Assignment 13

$\boxed{8.4}$

2) $\{(a,b) \mid a \neq b\} \cup \{(a,b) \mid a = b\}$

4) The reflexive closure of a relation is the relation and the elements (a,a) that were not already in the relation, So, just add all the possible self-loops to the graph.

12) $R \cup \Delta = M_{R \cup \Delta} = M_R \vee \Delta = M_R \vee I_n$

22) $R^* = R \cup R^2 \cup R^3 \cup \ldots \cup R^n$

If R is reflexive then R^2 will be, R^3 will be, and etc. So, R^* is reflexive.

26) a) $M_{R^*} = \begin{bmatrix} 1 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 1 & 0 \\ 1 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 & 0 \end{bmatrix}$

b) $M_{R^*} = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 & 1 \\ 0 & 1 & 1 & 0 & 1 \\ 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 & 1 \end{bmatrix}$

c) $M_{R^*} = \begin{bmatrix} 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 \end{bmatrix}$

d) $M_{R^*} = \begin{bmatrix} 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 \end{bmatrix}$

$\boxed{8.5}$

2) a) equivalence relation

b) equivalence relation

c) Not transitive

d) not transitive

e) Not transitive

6) 1) $R = \{(a,b) \mid$ a and b start before noon or a and b start after noon. $\}$

Two equivalence classes: classes that start before noon and classes that start after

2) $R = \{(a,b) \mid$ a and b have class the same amount of days per week $\}$

5 equivalence classes: 1 day of class, 2 days, 3 days, 4 days, 5 days

3) $R = \{(a,b) \mid$ a and b are taught by the same teacher $\}$

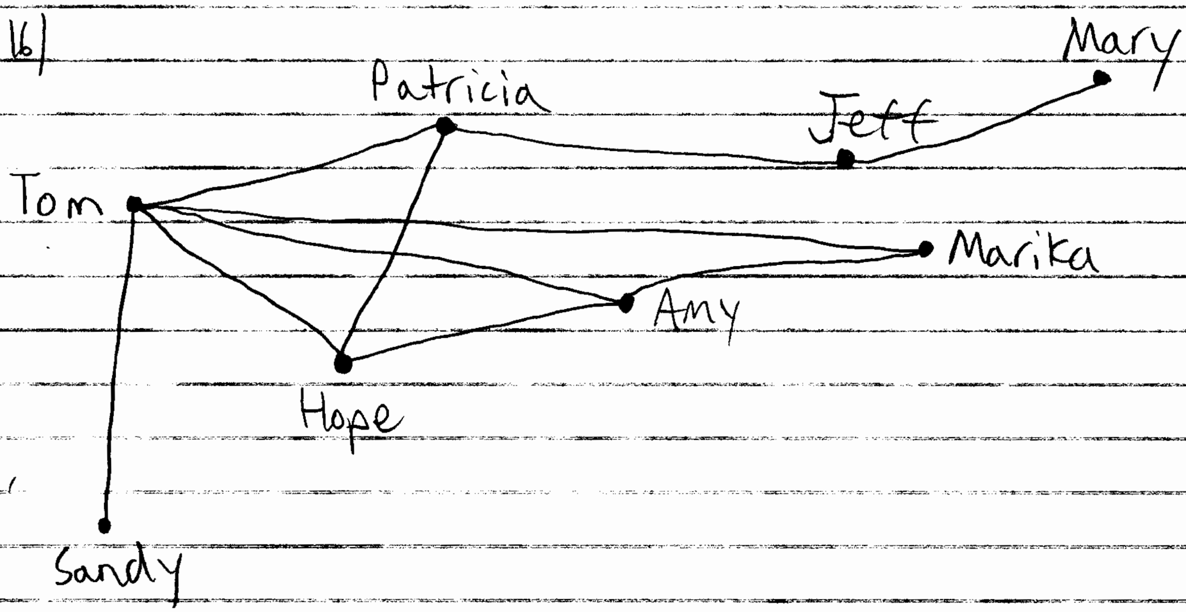
Infinite equivalence classes: Taught by x, Taught by y, Taught by w, etc...

$\boxed{9.1}$

2) a) Simple Graph b) Multigraph c) Pseudograph

4) undirected edges, multiple edges, no loops
$\boxed{\text{Multigraph}}$

16)



20) Team 4 beat: Team 3

Beat Team 4: Team 1, Team 2, Team 5, Team 6

$\boxed{9.2}$

2) $v = 5$, $e = 13$, $\deg(a) = 6$, $\deg(b) = 6$, $\deg(c) = 6$
$\deg(d) = 5$ $\deg(e) = 3$

→

4) 1) $e = 6$ sum of deg $= 12$

$2 \cdot 6 = 12$

2) $e = 13$ sum of deg $= 26$

$2 \cdot 13 = 26$

3) $e = 12$ sum of deg $= 24$

$12 \cdot 2 = 24$

12) $\deg(v)$ = how many acquaintences v has

Isolated = v has no acquaintences

Pendant = v has one acquaintence

If the average degree is 1000, then the average person has about 1000 acquaintences

18) 

$\deg(A) = 1$ $\deg(B) = 1$ $\deg(A) = 0$ $\deg(B) = 0$

So, if n is the number of vertices on a graph, there exists a vertex of degree $n-1$ or there doesn't. If there does then there are degrees from 1 to $n-1$. Since there are $n-1$ possible degrees and n vertices at least two vertices must have the same degree. If no vertex has degree $n-1$ then there are degrees from 0 to $n-2$ and so again at least two vertices must share a degree.