Assignment 9

5.4

2) a) $(x+y)(x+y)(x+y)(x+y)(x+y)$

$x^5 + x^4y + x^3y^2 + x^2y^3 + xy^4 + y^5$

$x^5 + 5x^4y + 10x^3y^2 + 10x^2y^3 + 5xy^4 + y^5$

$\binom{5}{5}\ \binom{5}{4}\ \binom{5}{3}\ \binom{5}{2}\ \binom{5}{1}\ \binom{5}{0}$

1 5 10 10 5 1

b) $\binom{5}{0}x^5 + \binom{5}{1}x^4y + \binom{5}{2}x^3y^2 + \binom{5}{3}x^2y^3 + \binom{5}{4}xy^4 + \binom{5}{5}$

$x^5 + 5x^4y + 10x^3y^2 + 10x^2y^3 + 5xy^4 + y^5$

4) $\binom{13}{8} = \frac{13!}{8!(5!)} = \frac{13 \cdot 12 \cdot 11 \cdot 10 \cdot 9}{5!} =$ **1287**

12) 1 10 45 120 210 252 210 120 45 10 1

1 11 55 165 330 462 462 330 165 55 11 1

5.5

2) $5^5 =$ **3,125** 4) $6^7 =$ **279,936**

6) $C(3+5-1, 5) = C(7,5) = \frac{7!}{5!(2!)} = \frac{7 \cdot 6}{2} =$ **21**

8) $C(21+12-1, 12) = C(32,12) = \frac{32!}{12!(20!)} = \frac{32 \cdot 31 \cdot 30 \cdot 29 \cdot 28 \cdot 27 \cdot 26 \cdot 25 \cdot 24 \cdot 23 \cdot 22}{12 \cdot 11 \cdot 10 \cdot 9 \cdot 8 \cdot 7 \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2}$

$=$ **225,792,840**

14) $C(4+17-1, 17) = C(20,17) = \frac{20!}{17!(3!)} =$ **1140**

16) a) $x_i \geq 2$, $y_i = x_i - 2$

$(y_1+2)+(y_2+2)+(y_3+2)+(y_4+2)+(y_5+2)+(y_6+2) = 29$

$y_1+y_2+y_3+y_4+y_5+y_6 = 17$

$C(6+17-1, 17) = C(22,17) = \frac{22!}{17!(5!)} = \boxed{26,334}$

b) $y_1 = x_1 - 1$, $y_2 = x_2 - 2$, $y_3 = x_3 - 3$, $y_4 = x_4 - 4$, $y_5 = x_5 - 5$, $y_6 = x_6$

$(y_1+1)+(y_2+2)+(y_3+3)+(y_4+4)+(y_5+5)+(y_6+6) = 29$

$y_1+y_2+y_3+y_4+y_5+y_6 = 8$

$C(6+8-1, 8) = C(13,8) = \frac{13!}{8!(5!)} = \boxed{1287}$

c) $C(6+29-1, 29) = C(34,29) = 278,256$

$x_1 \geq 6 \quad C(6+23-1, 23) = C(28,23) = -98,280$

$\boxed{179,976}$

d) $x_2 \geq 9 \quad C(6+20-1, 20) = C(25,20)$

$x_1 \geq 8, x_2 \geq 9 \quad C(6+12-1, 12) = C(17,12)$

$C(25,20) - C(17,12) = \boxed{46,942}$

44) a) $C(4+12-1, 12) = C(15,12) = \boxed{455}$

b) shelves = | | |

books = * * (12)

$4 \times 5 \times 6 \times 7 \times \ldots = \boxed{\frac{15!}{3!}}$

6.1

2) 1/6 4) 30/366 8) $\frac{1}{52}+\frac{1}{52}+\frac{1}{52}+\frac{1}{52}+\frac{1}{52}=\boxed{\frac{5}{52}}$

16) $\frac{C(4,1)\cdot C(13,5)}{C(52,5)}=\frac{33}{16,660}$

30) $\frac{C(6,5)\cdot C(34,1)}{C(40,6)}=\frac{17}{319,865}$

38)

Toss	E1	E2	E3
T_0	T	T	T
T_1	T	T	H
T_2	T	H	T
T_3	T	H	H
T_4	H	T	T
T_5	H	T	H
T_6	H	H	T
T_7	H	H	H

a) $E1=\{T_0,T_1,T_2,T_3\}$ $E2=\{T_2,T_3,T_6,T_7\}$ $p(E_1\cap E_2)=p(T_2,T_3)=p(E_1)\cdot p(E_2)=\frac{1}{4}$ $\frac{1}{2}\cdot\frac{1}{2}$

