Midterm

1) $z = a^{n^3}$

$(n+1)^3 = n^3 + 3n^2 + 3n + 1$

$uv = a's$

$|uv| \leq n$

$uw \notin L$

$uvvw = |uvw| + |v|$

$\leq n^3 + n$

2)

3a) M: $Q = \{q_0, q_1\}$
$\Sigma = \{a, b\}$
$\Gamma = \{A, B\}$
$F = \{q_1\}$

$\delta(q_0, a, \lambda) = \{[q_0, A]\}$
$\delta(q_0, b, \lambda) = \{[q_0, B]\}$
$\delta(q_0, \lambda, \lambda) = \{[q_1, \lambda]\}$
$\delta(q_1, a, A) = \{[q_1, \lambda]\}$
$\delta(q_1, b, B) = \{[q_1, \lambda]\}$

b) $[q_0, ab, \lambda]$
$[q_1, ab, \lambda]$

$[q_0, abb, \lambda]$
$[q_1, abb, \lambda]$

$[q_0, abb, \lambda]$
$[q_0, bb, A]$
$[q_0, b, BA]$
$[q_0, \lambda, BBA]$
$[q_1, \lambda, BBA]$

$[q_0, ab, \lambda]$
$[q_0, b, A]$
$[q_1, b, A]$

$[q_0, abb, \lambda]$
$[q_0, bb, A]$
$[q_1, bb, A]$

$[q_0, abb, \lambda]$
$[q_0, bb, A]$
$[q_0, b, BA]$
$[q_1, b, BA]$
$[q_1, \lambda, A]$

$[q_0, ab, \lambda]$
$[q_0, b, A]$
$[q_0, \lambda, BA]$
$[q_1, \lambda, BA]$

$[q_0, abbb, \lambda]$
$[q_1, abbb, \lambda]$

$[q_0, abbb, \lambda]$
$[q_0, bbb, A]$
$[q_0, bb, BA]$
$[q_0, b, BBA]$
$[q_1, b, BBA]$
$[q_1, \lambda, BA]$

$[q_0, abbb, \lambda]$
$[q_0, bbb, A]$
$[q_1, bbb, A]$

$[q_0, abbb, \lambda]$
$[q_0, bbb, A]$
$[q_0, bb, BA]$
$[q_0, b, BBA]$
$[q_0, \lambda, BBBA]$
$[q_1, \lambda, BBBA]$

$[q_0, abbb, \lambda]$
$[q_0, bbb, A]$
$[q_0, bb, BA]$
$[q_1, bb, BA]$
$[q_1, b, A]$

→

M accepts strings

c) [q0, aaaa, λ]
[q0, aaa, A]
[q0, aa, AA]
[q1, aa, AA]
[q1, a, A]
[q1, λ, λ]

[q0, baab, λ]
[q0, aab, B]
[q0, ab, AB]
[q1, ab, AB]
[q1, b, B]
[q1, λ, λ]

d) M doesn't accept strings. (ab in part b)

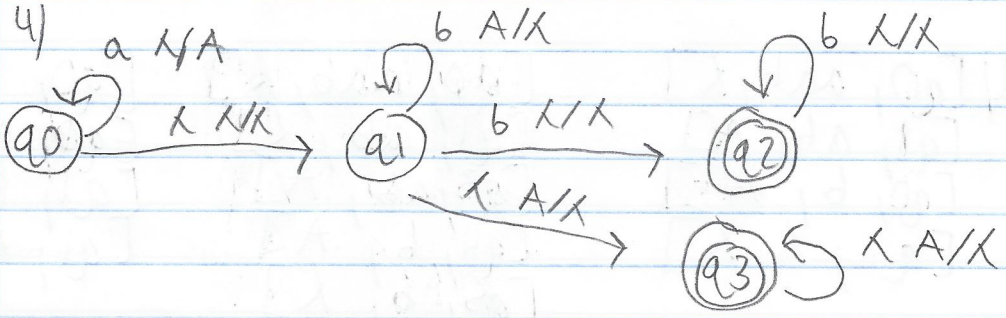
[q0, aaa, λ]
[q1, aaa, λ]

[q0, aaa, λ]
[q0, aa, A]
[q1, aa, A]
[q1, a, λ]

