ch. 6

14) a) $z = a^n bb \, a^n$

   $uv = a's$
   $w = a^j bb \, a^n$

   $uv^0 w \notin L$ So, L is not regular.
   $uv \, w \in L$
   $uv^2 w \notin L$

b) $z = a^n b^m \mid n < m$

   $uv = a's$
   $w = a^n b^m$

   $uv^0 w \in L$
   $uv \, w \in L$
   $uv^2 w \notin L$ So, L is not regular.

c) $z = a^i b^j c^{2j} \mid i \geq 0, \, j \geq 0$

   $uv = ab$
   $w = a^n b^j c^{2j}$

   $uv^0 w \in L$
   $uv \, w \in L$
   $uv^2 w \notin L$ So, L is not regular

d) $z = a^n b \, a^n b$

   $uv = a's$
   $w = a^j b \, a^n b$

   $uv^0 w \notin L$
   $uv \, w \in L$
   $uv^2 w \notin L$ So, L is not regular.

e) $\lambda, a, ab, aba, abaa, abaab$

$z = abaab \ldots ba^{k-1} ba^k b$

Case 1: v has no b's. uv^0w has sequences of a's in which the second sequence has at most the same number of a's as its predecessor. $uv^0w \notin L$

Case 2: v has one b. $uv = a^s ba^t$, $z = ba^j ba^{j+1} b$. $uv^2w = ba^j ba^{s+t} ba^{j+1} b$, so $uv^2w \notin L$

Case 3: $uv = ba^t b$. $uv^2w = ba^t b ba^t b$, so $uv^2w \notin L$.

f) $z = a^{n^3} b$

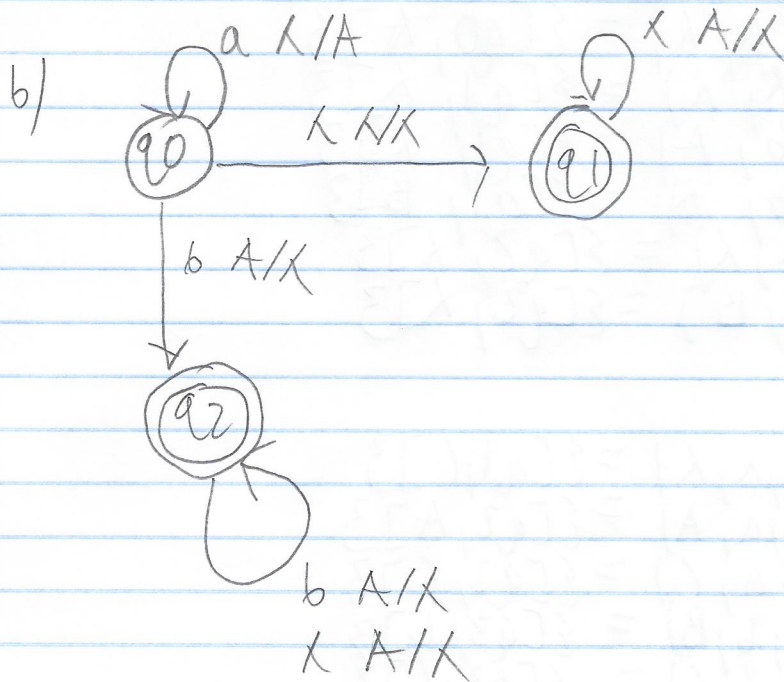$(n+1)^3 = n^3 + 3n^2 + 3n + 1$

$uv = a's$
$|uv| \leq n$

$uw \notin L$
$uv^2w = |uvw| + |v|$
$\quad \leq n^3 + n$

24)

27)

Chap 7/

a) $\{a^i b^j \mid 0 \leq j \leq i\}$

b)

c) $[q0, aab, \lambda]$ $[q0, abb, \lambda]$ $[q0, aba, \lambda]$
   $[q0, ab, A]$ $[q0, bb, A]$ $[q0, ba, A]$
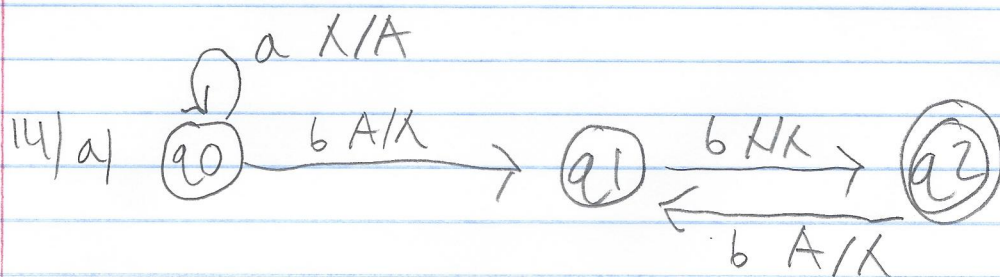 $[q0, b, AA]$ $[q2, b, \lambda]$ $[q2, a, \lambda]$
   $[q2, b, A]$ $[q2$
   $[q2, \lambda, \lambda]$ $\rightarrow$

d) [q0, aabb, λ]
   [q0, abb, A]
   [q0, bb, AA]
   [q2, b, A]
   [q2, λ, λ]

[q0, aaab, λ]
   [q0, aab, A]
   [q0, ab, AA]
   [q0, b, AAA]
   [q2, λ, AA]
   [q2, λ, A]
   [q2, λ, λ]

3) a) $\delta(q0, λ, λ) = \{[q0, λ]\}$
   $\delta(q0, a, λ) = \{[q1, A]\}$
   $\delta(q1, a, λ) = \{[q1, A]\}$
   $\delta(q1, b, λ) = \{[q2, λ]\}$
   $\delta(q2, b, λ) = \{[q2, λ]\}$
   $\delta(q2, λ, A) = \{[q2, λ]\}$

$Q = \{q0, q1, q2\}$
$F = \{q0, q2\}$

c) $(q0, a, λ) = \{[q0, A]\}$
   $(q0, λ, λ) = \{[q1, λ]\}$
   $(q1, b, A) = \{[q1, λ]\}$
   $(q1, b, λ) = \{[q1, B]\}$
   $(q1, λ, λ) = \{[q2, λ]\}$
   $(q1, c, B) = \{[q2, λ]\}$

d) $\delta(q0, λ, λ) = \{[q1, C]\}$
   $\delta(q1, a, A) = \{[q2, A]\}$
   $\delta(q1, a, C) = \{[q2, C]\}$
   $\delta(q1, b, B) = \{[q3, B]\}$
   $\delta(q1, b, C) = \{[q3, C]\}$
   $\delta(q1, a, B) = \{[q1, λ]\}$
   $\delta(q1, b, A) = \{[q1, λ]\}$
   $\delta(q1, λ, C) = \{[q5, λ]\}$
   $\delta(q2, λ, λ) = \{[q1, A]\}$
   $\delta(q3, λ, λ) = \{[q4, B]\}$
   $\delta(q4, λ, λ) = \{[q1, B]\}$

j)

14) a)

b) $\{a^i b^{2i} \mid i \geq 1\}$

d) $[q0, aabbbb, \lambda]$
$[q0, abbbb, A]$
$[q0, bbbb, AA]$
$[q1, bbb, A]$
$[q2, bb, A]$
$[q1, b, \lambda]$
$[q2, \lambda, \lambda]$

17) a) a^{k^2}

$\text{length}(z) = \text{length}(uv^2wx^2y)$
$= \text{length}(uvwxy) + \text{length}(u) + \text{length}(v)$
$= k^2 + \text{length}(u) + \text{length}(v)$
$\leq k^2 + k$
$< (k+1)^2$