                    activation='sigmoid',
                    kernel_regularizer=l2(shallow_alpha),
                    kernel_initializer='he_normal'))
    print(model.summary())
    model.compile(loss='binary_crossentropy',
                  optimizer='adam', metrics=['accuracy'])
    return model

# build shallow model
shallow_model = KerasClassifier(build_shallow_model)
```

Here's the definition of the deep NN model.

In [7]:

```
def build_deep_model():
    deep_alpha = 0.1
    model = Sequential()
    model.add(Dense(int(X.shape[1]*.75),
                    input_dim=X.shape[1],
                    activation='sigmoid',
                    kernel_regularizer=l2(deep_alpha),
                    kernel_initializer='he_normal'))
    model.add(BatchNormalization())
    model.add(Dropout(0.6))
    model.add(Dense(int(X.shape[1]*.5),
                    activation='sigmoid',
                    kernel_regularizer=l2(deep_alpha),
                    kernel_initializer='he_normal'))
    model.add(BatchNormalization())
    model.add(Dense(1,
                    activation='sigmoid',
                    kernel_regularizer=l2(deep_alpha),
                    kernel_initializer='he_normal'))
    print(model.summary())
    model.compile(loss='binary_crossentropy', optimizer='adam', metrics=['accuracy'])
    return model

# build deep model
deep_model = KerasClassifier(build_deep_model)
```

## Training

In [8]:

```
# train logistic regression with grid_search
grid.fit(X, y)

# report best hyperparameters
print('Best Hyperparameters:')
print(grid.best_params_)
print('')

# report the best score
print('Best Score:')
print(grid.best_score_)
print('')

# keep track of best model
best_log_reg_model = grid.best_estimator_
```

Fitting 10 folds for each of 30 candidates, totalling 300 fits

[Parallel(n\_jobs=4)]: Using backend LokyBackend with 4 concurrent workers.

[Parallel(n\_jobs=4)]: Done 42 tasks | elapsed: 13.9s

[Parallel(n\_jobs=4)]: Done 192 tasks | elapsed: 1.1min

Best Hyperparameters:

{'penalty': 'l1', 'C': 0.01}

Best Score:

0.7745155904918126

[Parallel(n\_jobs=4)]: Done 300 out of 300 | elapsed: 1.7min finished

In [9]:

```
# train shallow model
EPOCHS_SHALLOW = 50
BATCH_SIZE_SHALLOW = 64
shallow_model.fit(X_train, y_train, epochs=EPOCHS_SHALLOW, batch_size=BATCH_SIZE_SHALLOW, verbose=1)
```

Layer (type)	Output Shape	Param #
dense_1 (Dense)	(None, 1)	10001
Total params: 10,001		
Trainable params: 10,001		
Non-trainable params: 0		

None

Epoch 1/50

4609/4609 [=====] - 1s 158us/step - loss: 0.7889 - acc: 0.5613

Epoch 2/50

4609/4609 [=====] - 1s 112us/step - loss: 0.4577 - acc: 0.7924

Epoch 3/50

4609/4609 [=====] - 1s 113us/step - loss: 0.3642 - acc: 0.8720

Epoch 4/50

4609/4609 [=====] - 0s 108us/step - loss: 0.3162 - acc: 0.9006

Epoch 5/50

4609/4609 [=====] - 1s 114us/step - loss: 0.2865 - acc: 0.9212

Epoch 6/50

4609/4609 [=====] - 1s 109us/step - loss: 0.2678 - acc: 0.9362

Epoch 7/50

4609/4609 [=====] - 1s 110us/step - loss: 0.2538 - acc: 0.9408

Epoch 8/50

4609/4609 [=====] - 1s 113us/step - loss: 0.2434 - acc: 0.9466

Epoch 9/50

4609/4609 [=====] - 1s 130us/step - loss: 0.2375 - acc: 0.9521

Epoch 10/50

4609/4609 [=====] - 1s 121us/step - loss: 0.2313 - acc: 0.9540

Epoch 11/50

4609/4609 [=====] - 1s 119us/step - loss: 0.2266 - acc: 0.9568

Epoch 12/50

4609/4609 [=====] - 1s 121us/step - loss: 0.2241 - acc: 0.9599

Epoch 13/50

4609/4609 [=====] - 1s 139us/step - loss: 0.2214 - acc: 0.9625

Epoch 14/50

4609/4609 [=====] - 1s 130us/step - loss: 0.2193 - acc: 0.9640

Epoch 15/50

4609/4609 [=====] - 1s 112us/step - loss: 0.2176 - acc: 0.9655

Epoch 16/50

4609/4609 [=====] - 1s 123us/step - loss: 0.2166 - acc: 0.9657

Epoch 17/50

4609/4609 [=====] - 1s 128us/step - loss: 0.2159 - acc: 0.9668

Epoch 18/50  
4609/4609 [=====] - 1s 123us/step - loss:  
0.2155 - acc: 0.9653  
Epoch 19/50  
4609/4609 [=====] - 1s 122us/step - loss:  
0.2148 - acc: 0.9670  
Epoch 20/50  
4609/4609 [=====] - 1s 122us/step - loss:  
0.2141 - acc: 0.9681  
Epoch 21/50  
4609/4609 [=====] - 1s 122us/step - loss:  
0.2136 - acc: 0.9681  
Epoch 22/50  
4609/4609 [=====] - 1s 124us/step - loss:  
0.2136 - acc: 0.9692  
Epoch 23/50  
4609/4609 [=====] - 1s 113us/step - loss:  
0.2139 - acc: 0.9690  
Epoch 24/50  
4609/4609 [=====] - 1s 124us/step - loss:  
0.2141 - acc: 0.9703  
Epoch 25/50  
4609/4609 [=====] - 1s 115us/step - loss:  
0.2143 - acc: 0.9694  
Epoch 26/50  
4609/4609 [=====] - 1s 117us/step - loss:  
0.2154 - acc: 0.9711  
Epoch 27/50  
4609/4609 [=====] - 1s 111us/step - loss:  
0.2153 - acc: 0.9701  
Epoch 28/50  
4609/4609 [=====] - 0s 99us/step - loss: 0.  
2154 - acc: 0.9701  
Epoch 29/50  
4609/4609 [=====] - 1s 117us/step - loss:  
0.2155 - acc: 0.9707  
Epoch 30/50  
4609/4609 [=====] - 1s 114us/step - loss:  
0.2158 - acc: 0.9709  
Epoch 31/50  
4609/4609 [=====] - 1s 124us/step - loss:  
0.2164 - acc: 0.9716  
Epoch 32/50  
4609/4609 [=====] - 1s 117us/step - loss:  
0.2177 - acc: 0.9705  
Epoch 33/50  
4609/4609 [=====] - 1s 117us/step - loss:  
0.2169 - acc: 0.9709  
Epoch 34/50  
4609/4609 [=====] - 1s 114us/step - loss:  
0.2171 - acc: 0.9714  
Epoch 35/50  
4609/4609 [=====] - 1s 113us/step - loss:  
0.2171 - acc: 0.9716  
Epoch 36/50  
4609/4609 [=====] - 1s 109us/step - loss:  
0.2186 - acc: 0.9727  
Epoch 37/50  
4609/4609 [=====] - 1s 111us/step - loss:  
0.2181 - acc: 0.9722  
Epoch 38/50



