Asn2   1.4: 2, 4, 8
1.5: 2, 4, 10
1.6: 2, 4, 10, 18, 20

Sem 11   A - 110S

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1.4

2) a) There exists an x for all real numbers of y where the product of x and y equals y.

b) For all real numbers of x and y, if x is greater than or equal to zero and y is less than or equal to zero then x minus y is greater than zero.

c) For all real numbers of x and y, there exists a z where x equals the sum of y and z.

4) a) There is some student in your class who has taken a computer science course at your school.

b) There is some student in your class who has taken every computer science course at your school.

c) Every student in your class has taken atleast 1 computer science course at your school.

d) There is a computer science course at your school that has been taken by every student in your class.

e) Every computer science course at your school has been taken by atleast 1 student in your class.

f) Every student in your class has taken every computer science course at your school.

8) a) $\exists x \exists y\, Q(x,y)$ b) $\forall x \exists y\, \neg Q(x,y)$

c) $\exists x\, (Q(x, \text{Jeopardy}) \wedge Q(x, \text{Wheel of Fortune}))$

d) $\forall y \exists x\, Q(x,y)$ e) $\exists x\, Q(2x, \text{Jeopardy})$

1.5

2) Modus Tollens
Valid

$$\begin{array}{l} p \to q \\ \underline{\neg q} \\ \neg p \end{array}$$

the conclusion is true because the premises are true

4) a) $\dfrac{p \wedge q}{q}$ Simplification

b) $\begin{array}{l} p \vee q \\ \underline{\neg p} \\ q \end{array}$ Disjunctive syllogism

c) $\begin{array}{l} p \\ \underline{p \to q} \\ q \end{array}$ Modus Ponens

d) $\dfrac{p}{p \vee q}$ Addition

e) $\begin{array}{l} p \to q \\ \underline{q \to r} \\ p \to r \end{array}$ Hypothetical Syllogism

10) a) 1) $p \to q$
2) $q \to r$
3) $\underline{\neg r}$
4) $\neg q \to \neg r$ Contra (2)
5) $\neg q$ Modus Ponens (4,3)

"I am not sore"
"I did not play hockey"

b) 1) $p \to (q \vee r)$
2) p on M∨F
3) $\neg q$ on T
4) $\underline{\neg r \text{ on } F}$

No valid conclusions can be made

c) 1) $p \to q$
2) $D = p$
3) $S = \neg q$

4) $= \neg q \to \neg p$ contra(1)
5) $D = q$ Modus Ponens (1)
6) $S = \neg p$ Modus Tollens(3,4)

"Dragon flies have six legs"
"Spiders are not insects"

d) 1) $p \to q$
2) $H = \neg q$
3) $M = q$

4) $\neg q \to \neg p$ contra(1)
5) $H = \neg p$ Modus Tollens (2,4)

"Homer is not a student"

e) 1) $p \to q$
2) $T = p$
3) You $= \neg q$
4) You $= \neg T$
5) $C = \neg p$

6) $\neg q \to \neg p$ contra(1)
7) You $= \neg p$ Modus Tollens(3,6)

"You only eat unhealthy food"
"Tofu doesn't taste good"

f) 1) $p \vee q$
2) $\neg p$
3) $q \to r$

4) q Disjunctive syllogism (1,2)
5) r Modus Ponens (3,4)

"I am hallucinating"
"I see elephants running down the road."

$\boxed{1.6}$

2) n(even) + n(even) $\longrightarrow$ n(even)

$2x + 2x$

$4x$

$2(2x)$

Therefore, the sun of two even integers is even.

4) n(even) $\longrightarrow \pm$ n (even)

$2x$ $\quad -2x$

$-2x$ $\quad 2x$

$2(-x)$ $\quad 2x$

Therefore, the additive or negative inverse of an even number is even.

10) n(rational) · n(rational) → n(rational)

$\frac{p}{q} \cdot \frac{r}{s} = \frac{pr}{qs}$ $qs \neq 0$

Therefore, the sum of two rational numbers is rational.

18) a) n(odd) → (3n+2 (odd)

$3(2x+1)+2$

$6x+3+2$

$6x+4+1$

$2(3x+2)+1$

Therefore, if 3n+2 is even, n is even.

b) n(odd) → 3n+2 (odd) 3n+2 odd $\neg p$

$2(2x+1)+2$

$6x+3+2$ $(\neg p)$ 3n+2 odd, 3n+2 even (p)

$6x+4+1$

$2(3x+2)+1$

Therefore, if 3n+2 is even, n is even.

20) n(pos) → $n^2 \geq n$

$P(1) \rightarrow 1^2 \geq 1$

$P(1) \rightarrow 1 \geq 1$

$\boxed{P(1) = \text{True, Direct Proof}}$