So, E1 and E2 independent

b) $E_1=\{T_0,T_1,T_2,T_3\}$ $E_2=\{T_3,T_6\}$ $p(E_1\cap E_2)=p(T_3)=\frac{1}{8}$ $p(E1)\cdot p(E2)=\frac{1}{8}$ $\frac{1}{2}$ $\frac{1}{4}$

So, E1 and E2 independent

c) $E_1 = \{T_0, T_1, T_4, T_5\}$ $E_2 = \{T_3, T_6\}$ $P(E_1 \wedge E_2) = 0$

E1 and E2 not independent

6.2

2) $P(1) = P(2) = P(4) = P(5) = P(6) = 1/7$; $P(3) = 2/7$

b) {1,2,3} {1,3,2} {2,1,3} {2,3,1} {3,1,2} {3,2,1}

a) $3/6 = \boxed{\frac{1}{2}}$ b) $\boxed{\frac{1}{2}}$ c) $2/6 = \boxed{1/3}$

8) a) $P(n,2)/2$ to choose positions for 1,2 where 1 precedes
$(n-2)!$ to arrange the rest.
$n!$ permutations

$$\frac{1}{2} \cdot \frac{n!}{(n-2)!} \cdot \frac{(n-2)!}{n!} = \boxed{\frac{1}{2}}$$

b) $\boxed{\frac{1}{2}}$

c) $\frac{(n-1)!}{n!} = \boxed{\frac{1}{n}}$

d) $P(n,2)/2$ for n precede 1
$P(n-2,2)/2$ for n-1 precede
$(n-4)!$ ways to arrange the rest

$$\frac{\frac{1}{4} \cdot \frac{n!}{(n-2)!} \cdot \frac{(n-2)!}{(n-4)!} \cdot (n-4)!}{n!} = \boxed{\frac{1}{4}}$$

e) $P(n,3)/3$ for n precede 1,2
$(n-3)!$ to arrange rest

$$\frac{\frac{1}{3} \frac{n!}{(n-3)!} \cdot (n-3)!}{n!} = \boxed{\frac{1}{3}}$$

10) a) $P(26,13)$ to get alphabetical permutations $\quad \frac{26!}{13!} = \boxed{\frac{1}{13!}}$
$26!$

b) $\boxed{\frac{24!}{26!}}$ c) az, za 2 ways $\quad \frac{2 \cdot 25!}{26!} = \boxed{\frac{1}{13}}$

d) a/d adjacent $= \frac{1}{13} \quad \frac{13}{13} - \frac{1}{13} = \boxed{\frac{12}{13}}$

e) a,z (23) (1,25/2,26) 4 ways $\quad \frac{6 \cdot 24!}{26!} = \boxed{\frac{6}{26 \cdot 25}}$
a,z (24) (1,26) 2 ways

f) $P(26,3)/3$ for z precede a/b $\quad \frac{1}{3} \cdot \frac{26!}{23!} \cdot 23! = \boxed{\frac{1}{3}}$
$\cdot 23!$ for the rest
$26!$

12) $P(E \cup F) = p(E) + p(F) - p(E \wedge F)$
$P(E \cup F) + p(E \wedge F) = 1.4$
$1.4 = p(E \cup F) + p(E \wedge F) \leq 1 + p(E \wedge F)$
$+1 \qquad\qquad -1$
$\boxed{p(E \wedge F) \geq 0.4}$

$p(E \wedge F) \leq P(F) = 0.6$
$1.4 = p(E \cup F) + p(E \wedge F) \leq p(E \cup F) + 0.6$
$-0.6 \qquad\qquad -0.6$
$\boxed{p(E \cup F) \geq 0.8}$

18| a) $\frac{1}{7} \cdot \frac{1}{7} = \frac{1}{49}$ $C(7,1) = 7$ $\frac{1}{49} \cdot 7 = \boxed{\frac{1}{7}}$

b| 7^n $p(\text{distinct| days} = \frac{C(7,n)\,n!}{7^n}$

$$\boxed{p(\text{same}) = 1 - \frac{C(7,n)\,n!}{7^n}}$$

c)

n	probability of two the same for n ppl
1	0
2	0.1429
3	0.3878
4	0.6501

use formula from b

4 people