[q0, aaa, λ]
[q0, aa, A]
[q0, a, AA]
[q1, a, AA]
[q1, λ, A]

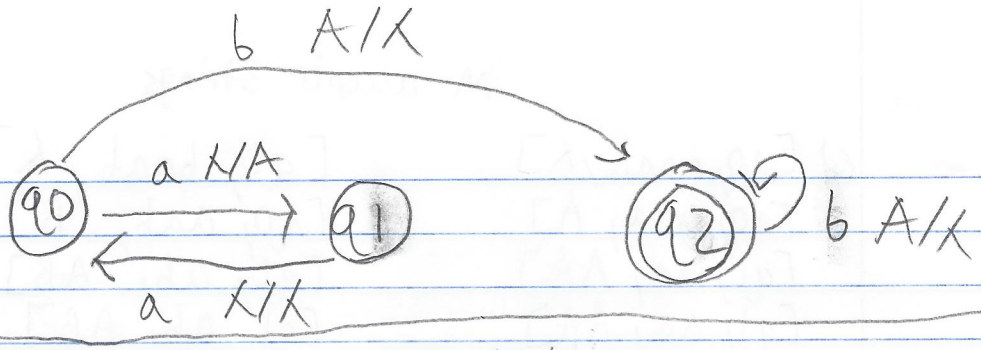
[q0, aaa, λ]
[q0, aa, A]
[q0, a, AA]
[q0, λ, AAA]
[q1, λ, AAA]

4)

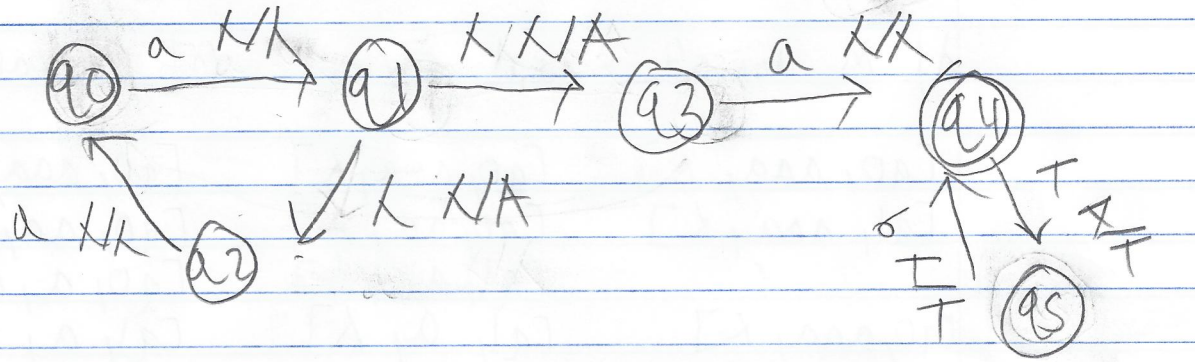


→

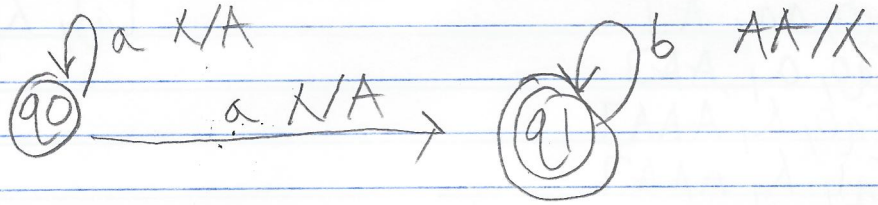
5) a)



b)



c)



d) [q0, aab, λ]
[q1, ab, A]
[q0, b, A]
[q2, λ, λ]

[q0, aab, λ]
[q1, ab, λ]
[q3, ab, A]
[q4, b, A]
[q5, b, λ]
[q4, λ, λ]

[q0, aab, λ]
[q0, ab, A]
[q1, b, AA]
[q1, λ, λ]

