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# Asn 4

2.3

2/ a/ Not a function, f(n) has two values for n.
b/ yes, function
c/ not a function, f(2) is undefined

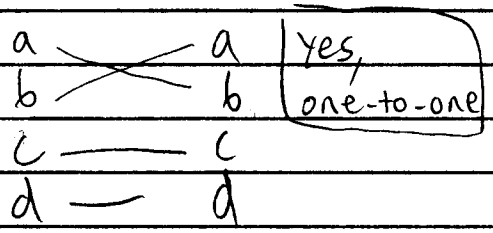
4/ a/ Domain: set of nonnegative integers
Range: set of nonnegative integers

b/ Domain: set of integers
Range: set of positive integers

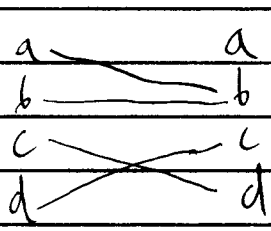
c/ Domain: set of bit strings
Range: set of nonnegative integers

d/ Domain: set of bit strings
Range: set of nonegative integers

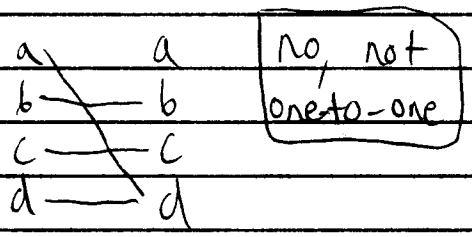
8/ a/ 1 b/ 2 c/ -1 d/ 0 e/ 3 f/ -2 g/ 1
h/ 1

10/ a/ 

yes, one-to-one

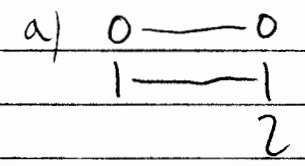
b/ 

no, not one-to-one

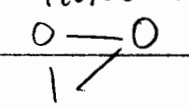
c/ 

no, not one-to-one

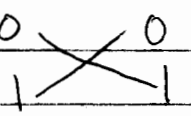
16) a) f from {0,1} to {0,1,2} with f(0)=0 and f(1)=1




b) f from {0,1} to {0} with f(0)=0 f(1)=0

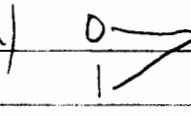



c)



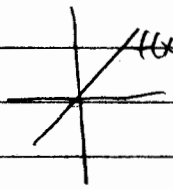

f from {0,1} to {0,1} with f(0)=1 and f(1)=0

d)




f from {0,1} to {0,1} with f(0)=0 and f(1)=0

20)




If $f(x)$ is increasing then $f(x) < f(y)$ whenever $x > y$. So, if $x > y$ then $g(x) = \frac{1}{f(x)} > \frac{1}{f(y)} = g(y)$ (so $g(x)$ is decreasing)

If $g(x)$ is decreasing then $g(x) > g(y)$ whenever $x > y$. So, if $x > y$ then $f(x) = \frac{1}{g(x)} < \frac{1}{g(y)} = f(y)$. (so $f(x)$ is increasing.)

32) $f(g(x)) = f(x+2) = x(x+2)+1 = x^2+2x+1$

$g(f(x)) = g(x^2+1) = 1(x^2+1)+2 = x^2+3$

46) $a = \lceil x \rceil$

$a-1 < x \leq a$

$a+m-1 < x+m \leq a+m$

$\lceil x+m \rceil = a+m$

$a = \lceil x \rceil$, so $a+m = \lceil x \rceil + m$

$\lceil x \rceil + m = a+m = \lceil x+m \rceil$

10) a) $a_n = 3 + 2n + n^2$
123, 146, 171

b) $a_n = 7 + 4n$
51, 55, 59

c) Starting with 1 in 1 bit binary continue to count in binary.
1100, 1101, 1110

d) Fibonacci's sequence with repetition: one 1, three 2's, five 3's, seven 5's, nine 8's, eleven 13's, etc.
8, 8, 8

e) $a_n = 3^n - 1$
59,048 177,146 531,440

f) The sequence of odd factorials: using only odd numbers, ie( $5! = 1 \cdot 3 \cdot 5 = 15$ / $9! = 1 \cdot 3 \cdot 5 \cdot 7 \cdot 9 = 945$)
$654,729,075$ $1.374931058^{10}$ $3.162341432^{11}$

g) one 1, two 0's, three 1's, 4 0's, 5 1's, etc.
0, 0, 0

h) $a^n = 2^{2^n}$ $2^{64} = 1.844674407^{10}$
$2^{128} = 3.402823669^{38}$
$2^{256} = 1.157920892$