```
4609/4609 [=====] - 1s 145us/step - loss:
0.2182 - acc: 0.9703
Epoch 39/50
4609/4609 [=====] - 1s 147us/step - loss:
0.2184 - acc: 0.9724
Epoch 40/50
4609/4609 [=====] - 1s 153us/step - loss:
0.2207 - acc: 0.9727
Epoch 41/50
4609/4609 [=====] - 1s 117us/step - loss:
0.2221 - acc: 0.9711
Epoch 42/50
4609/4609 [=====] - 0s 108us/step - loss:
0.2238 - acc: 0.9709
Epoch 43/50
4609/4609 [=====] - 0s 107us/step - loss:
0.2224 - acc: 0.9720
Epoch 44/50
4609/4609 [=====] - 1s 119us/step - loss:
0.2234 - acc: 0.9709
Epoch 45/50
4609/4609 [=====] - 1s 144us/step - loss:
0.2227 - acc: 0.9718
Epoch 46/50
4609/4609 [=====] - 1s 144us/step - loss:
0.2215 - acc: 0.9711
Epoch 47/50
4609/4609 [=====] - 1s 136us/step - loss:
0.2208 - acc: 0.9714
Epoch 48/50
4609/4609 [=====] - 1s 143us/step - loss:
0.2210 - acc: 0.9714
Epoch 49/50
4609/4609 [=====] - 1s 134us/step - loss:
0.2218 - acc: 0.9705
Epoch 50/50
4609/4609 [=====] - 1s 116us/step - loss:
0.2214 - acc: 0.9709
```

Out[9]:

<keras.callbacks.History at 0x7f7dee7ca278>

In [12]:

```
# train deep model
EPOCHS_DEEP = 1
BATCH_SIZE_DEEP = 256
deep_model.fit(X_train, y_train, epochs=EPOCHS_DEEP, batch_size=BATCH_SIZE_DEEP,
verbose=1)
```

Layer (type)	Output Shape	Param #
dense_2 (Dense)	(None, 7500)	75007500
batch_normalization_1 (Batch Normalization)	(None, 7500)	30000
dropout_1 (Dropout)	(None, 7500)	0
dense_3 (Dense)	(None, 5000)	37505000
batch_normalization_2 (Batch Normalization)	(None, 5000)	20000
dense_4 (Dense)	(None, 1)	5001

=====  
Total params: 112,567,501  
Trainable params: 112,542,501  
Non-trainable params: 25,000  
=====  
None  
Epoch 1/1  
4609/4609 [=====] - 44s 10ms/step - loss: 1  
088.9243 - acc: 0.5088

Out[12]:

<keras.callbacks.History at 0x7f7dec0c1cf8>

In [13]:

```
# evaluate linear regression model
y_pred_lr = cross_val_predict(best_log_reg_model, X, y, method='decision_function', cv=10)
fpr_lr, tpr_lr, thresholds_lr = roc_curve(y, y_pred_lr)
auc_lr = auc(fpr_lr, tpr_lr)
y_score_lr = best_log_reg_model.decision_function(X_test)
y_proba_pos_lr = (y_score_lr - y_score_lr.min()) / (y_score_lr.max() - y_score_lr.min())
precision_lr, recall_lr, _ = precision_recall_curve(y_test, y_score_lr)
average_precision_lr = average_precision_score(y_test, y_score_lr)
CM_lr = confusion_matrix(y, np.heaviside(y_pred_lr, 1).astype(int))
CM_lr = CM_lr.astype('float') / CM_lr.sum(axis=1)[:, np.newaxis]
```

In [14]:

```
# evaluate shallow model
y_pred_shallow = shallow_model.predict(X_test) > 0.5
fpr_shallow, tpr_shallow, thresholds_shallow = roc_curve(y_test, y_pred_shallow)
auc_shallow = auc(fpr_shallow, tpr_shallow)
y_proba_pos_shallow = shallow_model.predict_proba(X_test)[:, 1]
precision_shallow, recall_shallow, _ = precision_recall_curve(y_test, y_proba_pos_shallow)
average_precision_shallow = average_precision_score(y_test, y_proba_pos_shallow)
CM_shallow = confusion_matrix(y_test, y_pred_shallow > 0.5)
CM_shallow = CM_shallow.astype('float') / CM_shallow.sum(axis=1)[:, np.newaxis]
```

In [15]:

```
# evaluate deep model
y_pred_deep = deep_model.predict(X_test) > 0.5
fpr_deep, tpr_deep, thresholds_deep = roc_curve(y_test, y_pred_deep)
auc_deep = auc(fpr_deep, tpr_deep)
y_proba_pos_deep = deep_model.predict_proba(X_test)[:, 1]
precision_deep, recall_deep, _ = precision_recall_curve(y_test, y_proba_pos_deep)
average_precision_deep = average_precision_score(y_test, y_proba_pos_deep)
CM_deep = confusion_matrix(y_test, y_pred_deep > 0.5)
CM_deep = CM_deep.astype('float') / CM_deep.sum(axis=1)[:, np.newaxis]
```

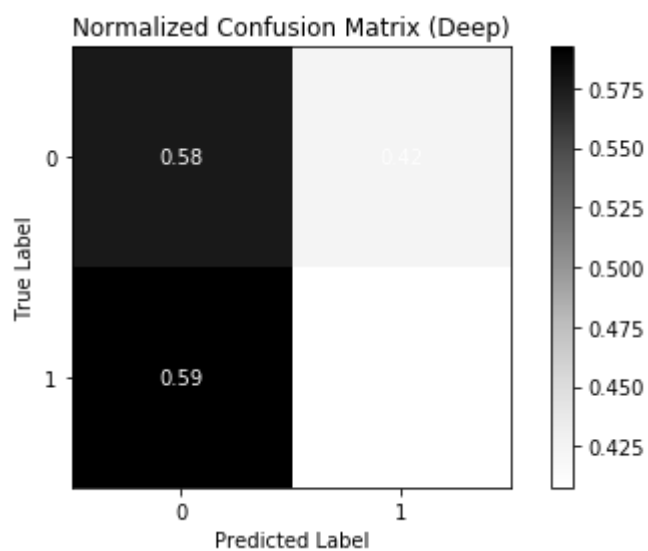
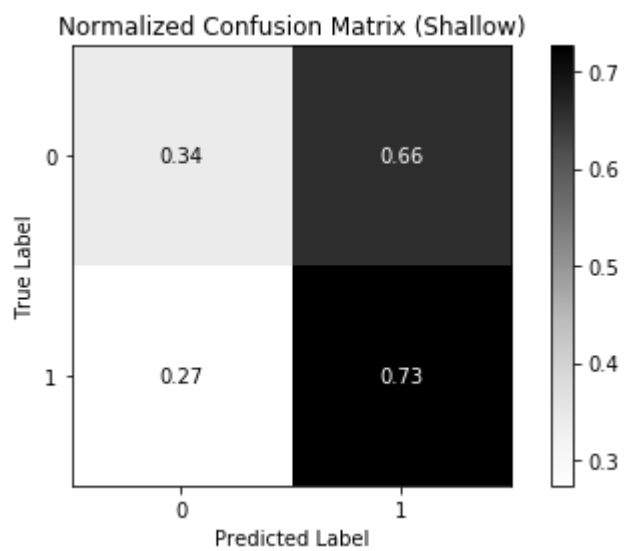
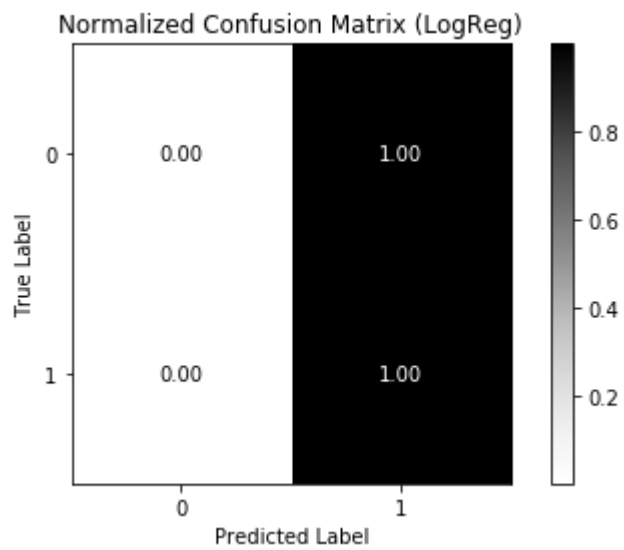
In [19]:

```
# function to create confusion matrix
def plot_confusion_matrix(CM, name):
    title = 'Normalized Confusion Matrix ('+name+')'
    classes = [0, 1]
    fig, ax = plt.subplots()
    im = ax.imshow(CM, interpolation='nearest', cmap=plt.cm.Greys)
    ax.figure.colorbar(im, ax=ax)
    ax.set(xticks=np.arange(CM.shape[1]),
           yticks=np.arange(CM.shape[0]),
           xticklabels=classes, yticklabels=classes,
           title=title,
           ylabel='True Label',
           xlabel='Predicted Label')

    # color text in confusion matrix appropriately
    fmt = '.2f'
    thresh = CM.max() / 2.
    for i in range(CM.shape[0]):
        for j in range(CM.shape[1]):
            ax.text(j, i, format(CM[i, j], fmt),
                    ha="center", va="center",
                    color="white" if CM[i, j] > thresh else "black")

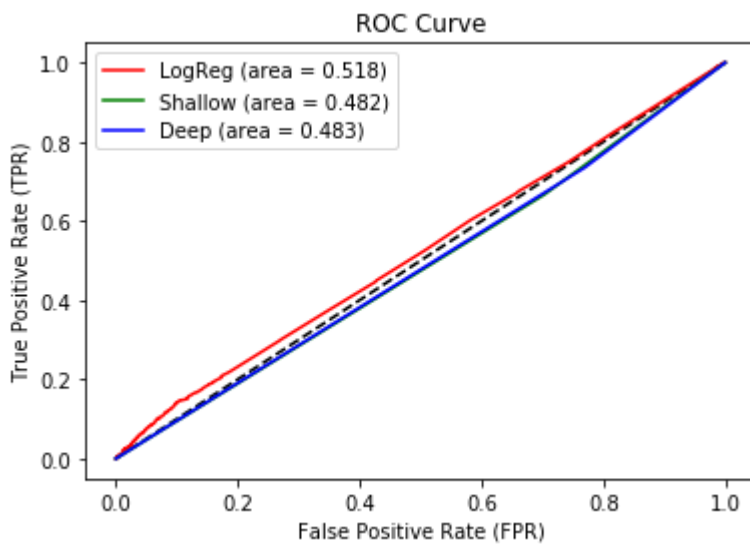
    fig.tight_layout()
    plt.show()

# create confusion matrices
plot_confusion_matrix(CM_lr, name='LogReg')
plot_confusion_matrix(CM_shallow, name='Shallow')
plot_confusion_matrix(CM_deep, name='Deep')
```



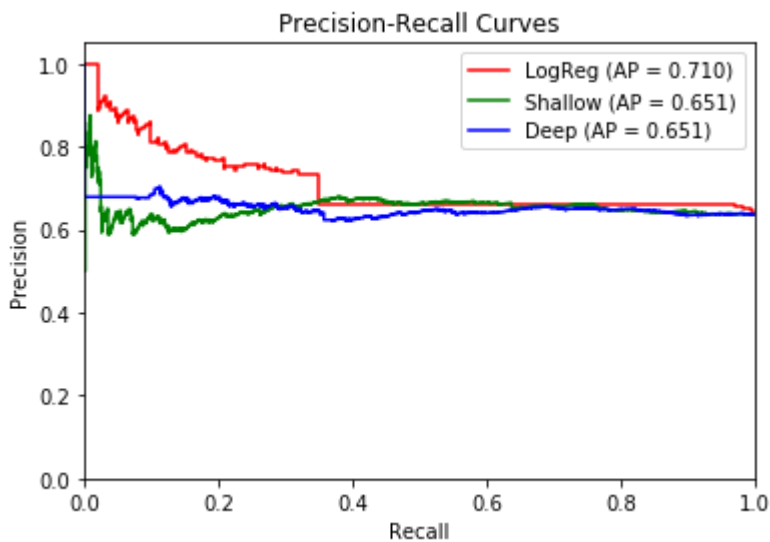
In [12]:

```
# create ROC plot
plt.figure(1)
plt.plot([0, 1], [0, 1], 'k--')
plt.plot(fpr_lr, tpr_lr, color='r', label='LogReg (area = {:.3f})'.format(auc_lr
))
plt.plot(fpr_shallow, tpr_shallow, color='g', label='Shallow (area = {:.3f})'.fo
rmat(auc_shallow))
plt.plot(fpr_deep, tpr_deep, color='b', label='Deep (area = {:.3f})'.format(auc_
deep))
plt.xlabel('False Positive Rate (FPR)')
plt.ylabel('True Positive Rate (TPR)')
plt.title('ROC Curve')
plt.legend(loc='best')
plt.show()
```



In [17]:

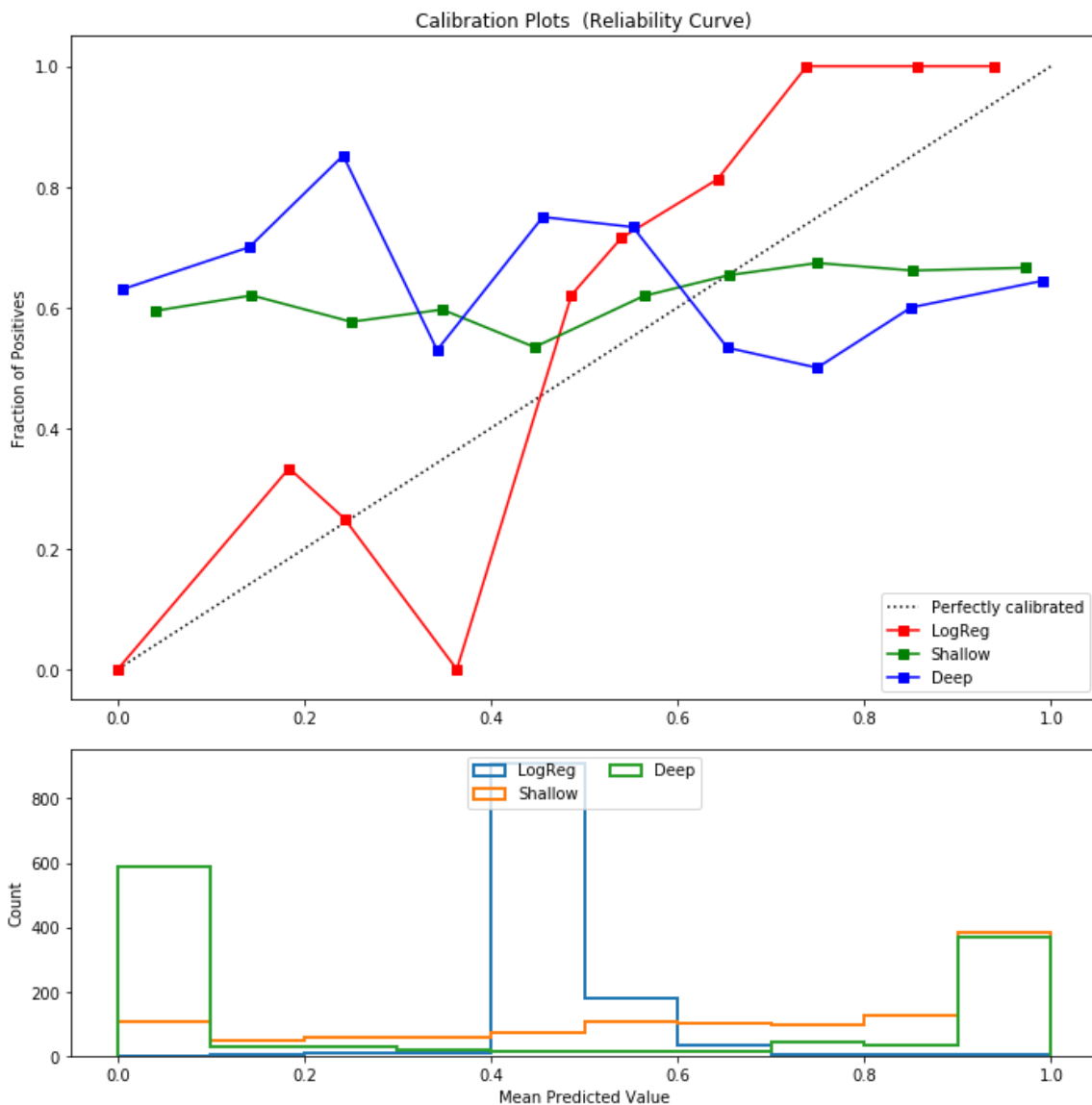
```
# create precision recall plot
plt.step(recall_lr, precision_lr, color='r', where='post', label='LogReg (AP = {:.3f})'.format(average_precision_lr))
plt.step(recall_shallow, precision_shallow, color='g', where='post', label='Shallow (AP = {:.3f})'.format(average_precision_shallow))
plt.step(recall_deep, precision_deep, color='b', where='post', label='Deep (AP = {:.3f})'.format(average_precision_deep))
plt.xlabel('Recall')
plt.ylabel('Precision')
plt.ylim([0.0, 1.05])
plt.xlim([0.0, 1.0])
plt.title('Precision-Recall Curves')
plt.legend(loc='best')
plt.show()
```



In [18]:

```
# create calibration plot
plt.figure(figsize=(10, 10))
ax1 = plt.subplot2grid((3, 1), (0, 0), rowspan=2)
ax2 = plt.subplot2grid((3, 1), (2, 0))
# create perfectly calibrated model
ax1.plot([0, 1], [0, 1], "k:", label="Perfectly calibrated")
# plot logistic regression
fraction_of_positives_lr, mean_predicted_value_lr = calibration_curve(y_test, y_
proba_pos_lr, n_bins=10)
ax1.plot(mean_predicted_value_lr, fraction_of_positives_lr, "s-", color='r', lab
el="%s" % ('LogReg', ))
ax2.hist(y_proba_pos_lr, range=(0, 1), bins=10, label='LogReg', histtype="step",
lw=2)
# plot shallow model
fraction_of_positives_shallow, mean_predicted_value_shallow = calibration_curve(
y_test, y_proba_pos_shallow, n_bins=10)
ax1.plot(mean_predicted_value_shallow, fraction_of_positives_shallow, "s-", colo
r='g', label="%s" % ('Shallow', ))
ax2.hist(y_proba_pos_shallow, range=(0, 1), bins=10, label='Shallow', histtype=
"step", lw=2)
# plot deep model
fraction_of_positives_deep, mean_predicted_value_deep = calibration_curve(y_test
, y_proba_pos_deep, n_bins=10)
ax1.plot(mean_predicted_value_deep, fraction_of_positives_deep, "s-", color='b',
label="%s" % ('Deep', ))
ax2.hist(y_proba_pos_deep, range=(0, 1), bins=10, label='Deep', histtype="step",
lw=2)
# name axes
ax1.set_ylabel("Fraction of Positives")
ax1.set_ylim([-0.05, 1.05])
ax1.legend(loc="lower right")
ax1.set_title('Calibration Plots (Reliability Curve)')
ax2.set_xlabel("Mean Predicted Value")
ax2.set_ylabel("Count")
ax2.legend(loc="upper center", ncol=2)
plt.tight_layout()
plt.show()
```





## Exercise 2

**Q:** Scale your n-gram frequencies while maintaining sparsity, as discussed in class. Train an elastic net model to predict log citations to a case (`log_cites`). Run `cross_val_predict()` to form model predictions and report a scatter plot of true and predicted values.

**A:** The sparsity-preserving scaling was done in the preprocessing (see `data_creation_and_loader.py`). Here's the code for the elastic net model: Previously, I had grid-searched over a large space of alpha values (0.001 to 10). I noticed that a very small alpha value is required, as we have a lot of parameters, so here the range is reduced from 0.3 to 2.0). Also, models with a smaller l1 ratio tended to work better.

In [4]:

```
%matplotlib inline
import matplotlib.pyplot as plt
from sklearn.linear_model import ElasticNet
from sklearn.model_selection import GridSearchCV
from sklearn.model_selection import cross_val_predict
from sklearn.utils import shuffle
import numpy as np
from data_creation_and_loader import get_exercise_2_dataset

# load dataset
X, y = get_exercise_2_dataset()

# use only 1'000 features (instead of 10'000)
# the features are sorted by their popularity
X = X[:,0:1000]

# shuffle the data
X, y = shuffle(X, y, random_state=71)

# use elastic net model
# increase max_iter to ensure convergence
elastic_net = ElasticNet(
    max_iter=10000,
    copy_X=False
)

# specify parameter grid_search
param_grid = {
    'alpha': [0.3, 0.4, 0.5, 0.7, 1.0, 1.5, 2.0],
    'l1_ratio': [0.01, 0.05, 0.1, 0.2, 0.3]
}

# specify grid_search search
grid_search = GridSearchCV(
    estimator=elastic_net,                # estimator to use
    param_grid=param_grid,                # parameters to do grid search over
    scoring='neg_mean_squared_error',     # use MSE score to evaluate models
    n_jobs=4,                             # use 4 cores
    iid=True,                             # ass. data was i.i.d. (to avoid warn.)
    cv=10,                                # use stratified 10-fold CV
    refit=True,                           # re-fit best model
    verbose=1,                             # do not print training progress
    return_train_score=True               # save training scores
)

# train with grid_search-search
grid_search.fit(X, y)
print('')

# report best hyperparameters
print('Best Hyperparameters:')
print(grid_search.best_params_)
print('')

# report the best score
print('Best Score:')
print(grid_search.best_score_)
print('')
```

```
# keep track of best model
best_elastic_net_model = grid_search.best_estimator_
```

Fitting 10 folds for each of 35 candidates, totalling 350 fits

```
[Parallel(n_jobs=4)]: Using backend LokyBackend with 4 concurrent wo  
rkers.
```

```
[Parallel(n_jobs=4)]: Done 42 tasks          | elapsed:    5.2s
```

```
[Parallel(n_jobs=4)]: Done 192 tasks        | elapsed:   20.4s
```

```
[Parallel(n_jobs=4)]: Done 350 out of 350 | elapsed:   35.2s finishe  
d
```

Best Hyperparameters:

```
{'alpha': 0.7, 'l1_ratio': 0.01}
```

Best Score:

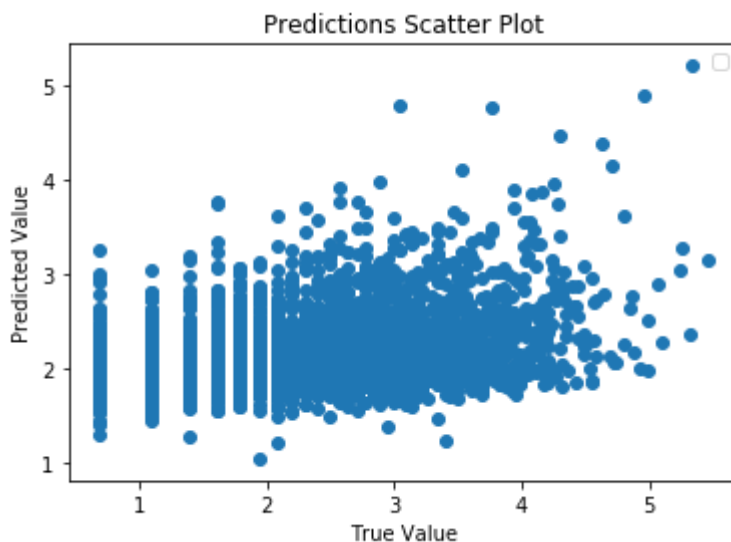
```
-0.7154045675790107
```

In [5]:

```
# run cross-val predictions
y_pred = cross_val_predict(best_elastic_net_model, X, y, cv=10)