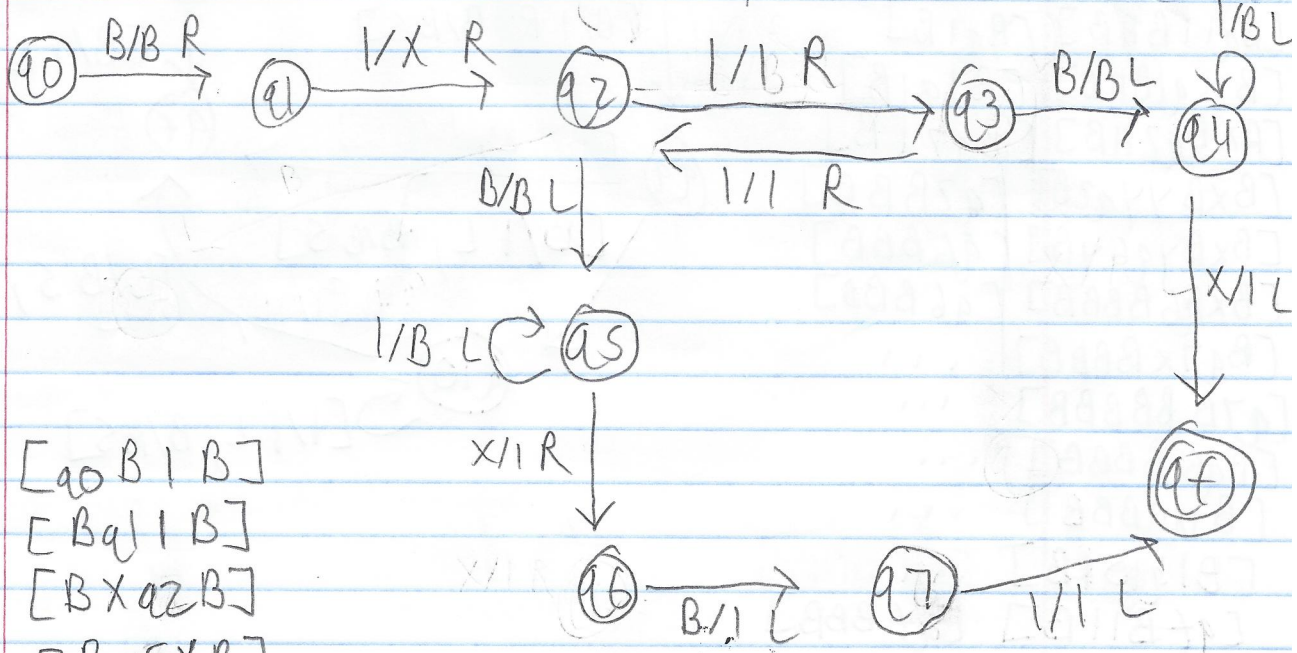
6/ $z = (a^k b^k)(a^k b^k)^R (a^k b^k) = a^k b^{2k} a^{2k} b^k$

length $vwx \leq k$
$uv^i wx^i y \in L$ for all $i \geq 0$

Assume an 'a' is in v or x. vwx cannot be longer than k, so the substring vwx can't contain a's from both sides of b^{2k}.

If a's are before b^{2k}, uv^2wx^2y increases the number of a's before b^{2k}, but not the a's after. So, $uv^2wx^2y \notin L$. If the a's are after b^{2k}, the same problem occurs. Therefore, L is not context free.