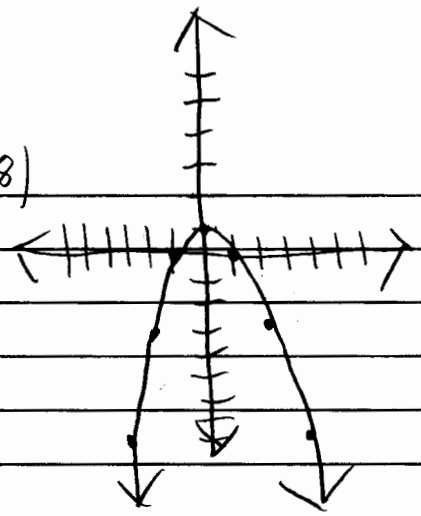
16) a) $2+0+2+0+2+0+2+0+2$
10

b) $0+1+5+19+65+211+665$
$+2059+6305$
9330

c) $5+12+30+78+210+582+1650$
$+4758+13890$
21,215

d) $1+2+4+8+16+32+64$
$+128+256$ 511

58)



66) $(f \circ g)^{-1} = g^{-1} \circ f^{-1}$

$((f \circ g) \circ (g^{-1} \circ f^{-1}))(x)$
$(f \circ g)((g^{-1} \circ f^{-1})(x))$
$(f \circ g)(g^{-1}(f^{-1}(x)))$
$(f(g(g^{-1}(f^{-1}(x)))))$
$(f(f^{-1}(x)))$
$x$

$((g \circ f) \circ (f^{-1} \circ g^{-1}))(z)$
$(g \circ f)(f^{-1} \circ g^{-1})(z))$
$(g \circ f)(f^{-1}(g^{-1}(z)))$
$(g(f(f^{-1}(g^{-1}(z)))))$
$(g(g^{-1}(z)))$
$z$

So $(f \circ g)^{-1} = g^{-1} \circ f^{-1}$

2.4

2) a) $2^8 - 1 =$ 255 b) 7 c) $1 + (-1)^8 =$ 2 d) $-(-2)^8 =$ -256

4) a) $a_0 = 1, a_1 = -2, a_2 = 4, a_3 = -8$

b) $a_0 = 3, a_1 = 3, a_2 = 3, a_3 = 3$

c) $a_0 = 8, a_1 = 11, a_2 = 23, a_3 = 71$

d) $a_0 = 2, a_1 = 0, a_2 = 8, a_3 = 0$

8) 3, 5, 7 1. A list of odd integers starting with 3.
2. $a_n = 3 + 2n$
3. A list of prime numbers starting with 3.

19) $\sum_{j=1}^{n} (a_j - a_{j-1})$ $\quad a_1 - a_0 + a_2 - a_1 + a_3 - a_2 + a_4 - a_3 + a_5 - a_4 + \ldots$

Because every term in the sequence except $a_0$ and $a_n$ will be canceled out

$$\sum_{j=1}^{n} (a_j - a_{j-1}) = a_n - a_0$$

20) $\sum_{k=1}^{n}$ $\quad \frac{1}{2} + \frac{1}{6} + \frac{1}{12} + \frac{1}{20}$

32) a) Countable $\quad f(n) = 10 + n = 10 + m$

$f(m) = m$

| 1 | 2 | 3 | 4 | 5 |
|---|---|---|---|---|
| ↕ | ↕ | ↕ | ↕ | ↕ |
| 11 | 12 | 13 | 14 | 15 |

b) Countable $\quad f(n) = -(2n+1) = -(2m+1)$

$f(m) = m$

| 1 | 2 | 3 | 4 | 5 |
|---|---|---|---|---|
| ↕ | ↕ | ↕ | ↕ | ↕ |
| -1 | -3 | -5 | -7 | -9 |

c) Uncountable

d) Countable $\quad f(n) = -10n$

$f(m) = 2m$

| 0 | 1 | 2 | 3 | 4 | 5 |
|---|---|---|---|---|---|
| ↕ | ↕ | ↕ | ↕ | ↕ | ↕ |
| 0 | 10 | -10 | 20 | -20 | 30 |

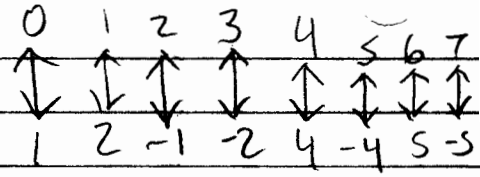
$f(x) = 10n$

$f(m) = 2m+1$

→

34/ a) countable $f(n) = 3n+1$

$f(m) = 2m$

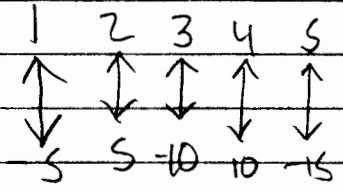



$f(n) = 3n+2$

$f(m) = 2m+1$

b) countable $f(n) = 5n ; n \neq 7n$

$f(m) = 2m$




$f(n) = -5n ; n \neq 7n$

$f(m) = 2m+1$

c) uncountable

d) uncountable