Formal Lang : Midterm

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1) a) $f(n) = 2n$ b) $f(0) = 0$, otherwise $f(n) = n-1$

c) $f(0) = 1$
$f(1) = 0$
otherwise $f(n) = n$

d) $f(n) = \frac{n}{2}$ if n is even

2) To be an equivalence relation $(<)$ must be symmetric, transitive, and reflexive.

$(<)$ is not symmetric.

Assume $a < b$.
To be symmetric $b < a$.
This can't be true because $a < b$.
So, $(<)$ is not symmetric. Therefore $(<)$ is not an equivalence relation.

3) Basis: $[1,0] \in GT$

Recursive: If $[m,n] \in GT$, then $[s(m), n] \in GT$ and $[s(m), s(n)] \in GT$.

Closure: $[m,n] \in GT$ only if it can be obtained from $[1,0]$ by a finite number of applications of the recursive step.

$\longrightarrow$

4) BC: LHS = $3n-1$ RHS = $n(3n+1)/2$

$3(1)-1$ $1(3(1)+1)/2$

2 2

IH: Assume $\underbrace{\sum_{i=1}^{n} 3n-1}_{LHS} = n(3n+1)/2 == \underbrace{\left(\frac{3n^2+n}{2}\right)}_{RHS}$

Prove for $k+1$

LHS = $\sum_{i=1}^{k+1} 3k-1 = \sum_{i=1}^{k} 3k-1 + (k+1)$ RHS = $\frac{3(k+1)^2+(k+1)}{2}$

$\xrightarrow{IH} \frac{3k^2+k}{2} + 3(k+1)-1$ RHS = $\frac{3k^2+7k+4}{2}$

LHS = $\frac{3k^2+k}{2} + 3k+2$

$\frac{3k^2+k}{2} + \frac{6k+4}{2}$

LHS = $\frac{3k^2+7k+4}{2}$

LHS == RHS

Therefore, $\sum_{i=1}^{n} 3n-1 = \frac{n(3n+1)}{2}$ for all $n > 0$.

5) Basis: $\lambda \in L$

Recursive: If $u \in L$ then $uauaubu \in L$, $uaubuau \in L$, $ubuaua \in L$

Closure: A string u is in L only if it can be obtained from λ using a finite number of applications of the recursive step.

6) i) $L_0 = aab$
$L_1 = aaaabb$
$L_2 = aaaaaabbb$

ii) $\{(aa)^n (aab)^m b^n\}$ $n > 0$, $m \geq 0$

iii) BC: $L_0 = aab$
$n_a(u)$ is twice $n_b(u)$

IH: Assume $n_a(u)$ is twice $n_b(u)$ for L_n

Prove $n_a(u)$ is twice $n_b(u)$ for $L_{(n+1)}$

So, the base case is aab. Any other string in the language is a finite number of applications of the recursive step aaub. Because, our base case has twice as many a's as b's and our recursive case has twice as many a's as b's, Any number of Recursive steps from the base case will have twice as many a's as b's. Therefore, $L_{(n+1)} = n_a(u)$ is twice $n_b(u)$

7) a) $\{(aa)^n b^m\}$
$n \geq 0$ and ($m = 0$ if $n = 0$, otherwise $m \geq 1$)

b) $\{a^n c^m (bb)^n\}$ $1 \leq n \leq m$

c) $\{(ab)^n (cd)^m (ba)^m (dc)^n\}$ $n \geq 1$, $m \geq 0$

d) $\{a^n c^m a^L b^L d^m b^n\}$ $1 \leq n \leq m \leq L$

e) $\{a^n b^m\}$ $1 \leq n \leq m$

$\rightarrow$

8) $S \to aA \mid bA \mid bB \mid bA_2 \mid \lambda$
$A \to aB \mid \lambda$
$A_2 \to aA \mid \lambda$
$B \to bB \mid \lambda$

9) $L_0 = b$ $\{a^n b^m \mid 0 \le n < m\}$
$L_1 = abb, bb$

So, the only string with one recursive case is "b". So, m must be $m > 0$.

"a" isn't in the first recursive case, but is in the second, so n must be $n \le 0$.

There are always more b's than a's So, $m > n$.

Therefore $\{a^n b^m \mid 0 \le n < m\}$ for those two cases.

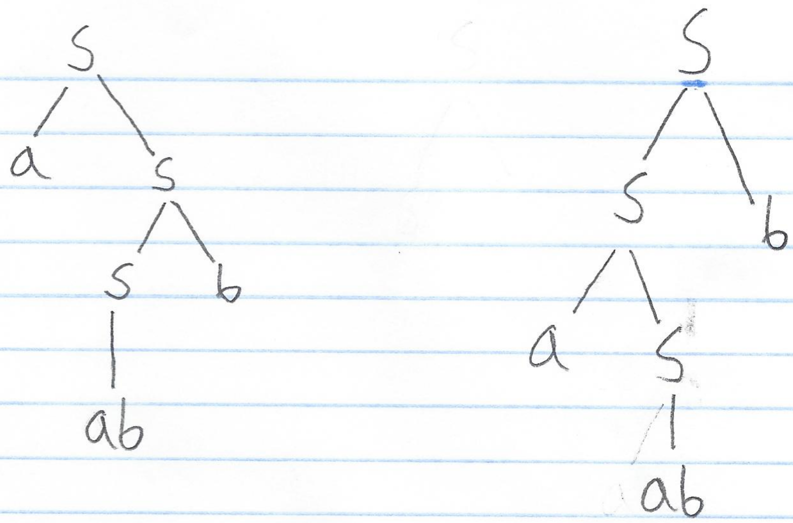
for n+1 Recursive cases you must always terminate with B. So, m will always be $> n$.

10) a) $a^*(ab)^+ b^*$

b) $S \Rightarrow aS$ $S \Rightarrow Sb$
asb aSb
$aabb$ $aabb$

$\longrightarrow$

c)



d)

$S \to aA \mid Bb \mid C$

$A \to aA \mid C$

$C \to abC \mid B$

$B \to Bb \mid \lambda$