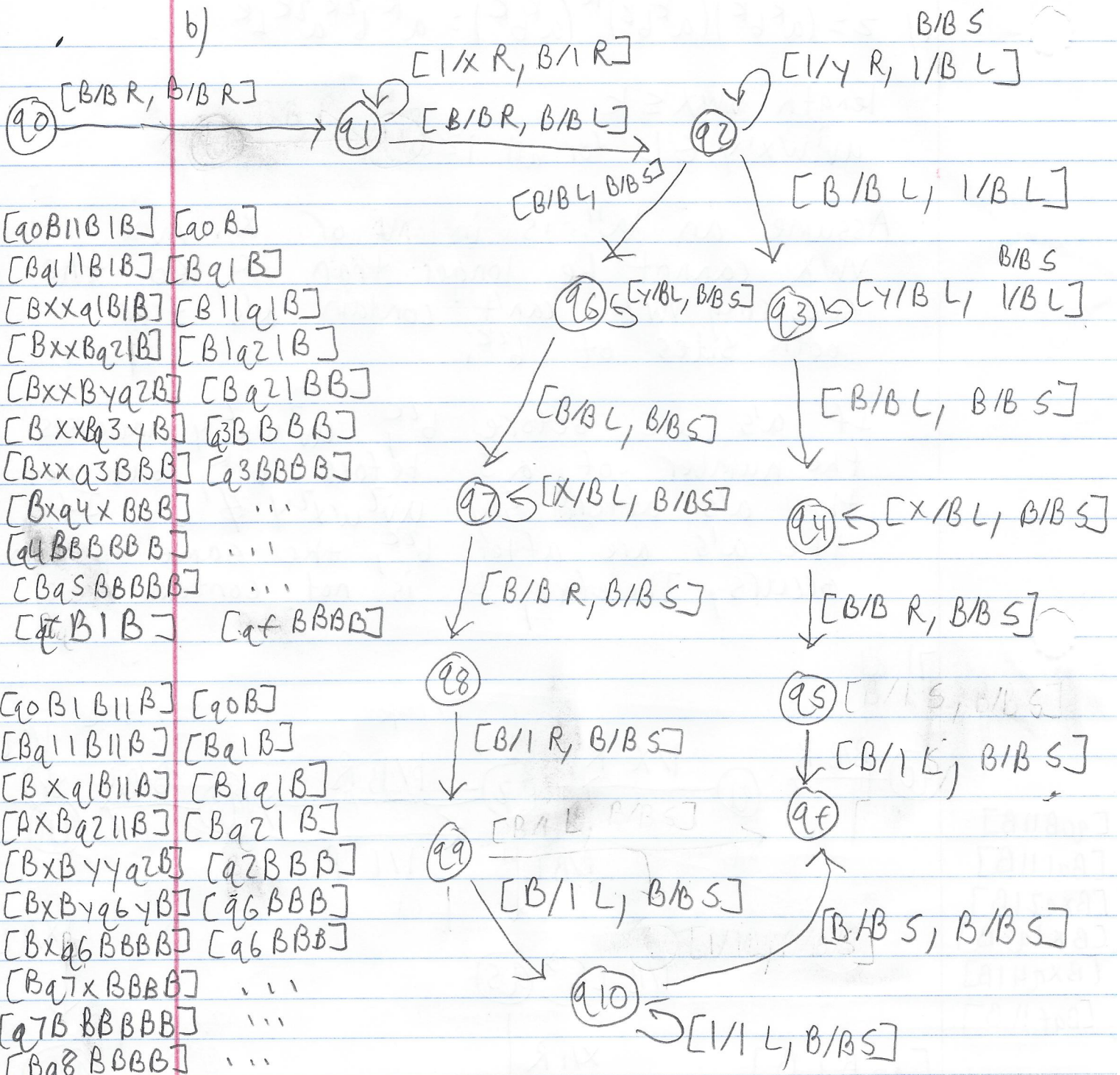
7) a/



[q0B11B]
[Bq111B]
[BXq211B]
[BX1q3B]
[BXq411B]
[Bqf11B]

[q0B1B]
[Bq11B]
[BXq2B]
[Bq5XB]
[B1q6B]
[B1q71B]
[Bqf11B]

b)

