Asn 8

$\boxed{4.1}$

4) a) $P(1) = 1^3 = 1$

b) $1^3 = 1 \quad \left(\frac{1(1+1)}{2}\right)^2 = \left(\frac{2}{2}\right)^2 = 1$

c) Assume $1^3 + 2^3 + \ldots + n^3 = \left(\frac{n(n+1)}{2}\right)^2$ for $n \geq 1$

d) Prove $1^3 + 2^3 + \ldots + n^3 + (n+1)^3 = \left(\frac{(n+1)(n+2)}{2}\right)^2 = \frac{n^4}{4} + \frac{3}{2}n^3 + \frac{13}{4}n^2 + 3n + 1$

e) $\left(\frac{n(n+1)}{2}\right)^2 + (n+1)^3$

$\frac{n^4}{4} + \frac{1}{2}n^3 + \frac{n^2}{4} + n^3 + 3n^2 + 3n + 1$

$\frac{n^4}{4} + \frac{3}{2}n^3 + \frac{13}{4}n^2 + 3n + 1$

f) These steps show that for any positive integer n the formula works because induction proved that the formula works for any n and n+1 (so any n where $n \geq 1$).

10) a) $\frac{1}{1 \cdot 2} + \frac{1}{2 \cdot 3} + \ldots + \frac{1}{n(n+1)} = \frac{n}{n+1}$

b) $\frac{1}{1(1+1)} = \frac{1}{2} \quad \frac{1}{1+1} = \frac{1}{2}$ BC $n = 1$ ✓

IH Assume $\frac{1}{1 \cdot 2} + \frac{1}{2 \cdot 3} + \ldots + \frac{1}{n(n+1)} = \frac{n}{n+1}$ for $n \geq 1$

$\rightarrow$

$$\text{LHS} \qquad\qquad = \quad \text{RHS}$$

Prove $\underbrace{\frac{1}{1\cdot 2}+\frac{1}{2\cdot 3}+\dots+\frac{1}{n(n+1)}} + \frac{1}{n+1(n+2)} = \frac{n+1}{n+2}$

$$\frac{n}{n+1} + \frac{1}{(n+1)(n+2)}$$

$$\frac{n(n+2)}{(n+1)(n+2)} + \frac{1}{(n+1)(n+2)}$$

$$\frac{n(n+2)+1}{(n+1)(n+2)} = \frac{n^2+2n+1}{(n+1)(n+2)} = \frac{\cancel{(n+1)}(n+1)}{\cancel{(n+1)}(n+2)}$$

$$\frac{n+1}{n+2} = \text{RHS}$$

32) $3 \mid 1^3 + 1\cdot 2$

$3 \mid 3$ $\qquad n=1$ BC ✓

$n^3+2n=3m$

IH Assume for $n\geq 1$ that $3 \mid n^3+2n$

Prove $3 \mid (n+1)^3+2(n+1)$

$$n^3+3n^2+3n+1+2n+2=3m$$

$$n^3+3n^2+5n+3$$

$$n^3+3n^2+2n+3n+3$$

$$n^3+2n+3(n^2+n+1)$$

$$3 \mid n^3+2n \qquad 3 \mid 3(n^2+n+1)$$

$$3 \mid n^3+2n+3(n^2+n+1)$$

44) BC. $P(2) = (A_1 - B) \cup (A_2 - B)$ $\qquad (A_1 \cup A_2) - B$

$(A_1 \cap \overline{B}) \cup (A_2 \cap \overline{B})$

$(A_1 \cup A_2) \cap \overline{B}$ ✓

$(A_1 \cup A_2) - B$

IH. Assume $(A_1 - B) \cup (A_2 - B) \cup \ldots \cup (A_n - B) =$

$(A_1 \cup A_2 \cup \ldots \cup A_n) - B$

for $n \geq 2$

RC. prove for $n+1 = (A_1 \cup A_2 \cup \ldots \cup A_n \cup A_{n+1}) - B$

$(A_1 \cup A_2 \cup \ldots, \cup A_n) - B \quad \cup (A_{n+1} - B)$

$(A_1 \cup A_2 \cup \ldots, \cup A_n) \cap \overline{B} \quad \cup (A_{n+1} \cap \overline{B})$

$(A_1 \cup A_2 \cup \ldots, \cup A_n \cup A_{n+1}) \cap \overline{B}$

$(A_1 \cup A_2 \cup \ldots, \cup A_n \cup A_{n+1}) - B$

$\boxed{4.2}$

4) a) $P(18) = 2(7) + 1(4) \qquad P(19) = 1(7) + 3(4)$

$P(20) = 0(7) + 5(4) \qquad P(21) = 3(7) + 0(4)$

b) $P(j)$ is true for $18 \leq j \leq k$ where k is an integer with $k \geq 21$

c) Prove that $P(k+1)$ is true

d) $P(k-3)$ True because $k-3 \geq 18$

$P(k+1)$ True because you only need to add a 4-cent stamp to the stamps used in the postage of $k-3$ cents

e) the proof showed that the statement is true for every integer n greater than or equal to 18.

8) P(j) true for $145+n$ where n is an integer

BC. $P(0) = 3(40) + 1(25)$ $P(1) = 0(40) + 6(25)$
$P(2) = 2(40) + 3(25)$ $P(3) = 4(40) + 0(25)$
$P(4) = 1(40) + 5(25)$

IH. P(j) true for $145 \leq 145 + 5j \leq 145 + 5k$ where k is an integer with $k \geq 4$.

RC. Prove $P(k+1)$

$P(k-4)$ true because $145 + 5(k-4) \geq 145$

For $P(k+1)$ you only need to add another 25 dollars to the dollars used in the dollars for $k-4$.
So, $P(k+1)$ true

14)

$\boxed{4.3}$

4) a) $f(2) = 0$ $f(3) = -1$ $f(4) = -1$ $f(5) = 0$

b) $f(2) = 1$ $f(3) = 1$ $f(4) = 1$ $f(5) = 1$

c) $f(2) = 2$ $f(3) = 5$ $f(4) = 33$ $f(5) = 1214$

d) $f(2) = 1$ $f(3) = 1$ $f(4) = 1$ $f(5) = 1$

8) a) $a_{n+1} = a_n + 4$, for $n \geq 1$ $a_1 = 2$

b) $a_{n+1} = a_n + 2(1 - a_n)$, for $n \geq 1$ $a_1 = 0$

c) $a_{n+1} = a_n + 2(n+1)$, for $n \geq 1$ $a_1 = 2$

d) $a_{n+1} = a_n + 2n + 1$ for $n \geq 1$ $a_1 = 1$

24) a) $a_{n+1} = a_n + 2$ for $n \geq 1$, $a_1 = 1$

b) $a_{n+1} = a_n \cdot 3$ for $n \geq 1$, $a_1 = 1$

c) $a_{n+1} = \{xy + z$, x, z integers, y an element of $a_n\}$

$a_1 = \{xy + z$, x, z integers$\}$

44) BC: $\bullet$ $l(T)=1$ $i(T)=0$

IH: Let T be a tree where $l(T)\geq 2$

$l(T)=l(T_1)+l(T_2)$

$i(T)=i(T_1)+i(T_2)+1$

RC: Prove $l(T)=i(T)+1$

$l(T_1)=i(T_1)+1$ $l(T_2)=i(T_2)+1$

$l(T)=i(T_1)+i(T_2)+2$

$l(T)=i(T)+1$