Ch. 1

1) a) $X \cup Y = \{0,1,2,3,4,6\}$ b) $X \cap Y = \{2,4\}$

c) $X - Y = \{1,3\}$ d) $Y - X = \{0,6\}$

e) $P(X) = \{\emptyset, \{1\}, \{2\}, \{3\}, \{4\}, \{1,2\}, \{2,3\}, \{3,4\}, \{1,4\}, \{1,3\}, \{2,4\}, \{1,2,3\}, \{1,2,4\}, \{2,3,4\}, \{1,3,4\}, \{1,2,3,4\}\}$

4) $X = (n_0)^3 + 3(n_0)^2 + 3n_0$

6) a) $f(n) = 2n$ b) $f(n) = \begin{cases} 0 & \text{if } n=0 \\ n-1 & \text{otherwise} \end{cases}$

c) $f(n) = \begin{cases} 1 & \text{if } n=0 \\ 0 & \text{if } n=1 \\ n & \text{otherwise} \end{cases}$ d) $f(n) = \begin{cases} n/2 & \text{if } n \text{ is even} \\ \uparrow & \text{otherwise} \end{cases}$

10) $n = \{1,2,3,4 \ldots\}$ $m = \{1,2,3,4 \ldots\}$

Refexive: if $n=m$ $a \equiv a$ for all $a \in n$ or m

Symmetric: if $n=m$, $m=n$ $a \equiv b$, $b \equiv a$ for all $a/b \in n$ or m

Transitive: if $n=m$, $m=n$,

III. 22) Assume that the set of monotone increasing functions is countable. Then these functions can be listed as $f(0), f(1), f(2) \ldots, f(n) \ldots$

Now consider the function as follows:

$f(0) = f_0(0) + 1$ for $i > 0$

$f(i) = f_i(i) + f(i-1)$

Since $f_i(i) > 0$, it follows that $f(i) > f(i-1)$ for all i and f is monotone increasing.

Clearly $f(i) \neq f_i(i)$ for any i, contradicting the assumption that $f(0), f(1), f(2) \ldots f(n) \ldots$ exhaustively enumerates the monotone increasing functions. Therefore, we can conclude that the set is uncountable.

29) Basis: If $n = 0$ and $m = 0$, then $m = n$.

Recursive: $S(n) = S(m)$

closure: $m = n$ only if m an n are given the same number of applications of the operation S.

34) Basis:

Recursive:

closure:

38) Basis: $n(1) = 2$ $\quad 3(1) - 1 = 2$ $\quad \frac{1(3(1)+1)}{2} = 2$

IH: Assume for all $k > 0$ that

$$\sum_{i=0}^{k} i = \frac{k(3k+1)}{2} = \frac{3k^2+k}{2}$$

Prove: $$\sum_{i=0}^{n+1} n = \frac{n+1(3(n+1)+1)}{2} = \frac{3n^2+n}{2} + (n+1)$$

$(3n+4)(n+1)$

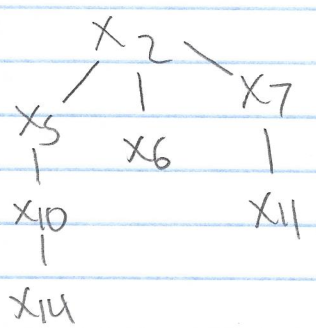
$3n^2 + 7n + 4$

42)

46) a) 4   b) $\{X_{11}, X_7, X_2, X_1\}$

c) X_{14} and $X_{11} = X_2$, X_{15} and $X_{11} = X_1$

d)



e) $\{X_{14}, X_6, X_{11}, X_3, X_8, X_{12}, X_{15}, X_{16}\}$