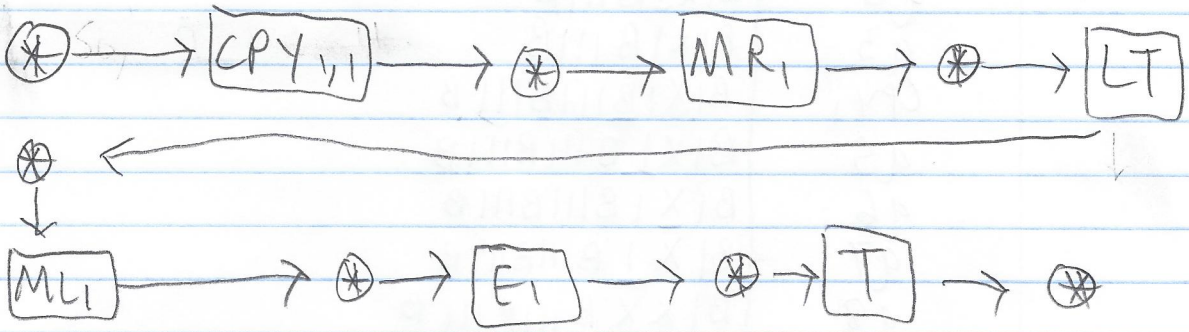


[q0B11B1B] [q0B]
[Bq111B1B] [Bq1B]
[BXXq1B1B] [B11q1B]
[BXXBq21B] [B1q21B]
[BXXBYq2B] [Bq21BB]
[BXXBq3YB] [q3BBBB]
[BXXq3BBB] [q3BBBB]
[BXq4XBBB] ...
[q4BBBBBB] ...
[Bq5BBBBB] ...
[qfB1B] [qfBBBB]

[q0B1B11B] [q0B]
[Bq11B11B] [Bq1B]
[BXq1B11B] [B1q1B]
[BXBq211B] [Bq21B]
[BXBYYq2B] [q2BBB]
[BXBYq6YB] [q6BBB]
[BXq6BBBB] [q6BBB]
[Bq7XBBBB] ...
[q7BBBBBB] ...
[Bq8BBBB] ...
[B1q9BBB] ...
[B1q10 1B] ...
[qfB11B] [qfBBB]

8) To make the gt(n,m) machine, I would simply use my Lt machine from problem 7.

Because $gt(n,m) = lt(m,n)$



Machine	Configuration
	B n B m B
CPY1,1	B n B m B n
MR1	B n B m B n
LT	B n B lt(m,n) B
ML1	B n B lt(m,n) B
E1	B B lt(m,n) B
T	B lt(m,n) B

9) a)

Machine	Config
q0	B 1 B 11111 B
q1	B 1 B 11111 B
q2	B 1 B 11111 B
E1	B 1 B B B
ML1	B 1 B B B

b)

Machine	Config
q0	B 11 B 1 B
q1	B 1 1 B 1 B
q2	B 1 1 B 1 B
q3	B 1 X B 1 B
CPY1	B 1 X B 1 B 1 B
q5	B 1 X B 1 B 1 B
q4	B 1 B B 1 B 1 B
q4	B B B B 1 B 1 B
T	B 1 B 1 B
E1	B B B 1 B
T	B 1 B

c)

Machine	Config
q_0	$\underline{B}111B11B$
q_1	$B\underline{1}11B111B$
q_2	$B1\underline{1}1B111B$
q_3	$B1X\underline{1}B111B$
q_3	$B1X1\underline{B}111B$
CPY_1	$B1X1\underline{B}111B111B$
q_5	$B1X\underline{1}B111B111B$
q_6	$B1\underline{X}1B111B111B$
q_7	$B1X\underline{1}B111B111B$
q_8	$B1XX\underline{B}111B111B$
CPY	$B1XX\underline{B}111B111B111B$
MR_1	$B1XX\ B111\ \underline{B}\ 111B111B$
A	$B1XXB111\underline{B}11111B$
ML_1	$B1XX\underline{B}111B11111B$
q_5	$B1X\underline{X}B111B11111B$
q_4	$B1\underline{X}BB111B11111B$
q_4	$B\underline{1}BBB111B11111B$
q_4	$\underline{B}BBBB111B11111B$
T	$\underline{B}111B11111B$
E_1	$\underline{B}BB1111B$
T	$\underline{B}11111B$

