Ch:2

1) Basis: length $\lambda = 0$

Recursive: length (x:xs) = 1 + length xs

Closure: The length of a string can only be determined through a finite quantity of applications of the recursive case.

5) a) $L_0 = \{b\}$
$L_1 = \{bb, bab, bba\}$
$L_2 = \{bbb, bbab, bbba, babb, babab, babba, bbaba, bbaab, bbbaa\}$

b) No, because you can build up bbaab, but you can't ge that extra a on the end without another b.

c) No, because you can't get that many consecutive a's without adding on more b's.

6) Basis: $b \in L$

Recursive: if u is in L then $aau \in L$ and $uab \in L$.

Closure: a string v is in L only if it can be obtained from the basis by a finit number of iterations of the recursive step.

→

10/ Basis: $n_a(u) \geq n_b(u)$ at L_1

$L_1 = a, ab$

IH: Assume $n_a(u) \geq n_b(u)$ for L_n

Prove $n_a(u) \geq n_b(u)$ for $L(n+1)$

So, our base cases are $u = a$ and $u = ab$. We can only apply ua and uab to every string. Therefore, everytime a "b" is added, so is an "a".
From our basis $n_a(u) \geq n_b(u)$ therefore $L(n+1) = n_a(u) \geq n_b(u)$ because oure recursive definitions maintain an "a" for every "b".

12/ $P \subseteq W$

Basis: λ and a, for all $a \in \Sigma$. $W = W^R$ for every such string.

IH: Assume that every string generated by n or fewer applications of the recursive step is in W.

Let u be the string generated by $n+1$ applications of the recursive step. $u = awa$ for some string w and symbol $a \in \Sigma$, where w is generated by n applications of the rc step.

→

$u^R = (awa)^R$
$a^R w^R a^R$
$a w^R a$
$awa = v \in W$

$W \subseteq P$

Basis: If length$(u) = 0$, then $w = \lambda$ and $\lambda \in P$, and strings of length one in W are in P.

IH: Assume every string $w \in W$ with length n or less in P

Let $w \in W$ be a string of length $n+1$, $n \geq 1$. Then w can be written ua where length$(u) = n$.

$$w = w^R = (ua)^R = au^R$$

$$w^R = (ava)^R$$
$$a^R v^R a^R$$
$$a v^R a$$

$w = ava$ is also in P

13) L_2 consists of all strings over $\{a,b\}$ of length 4.

L_3 contains all the strings over $\{a,b\}$ of with length divisible by 4.

$L_4 \cap L_3 = (a^{12})^*$ only a's with length divisible by a

14) $L = a^* b^* c^*$

23) $a(a \cup c)^* b (a \cup c)^* b (a \cup c)^* cc$