Ch.1

14) Reflexive: if $a \in X$, $a \equiv a$

Symmetric: if $a \in X$ and $b \in X$ then (a,b) and (b,a) are in the partitions of X.

Transitive: if $a \in X$ and $b \in X$ and $c \in X$ then (a,b), (b,c) and (c,a) are in the partitions of X.

20) Assume $f_i(x)$ from N to $\{0,1\}$ is countable

	1	2	3	4	5	6	7
1	$f_1(1)$	$f_1(2)$	$f_1(3)$	$f_1(4)$			
2	$f_2(1)$	$f_2(3)$	$f_2(4)$				
3							
4							
5							
6							
7							

Now consider the function $f_{(i)}\, X = (f_{i-1}) \pm 1$

So, that $\pm$ == + or − to get 1 or 0.

This function is not on the graph, there the number of total functions is not countable.

30. Basis: $[1,0] \in GT$

Recursive: If $[m,n] \in GT$, then $[s(m), n] \in GT$ and $[s(m), s(n)] \in GT$.

→

Closure: $[m,n] \in GT$ only if it can be obtained from $[1,0]$ by a finite number of applications of the operations in the recursive step.

33. Basis: if $n=0$ then $m \cdot n = 0$

Recursive: $m \cdot S(n) = m + (m \cdot n)$

Closure: $m \cdot n = K$ only if this equality can be obtained from $m \cdot 0 = 0$ using finitely many applications of the recursive step.

40. Base Case: $LHS = 1+2^3 = 9$ $RHS = 3^3 = 27$

$LHS < RHS$ for $n=3$

IH: Assume $1+2^n < 3^n$ for n where $n > 2$.

Prove: for $n+1$

$LHS = 1+2^{n+1}$

$1+2^n \cdot 2 < 2 \cdot 3^n$ (IH)

$RHS = 3^{n+1}$

$3 \cdot 3^n$

Since $LHS < 2 \cdot 3^n$, $LHS < RHS$ ($RHS = 3 \cdot 3^n$)

So, the statement $1+2^{n+1} < 3^{n+1}$ is True when $n > 2$.

Therefore the statement is true for all $n > 2$.

47. Base case: n leaves $= 2n-1$ nodes

1 leaf $=$ 1 node 

IH: Assume n leaves $= 2n-1$ nodes

Prove for $n+1$