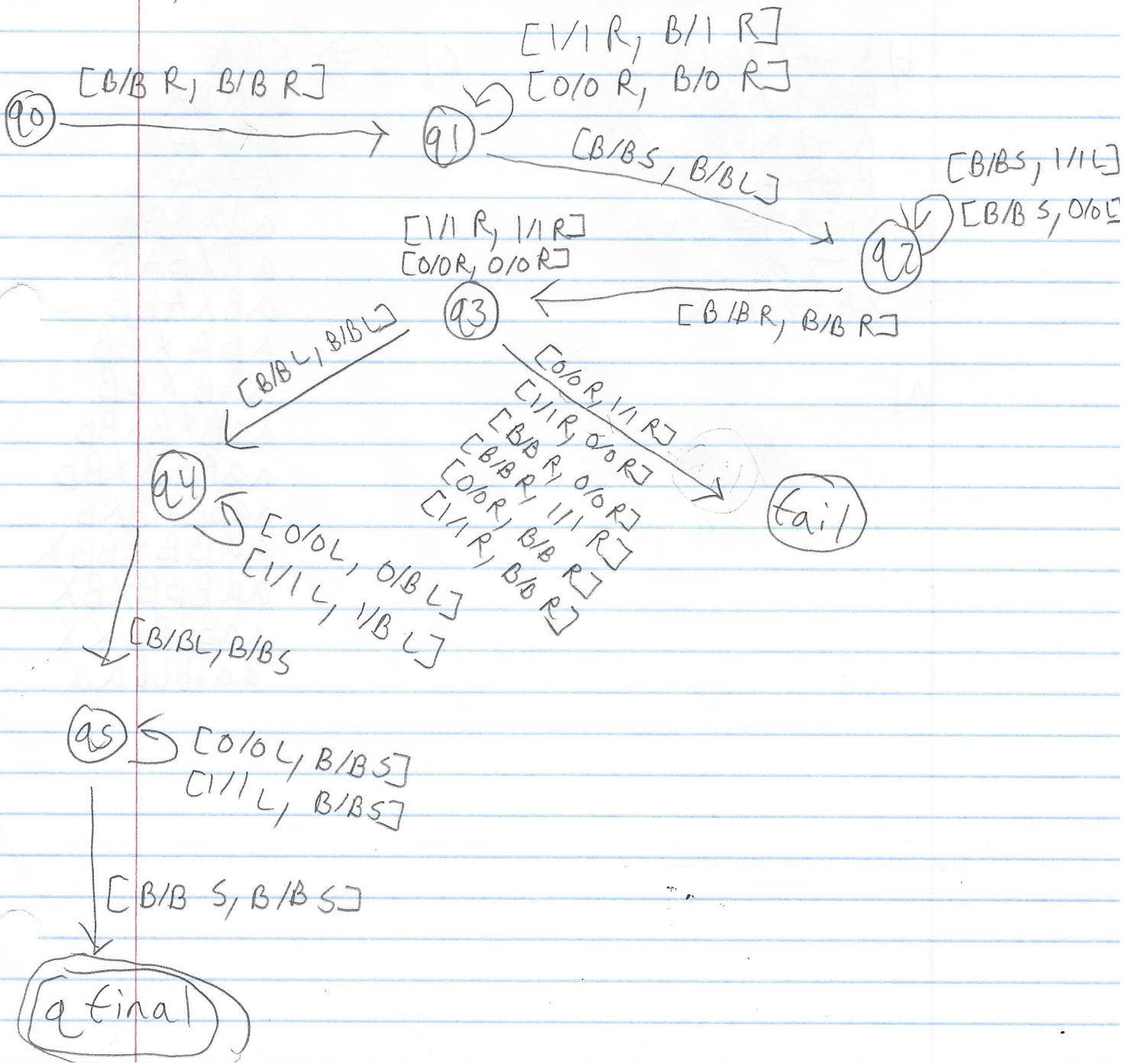
10)

11) a) $S \Rightarrow SBA$
aBA
aAB
$aaBB$
$aabB$
$aabb$

b) $L(G) = \{a^{i+1} b^{2i} \mid i \geq 0\}$

c) $S \rightarrow aSbb \mid a$

12)



13) a) $z = a^i b^{2i} a^i$

$uv = a's$

$w = a^i b^{2n} a^n$

If the left side a's are pumped then $uvw \notin L$.

b) $S \to SBA \mid X$

$BA \to AB$

$A \to aBX$

$B \to b$

$XB \to BX$

$X \to a$

$XA \to AX$

c) $S \Rightarrow SBA$

$SBABA$

$BABA$

$ABBA$

$aBXBBA$

$aBXBAB$

$aBXABB$

$aBAXBB$

$aABXBB$

$aaBXBXBB$

$aaBBXXBB$

$aaBBXBXB$

$aaBBXBBX$

$aaBBBXBX$

$aaBBBBXX$

$aabbbbaa$

d)