# print scatter plot
plt.scatter(y, y_pred)
plt.xlabel('True Value')
plt.ylabel('Predicted Value')
plt.title('Predictions Scatter Plot')
plt.legend(loc='best')
plt.show()
```

No handles with labels found to put in legend.



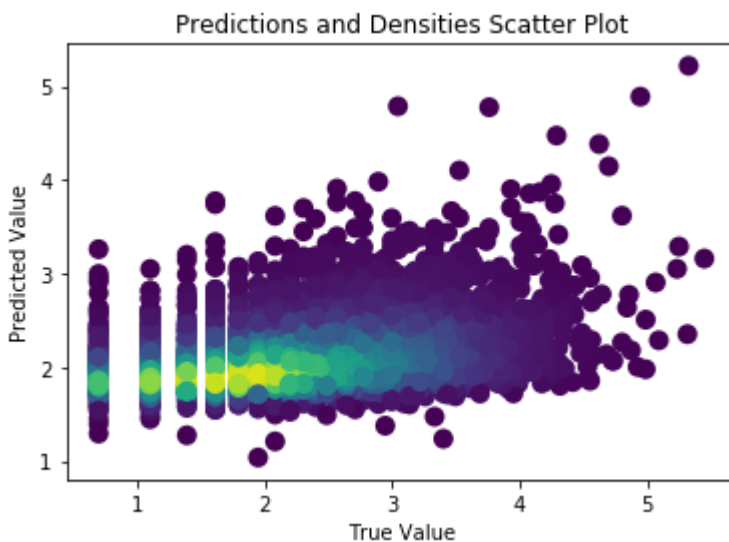
This is the scatter plot produced by the best model. As we can see we have a lot of variance in the predictions. It also seems that the model It might be that the features aren't predictive for the number of cites. Further enhancing the scatter plot with the density shows us that the model doesn't really succeed well to predict higher `log_cites` values.

In [7]:

```
import numpy as np
import matplotlib.pyplot as plt
from scipy.stats import gaussian_kde

xy = np.vstack([y, y_pred])
z = gaussian_kde(xy)(xy)

fig, ax = plt.subplots()
ax.scatter(y, y_pred, c=z, s=100, edgecolor='')
plt.xlabel('True Value')
plt.ylabel('Predicted Value')
plt.title('Predictions and Densities Scatter Plot')
plt.show()
```



## Exercise 3

**Q:** Use the judge identifiers (`judge_id`) to produce the average citations to cases for each judge. Then estimate a two-stage least-squares model with  $Z$  as average citations of the judge,  $X$  as citations to the case, and  $Y$  as whether the case was reversed. Include year fixed effects. You can do this in two stages, by regressing  $X$  on  $Z$  to get  $X_{\text{pred}}$ , and then regressing  $Y$  on  $X_{\text{pred}}$ . Report estimates for coefficient and standard error on  $X_{\text{pred}}$ . Compare to the parameter estimates for the OLS regression of  $Y$  on  $X$ .

[https://github.com/elliott/fiscal\\_policy\\_course/blob/master/notebooks/20-Instrumental-Variables.ipynb](https://github.com/elliott/fiscal_policy_course/blob/master/notebooks/20-Instrumental-Variables.ipynb)  
([https://github.com/elliott/fiscal\\_policy\\_course/blob/master/notebooks/20-Instrumental-Variables.ipynb](https://github.com/elliott/fiscal_policy_course/blob/master/notebooks/20-Instrumental-Variables.ipynb))

In [ ]:

```
from linearmodels.iv import IV2SLS
from data_creation_and_loader import get_exercise_3_dataframe

# get the data
df = get_exercise_3_dataframe()

# 2SLS
# Ypred := Y regressed on Xpred := (X regressed on Z)
formula_2sls = 'case_reversed ~ 1 + C(year) + [cites_case ~ avg_cites_judge]'
predictor_2sls = IV2SLS.from_formula(formula_2sls, data=df)
results_2sls = predictor_2sls.fit()

# OLS
# Ypred := Y regressed on X
formula_ols = 'case_reversed ~ 1 + C(year) + cites_case'
predictor_ols = IV2SLS.from_formula(formula_ols, data=df)
results_ols = predictor_ols.fit()
```

**A:** The tables below show the results of the parameter estimates.

The estimates for the coefficient and standard error for Xpred (cites\_case) in the 2SLS are:

Parameter	Std. Err.	T-stat	P-value	Lower CI	Upper CI
-0.0070	0.0012	-5.8450	0.0000	-0.0093	-0.0046

The estimates for the coefficient and standard error for X (cites\_case) in the 2SLS are:

Parameter	Std. Err.	T-stat	P-value	Lower CI	Upper CI
-0.0032	0.0005	-6.5101	0.0000	-0.0042	-0.0023

As we can see the p-values are very low for both regressions. It's unlikely that there is some linear relationship (or two-stage linear relationship) between the variables (as our model (or null hypothesis) assumed). However, the parameters can be estimated quite well in both cases with a very low standard error (and thus we have tight confidence intervals).

In [12]:

```
results_2sls
```

Out[12]:

#### IV-2SLS Estimation Summary

<b>Dep. Variable:</b>	case_reversed	<b>R-squared:</b>	0.0415
<b>Estimator:</b>	IV-2SLS	<b>Adj. R-squared:</b>	0.0215
<b>No. Observations:</b>	4399	<b>F-statistic:</b>	295.11
<b>Date:</b>	Wed, Apr 17 2019	<b>P-value (F-stat)</b>	0.0000
<b>Time:</b>	19:37:01	<b>Distribution:</b>	chi2(90)
<b>Cov. Estimator:</b>	robust		

#### Parameter Estimates

	Parameter	Std. Err.	T-stat	P-value	Lower CI	Upper CI
Intercept	0.3750	0.2808	1.3358	0.1816	-0.1752	0.9253
C(year)[T.1925.0]	0.2083	0.2991	0.6964	0.4862	-0.3780	0.7947
C(year)[T.1926.0]	0.2017	0.2983	0.6760	0.4991	-0.3831	0.7864
C(year)[T.1927.0]	0.4455	0.2927	1.5222	0.1280	-0.1281	1.0192
C(year)[T.1928.0]	0.3076	0.3007	1.0232	0.3062	-0.2817	0.8969
C(year)[T.1929.0]	0.0989	0.2970	0.3329	0.7392	-0.4832	0.6810
C(year)[T.1930.0]	0.1339	0.2955	0.4532	0.6504	-0.4453	0.7132
C(year)[T.1931.0]	0.0243	0.2945	0.0825	0.9342	-0.5529	0.6015
C(year)[T.1932.0]	0.0076	0.2890	0.0262	0.9791	-0.5589	0.5740
C(year)[T.1933.0]	0.2035	0.2915	0.6979	0.4852	-0.3679	0.7748
C(year)[T.1934.0]	0.1086	0.2891	0.3758	0.7071	-0.4579	0.6752
C(year)[T.1935.0]	-0.0383	0.2903	-0.1319	0.8951	-0.6074	0.5308
C(year)[T.1936.0]	0.1424	0.2898	0.4912	0.6233	-0.4257	0.7104
C(year)[T.1937.0]	0.3026	0.2890	1.0473	0.2950	-0.2637	0.8690
C(year)[T.1938.0]	0.0296	0.2893	0.1023	0.9185	-0.5374	0.5965
C(year)[T.1939.0]	0.2644	0.2880	0.9180	0.3586	-0.3001	0.8289
C(year)[T.1940.0]	0.2456	0.2865	0.8573	0.3913	-0.3159	0.8070
C(year)[T.1941.0]	0.3122	0.2891	1.0800	0.2801	-0.2544	0.8788
C(year)[T.1942.0]	0.2029	0.2896	0.7006	0.4835	-0.3647	0.7706
C(year)[T.1943.0]	0.2061	0.2902	0.7100	0.4777	-0.3628	0.7750
C(year)[T.1944.0]	0.2325	0.2891	0.8044	0.4212	-0.3341	0.7991
C(year)[T.1945.0]	0.2405	0.2905	0.8278	0.4078	-0.3289	0.8099
C(year)[T.1946.0]	0.1365	0.2909	0.4693	0.6389	-0.4337	0.7067
C(year)[T.1947.0]	0.3026	0.2915	1.0381	0.2992	-0.2687	0.8739
C(year)[T.1948.0]	0.2639	0.2888	0.9139	0.3608	-0.3021	0.8299
C(year)[T.1949.0]	0.2807	0.2919	0.9615	0.3363	-0.2915	0.8529
C(year)[T.1950.0]	0.3361	0.2889	1.1631	0.2448	-0.2302	0.9024

C(year)[T.1951.0]	0.1280	0.2918	0.4388	0.6608	-0.4438	0.6999
C(year)[T.1952.0]	0.2668	0.2920	0.9138	0.3608	-0.3054	0.8390
C(year)[T.1953.0]	0.2313	0.2916	0.7933	0.4276	-0.3402	0.8027
C(year)[T.1954.0]	0.3346	0.2903	1.1525	0.2491	-0.2344	0.9037
C(year)[T.1955.0]	0.2878	0.2892	0.9951	0.3197	-0.2791	0.8546
C(year)[T.1956.0]	0.2477	0.2869	0.8635	0.3879	-0.3146	0.8101
C(year)[T.1957.0]	0.2407	0.2897	0.8308	0.4061	-0.3272	0.8086
C(year)[T.1958.0]	0.3178	0.2877	1.1048	0.2692	-0.2460	0.8817
C(year)[T.1959.0]	0.3842	0.2871	1.3381	0.1809	-0.1786	0.9470
C(year)[T.1960.0]	0.3185	0.2892	1.1013	0.2708	-0.2483	0.8852
C(year)[T.1961.0]	0.5451	0.2856	1.9084	0.0563	-0.0147	1.1049
C(year)[T.1962.0]	0.5201	0.2854	1.8220	0.0684	-0.0394	1.0795
C(year)[T.1963.0]	0.3471	0.2887	1.2025	0.2292	-0.2187	0.9129
C(year)[T.1964.0]	0.3394	0.2892	1.1736	0.2405	-0.2274	0.9062
C(year)[T.1965.0]	0.3811	0.2889	1.3190	0.1872	-0.1852	0.9474
C(year)[T.1966.0]	0.5070	0.2867	1.7682	0.0770	-0.0550	1.0690
C(year)[T.1967.0]	0.5620	0.2849	1.9728	0.0485	0.0036	1.1204
C(year)[T.1968.0]	0.4506	0.2883	1.5628	0.1181	-0.1145	1.0158
C(year)[T.1969.0]	0.4252	0.2882	1.4752	0.1402	-0.1397	0.9900
C(year)[T.1970.0]	0.3048	0.2894	1.0531	0.2923	-0.2625	0.8720
C(year)[T.1971.0]	0.4225	0.2897	1.4587	0.1446	-0.1452	0.9903
C(year)[T.1972.0]	0.3479	0.2877	1.2094	0.2265	-0.2159	0.9118
C(year)[T.1973.0]	0.4413	0.2857	1.5443	0.1225	-0.1188	1.0013
C(year)[T.1974.0]	0.3513	0.2878	1.2210	0.2221	-0.2126	0.9153
C(year)[T.1975.0]	0.4760	0.2869	1.6589	0.0971	-0.0864	1.0384
C(year)[T.1976.0]	0.4655	0.2864	1.6253	0.1041	-0.0958	1.0268
C(year)[T.1977.0]	0.4136	0.2880	1.4363	0.1509	-0.1508	0.9781
C(year)[T.1978.0]	0.4305	0.2866	1.5021	0.1331	-0.1312	0.9921
C(year)[T.1979.0]	0.3333	0.2875	1.1592	0.2464	-0.2303	0.8969
C(year)[T.1980.0]	0.3580	0.2862	1.2507	0.2111	-0.2030	0.9189
C(year)[T.1981.0]	0.3970	0.2850	1.3931	0.1636	-0.1616	0.9556
C(year)[T.1982.0]	0.4216	0.2874	1.4671	0.1424	-0.1416	0.9848
C(year)[T.1983.0]	0.4110	0.2866	1.4342	0.1515	-0.1507	0.9727
C(year)[T.1984.0]	0.3537	0.2857	1.2380	0.2157	-0.2062	0.9135
C(year)[T.1985.0]	0.3795	0.2859	1.3272	0.1844	-0.1809	0.9399
C(year)[T.1986.0]	0.3301	0.2864	1.1527	0.2490	-0.2312	0.8914
C(year)[T.1987.0]	0.3742	0.2879	1.2997	0.1937	-0.1901	0.9385
C(year)[T.1988.0]	0.3657	0.2872	1.2730	0.2030	-0.1973	0.9286



<b>C(year)[T.1989.0]</b>	0.3228	0.2869	1.1254	0.2604	-0.2394	0.8851
<b>C(year)[T.1990.0]</b>	0.3156	0.2866	1.1009	0.2709	-0.2462	0.8774
<b>C(year)[T.1991.0]</b>	0.2789	0.2885	0.9667	0.3337	-0.2866	0.8443
<b>C(year)[T.1992.0]</b>	0.3840	0.2885	1.3312	0.1831	-0.1814	0.9495
<b>C(year)[T.1993.0]</b>	0.3099	0.2921	1.0607	0.2888	-0.2627	0.8824
<b>C(year)[T.1994.0]</b>	0.3558	0.2883	1.2342	0.2171	-0.2093	0.9209
<b>C(year)[T.1995.0]</b>	0.4102	0.2884	1.4225	0.1549	-0.1550	0.9753
<b>C(year)[T.1996.0]</b>	0.3696	0.2892	1.2782	0.2012	-0.1971	0.9363
<b>C(year)[T.1997.0]</b>	0.4158	0.2885	1.4416	0.1494	-0.1495	0.9812
<b>C(year)[T.1998.0]</b>	0.4555	0.2879	1.5823	0.1136	-0.1087	1.0197
<b>C(year)[T.1999.0]</b>	0.3477	0.2887	1.2043	0.2285	-0.2182	0.9136
<b>C(year)[T.2000.0]</b>	0.3694	0.2902	1.2731	0.2030	-0.1993	0.9381
<b>C(year)[T.2001.0]</b>	0.4952	0.2907	1.7034	0.0885	-0.0746	1.0650
<b>C(year)[T.2002.0]</b>	0.5604	0.2864	1.9564	0.0504	-0.0010	1.1218
<b>C(year)[T.2003.0]</b>	0.4985	0.2948	1.6910	0.0908	-0.0793	1.0764
<b>C(year)[T.2004.0]</b>	0.5162	0.2889	1.7867	0.0740	-0.0501	1.0824
<b>C(year)[T.2005.0]</b>	0.3914	0.2923	1.3392	0.1805	-0.1814	0.9643
<b>C(year)[T.2006.0]</b>	0.4049	0.2944	1.3755	0.1690	-0.1721	0.9819
<b>C(year)[T.2007.0]</b>	0.5456	0.2883	1.8924	0.0584	-0.0195	1.1107
<b>C(year)[T.2008.0]</b>	0.4617	0.2883	1.6015	0.1093	-0.1033	1.0266
<b>C(year)[T.2009.0]</b>	0.3849	0.2918	1.3188	0.1872	-0.1871	0.9569
<b>C(year)[T.2010.0]</b>	0.2592	0.2968	0.8736	0.3823	-0.3224	0.8409
<b>C(year)[T.2011.0]</b>	0.4088	0.2971	1.3759	0.1689	-0.1735	0.9911
<b>C(year)[T.2012.0]</b>	0.4346	0.2917	1.4900	0.1362	-0.1371	1.0064
<b>C(year)[T.2013.0]</b>	0.4661	0.3052	1.5269	0.1268	-0.1322	1.0643
<b>cites_case</b>	-0.0070	0.0012	-5.8450	0.0000	-0.0093	-0.0046

Endogenous: cites\_case

Instruments: avg\_cites\_judge

Robust Covariance (Heteroskedastic)

Debiased: False

id: 0x7fa6c8d80b38

In [11]:

```
results_ols
```

Out[11]:

#### OLS Estimation Summary

<b>Dep. Variable:</b>	case_reversed	<b>R-squared:</b>	0.0566
<b>Estimator:</b>	OLS	<b>Adj. R-squared:</b>	0.0369
<b>No. Observations:</b>	4399	<b>F-statistic:</b>	297.87
<b>Date:</b>	Wed, Apr 17 2019	<b>P-value (F-stat)</b>	0.0000
<b>Time:</b>	19:37:01	<b>Distribution:</b>	chi2(90)
<b>Cov. Estimator:</b>	robust		

#### Parameter Estimates

	Parameter	Std. Err.	T-stat	P-value	Lower CI	Upper CI
<b>Intercept</b>	0.3527	0.2761	1.2774	0.2015	-0.1885	0.8940
<b>C(year)[T.1925.0]</b>	0.2104	0.2954	0.7122	0.4763	-0.3686	0.7893
<b>C(year)[T.1926.0]</b>	0.2052	0.2942	0.6977	0.4854	-0.3713	0.7818
<b>C(year)[T.1927.0]</b>	0.4404	0.2882	1.5279	0.1265	-0.1245	1.0053
<b>C(year)[T.1928.0]</b>	0.3125	0.2962	1.0550	0.2914	-0.2680	0.8930
<b>C(year)[T.1929.0]</b>	0.1030	0.2931	0.3516	0.7251	-0.4713	0.6774
<b>C(year)[T.1930.0]</b>	0.1323	0.2915	0.4540	0.6498	-0.4390	0.7037
<b>C(year)[T.1931.0]</b>	0.0240	0.2904	0.0828	0.9340	-0.5451	0.5932
<b>C(year)[T.1932.0]</b>	0.0035	0.2847	0.0124	0.9901	-0.5546	0.5616
<b>C(year)[T.1933.0]</b>	0.2105	0.2871	0.7332	0.4634	-0.3522	0.7733
<b>C(year)[T.1934.0]</b>	0.0998	0.2850	0.3503	0.7261	-0.4587	0.6584
<b>C(year)[T.1935.0]</b>	-0.0371	0.2861	-0.1296	0.8968	-0.5977	0.5236
<b>C(year)[T.1936.0]</b>	0.1442	0.2854	0.5053	0.6133	-0.4151	0.7036
<b>C(year)[T.1937.0]</b>	0.3048	0.2844	1.0716	0.2839	-0.2527	0.8623
<b>C(year)[T.1938.0]</b>	0.0331	0.2850	0.1163	0.9074	-0.5255	0.5917
<b>C(year)[T.1939.0]</b>	0.2599	0.2837	0.9161	0.3596	-0.2961	0.8159
<b>C(year)[T.1940.0]</b>	0.2355	0.2820	0.8351	0.4036	-0.3172	0.7881
<b>C(year)[T.1941.0]</b>	0.3093	0.2845	1.0871	0.2770	-0.2483	0.8668
<b>C(year)[T.1942.0]</b>	0.1952	0.2855	0.6836	0.4942	-0.3644	0.7547
<b>C(year)[T.1943.0]</b>	0.2037	0.2861	0.7118	0.4766	-0.3571	0.7645
<b>C(year)[T.1944.0]</b>	0.2179	0.2846	0.7657	0.4439	-0.3399	0.7756
<b>C(year)[T.1945.0]</b>	0.2321	0.2861	0.8112	0.4173	-0.3287	0.7929
<b>C(year)[T.1946.0]</b>	0.1331	0.2867	0.4642	0.6425	-0.4289	0.6950
<b>C(year)[T.1947.0]</b>	0.2987	0.2875	1.0387	0.2989	-0.2649	0.8622
<b>C(year)[T.1948.0]</b>	0.2547	0.2846	0.8950	0.3708	-0.3030	0.8124
<b>C(year)[T.1949.0]</b>	0.2643	0.2878	0.9184	0.3584	-0.2997	0.8283
<b>C(year)[T.1950.0]</b>	0.3081	0.2845	1.0830	0.2788	-0.2495	0.8656

C(year)[T.1951.0]	0.1126	0.2876	0.3914	0.6955	-0.4511	0.6762
C(year)[T.1952.0]	0.2578	0.2878	0.8957	0.3704	-0.3063	0.8219
C(year)[T.1953.0]	0.1827	0.2872	0.6360	0.5248	-0.3803	0.7456
C(year)[T.1954.0]	0.3029	0.2855	1.0610	0.2887	-0.2567	0.8625
C(year)[T.1955.0]	0.2787	0.2849	0.9784	0.3279	-0.2796	0.8370
C(year)[T.1956.0]	0.2349	0.2824	0.8319	0.4054	-0.3186	0.7885
C(year)[T.1957.0]	0.2296	0.2854	0.8044	0.4212	-0.3298	0.7889
C(year)[T.1958.0]	0.3083	0.2831	1.0890	0.2761	-0.2466	0.8632
C(year)[T.1959.0]	0.3714	0.2824	1.3151	0.1885	-0.1821	0.9249
C(year)[T.1960.0]	0.3082	0.2846	1.0829	0.2789	-0.2496	0.8661
C(year)[T.1961.0]	0.5355	0.2811	1.9049	0.0568	-0.0155	1.0865
C(year)[T.1962.0]	0.5093	0.2808	1.8139	0.0697	-0.0410	1.0597
C(year)[T.1963.0]	0.3293	0.2841	1.1589	0.2465	-0.2276	0.8861
C(year)[T.1964.0]	0.3243	0.2849	1.1381	0.2551	-0.2342	0.8827
C(year)[T.1965.0]	0.3518	0.2844	1.2369	0.2161	-0.2057	0.9092
C(year)[T.1966.0]	0.4878	0.2822	1.7284	0.0839	-0.0653	1.0408
C(year)[T.1967.0]	0.5388	0.2803	1.9221	0.0546	-0.0106	1.0882
C(year)[T.1968.0]	0.4294	0.2839	1.5124	0.1304	-0.1270	0.9858
C(year)[T.1969.0]	0.4032	0.2838	1.4206	0.1554	-0.1531	0.9594
C(year)[T.1970.0]	0.2691	0.2848	0.9448	0.3448	-0.2892	0.8274
C(year)[T.1971.0]	0.3860	0.2844	1.3573	0.1747	-0.1714	0.9433
C(year)[T.1972.0]	0.3178	0.2829	1.1237	0.2611	-0.2365	0.8722
C(year)[T.1973.0]	0.4061	0.2809	1.4458	0.1482	-0.1444	0.9566
C(year)[T.1974.0]	0.3104	0.2823	1.0994	0.2716	-0.2429	0.8637
C(year)[T.1975.0]	0.4354	0.2821	1.5434	0.1227	-0.1175	0.9882
C(year)[T.1976.0]	0.4250	0.2816	1.5093	0.1312	-0.1269	0.9768
C(year)[T.1977.0]	0.3707	0.2829	1.3106	0.1900	-0.1837	0.9251
C(year)[T.1978.0]	0.3883	0.2816	1.3789	0.1679	-0.1636	0.9402
C(year)[T.1979.0]	0.2900	0.2824	1.0271	0.3044	-0.2634	0.8435
C(year)[T.1980.0]	0.3132	0.2814	1.1128	0.2658	-0.2384	0.8648
C(year)[T.1981.0]	0.3543	0.2801	1.2650	0.2059	-0.1947	0.9033
C(year)[T.1982.0]	0.3825	0.2823	1.3551	0.1754	-0.1707	0.9357
C(year)[T.1983.0]	0.3593	0.2811	1.2785	0.2011	-0.1916	0.9103
C(year)[T.1984.0]	0.3060	0.2807	1.0900	0.2757	-0.2442	0.8562
C(year)[T.1985.0]	0.3386	0.2813	1.2036	0.2287	-0.2128	0.8900
C(year)[T.1986.0]	0.2819	0.2817	1.0008	0.3169	-0.2702	0.8340
C(year)[T.1987.0]	0.3269	0.2830	1.1553	0.2480	-0.2277	0.8815
C(year)[T.1988.0]	0.3191	0.2824	1.1301	0.2584	-0.2343	0.8726

<b>C(year)[T.1989.0]</b>	0.2897	0.2823	1.0263	0.3047	-0.2635	0.8429
<b>C(year)[T.1990.0]</b>	0.2863	0.2822	1.0146	0.3103	-0.2668	0.8394
<b>C(year)[T.1991.0]</b>	0.2316	0.2834	0.8174	0.4137	-0.3238	0.7870
<b>C(year)[T.1992.0]</b>	0.3448	0.2834	1.2169	0.2236	-0.2106	0.9002
<b>C(year)[T.1993.0]</b>	0.2614	0.2873	0.9099	0.3629	-0.3017	0.8246
<b>C(year)[T.1994.0]</b>	0.3215	0.2843	1.1311	0.2580	-0.2356	0.8786
<b>C(year)[T.1995.0]</b>	0.3754	0.2833	1.3252	0.1851	-0.1798	0.9307
<b>C(year)[T.1996.0]</b>	0.3350	0.2846	1.1773	0.2391	-0.2227	0.8928
<b>C(year)[T.1997.0]</b>	0.3816	0.2834	1.3465	0.1782	-0.1739	0.9371
<b>C(year)[T.1998.0]</b>	0.4192	0.2838	1.4771	0.1396	-0.1370	0.9754
<b>C(year)[T.1999.0]</b>	0.2963	0.2841	1.0431	0.2969	-0.2604	0.8531
<b>C(year)[T.2000.0]</b>	0.3145	0.2853	1.1022	0.2704	-0.2447	0.8737
<b>C(year)[T.2001.0]</b>	0.4434	0.2851	1.5554	0.1198	-0.1153	1.0022
<b>C(year)[T.2002.0]</b>	0.5408	0.2816	1.9203	0.0548	-0.0112	1.0928
<b>C(year)[T.2003.0]</b>	0.4426	0.2904	1.5243	0.1274	-0.1265	1.0117
<b>C(year)[T.2004.0]</b>	0.4744	0.2843	1.6689	0.0951	-0.0827	1.0316
<b>C(year)[T.2005.0]</b>	0.3507	0.2873	1.2209	0.2221	-0.2123	0.9137
<b>C(year)[T.2006.0]</b>	0.3551	0.2892	1.2278	0.2195	-0.2118	0.9220
<b>C(year)[T.2007.0]</b>	0.4978	0.2834	1.7563	0.0790	-0.0577	1.0533
<b>C(year)[T.2008.0]</b>	0.4294	0.2834	1.5153	0.1297	-0.1260	0.9849
<b>C(year)[T.2009.0]</b>	0.3436	0.2868	1.1981	0.2309	-0.2185	0.9057
<b>C(year)[T.2010.0]</b>	0.2204	0.2928	0.7529	0.4515	-0.3534	0.7943
<b>C(year)[T.2011.0]</b>	0.3939	0.2930	1.3446	0.1788	-0.1803	0.9681
<b>C(year)[T.2012.0]</b>	0.4296	0.2873	1.4954	0.1348	-0.1335	0.9928
<b>C(year)[T.2013.0]</b>	0.4545	0.3049	1.4907	0.1360	-0.1431	1.0521
<b>cites_case</b>	-0.0032	0.0005	-6.5101	0.0000	-0.0042	-0.0023

id: 0x7fa6c8d55f98

## Exercise 4

**Q:** Train LDA on the cases. Produce word clouds for the topics. Inspect the word clouds for different models to decide on the right number of topics.

**A:** I've tried out various numbers of topics.

- **Many topics (e.g., 10, 15 or 20):** I noticed that by increasing the number of topics, suddenly, smaller topics (that aren't as dominant in the dataset as others) started to appear. However, I also noticed that I had a lot of overlapping topics.
- **Few topics (e.g., 3, 5, 7):** When choosing too few topics ( $\approx 3$ ) then the topics weren't representative and distinguishable enough. I realized that by increasing the number of topics (to 5 or 7) the topics became enough diverse.

I decided to use 5 topics as it seemed to be a good choice for the trade-off between *having enough diversity* and *not having too many overlapping topics*.

As one can see in the results below the topics that can be identified from the clouds are: business related topics (patents, contracts, taxes), law related terms (law, evidence, claim), public services and politics (school, constitution, ...).

In [15]:

```
%matplotlib inline
import numpy as np
from gensim.corpora import Dictionary
from gensim.models import LdaMulticore
from numpy.random import randint
from wordcloud import WordCloud
import matplotlib.pyplot as plt
from sklearn.utils import shuffle
from data_creation_and_loader import get_exercise_4_list

NUM_TOPICS = 5
NUM_PASSES = 30
NUM_WORDS_PER_WORD_CLOUD = 100

# get dataset
case_token_lists = get_exercise_4_list()

# define function to filter list of words
BAD_WORDS = ['state', 'court', 'case', 'section', 'district']
def filter_words(word):
    if word in BAD_WORDS:
        return False
    return True
filter_list = lambda ls : list(filter(filter_words, ls))
# filter out some words that appear huge in every word cloud
case_token_lists = list(map(filter_list, case_token_lists))

# randomize document order
shuffle(case_token_lists)

# creating the term dictionary
dictionary = Dictionary(case_token_lists)

# creating the document-term matrix
doc_term_matrix = [dictionary.doc2bow(case) for case in case_token_lists]

# train LDA
lda = LdaMulticore(doc_term_matrix,
                   num_topics=NUM_TOPICS,
                   id2word=dictionary,
                   passes=NUM_PASSES)

# get the list of topics
topics = lda.show_topics(num_topics=NUM_TOPICS,
                        num_words=NUM_WORDS_PER_WORD_CLOUD,
                        formatted=False)

# show the word cloud for each topic
for topic_idx, word_weights in topics:

    # use logarithmic weights to balance out outliers
    logweights = [(w[0], np.log(w[1])) for w in word_weights]

    # sample a random color for the topic
    maincol = randint(0, 360)
    # create a coloring function
    def colorfunc(word=None,
                  font_size=None,
                  position=None,
```

```

        orientation=None,
        font_path=None,
        random_state=None):
    # get a color near to the main color
    color = randint(maincol - 10, maincol + 10)
    # adjust color via modulus
    if color < 0:
        color = 360 + color
    # generate color string
    # format (hue (color), saturation%, lightness%, alpha%)
    color_string = 'hsl(%d, %d%%, %d%%)' % \
        (color,
         randint(65, 75) + font_size / 7,
         randint(35, 45) - font_size / 10)
    return color_string

# initialize word cloud for topic
# specify color function
wordcloud = WordCloud(background_color='white',
                      ranks_only=False,
                      max_font_size=120,
                      color_func=colorfunc,
                      height=600,
                      width=800)

# generate the word cloud
wordcloud.generate_from_frequencies(dict(logweights))

# plot the word cloud
plt.clf()
plt.imshow(wordcloud, interpolation='bilinear')
plt.axis('off')
plt.show()

```



feder order requir decis cir issu deni patent  
claim defend  
trial fact  
appell rule  
unit evid  
convict govern  
present sentenc offic

statut paid trust payment asset  
act compani  
year time co land judgment  
taxpay commission fact provide  
unit tax bank  
right claim incom  
proper ti stock  
interest corpor said  
provis order account rule

nation matter general motion circuit benefit  
sct union requir  
employ act  
board employe  
action law rule  
plaintiff  
order claim  
violat agreement repres secur administr enforc class

interest school constitution amend statute law public requirement protect reason

congress act law regul feder commiss requir statute