andbra 16
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1.7 2| 1, 8, 27, 64, 125, 216, 343, 512, 729, 1000

$1+8=9$, $8+27=35$, $27+64=91$, $125+64=189$, $125+216=341$,
$216+343=559$, $343+512=855$ $729+343=1072$
$343+125=468$ $512+216=728$ $729+216=945$

No, positive perfect cube will equal the sum of the cubes of two perfect integers.

6| $n=3$ $1+2=3$
constructive

10| $x(\text{pos}) \cdot y(\text{pos}) = n(\text{non neg})$ $x(\text{neg}) \cdot y(\text{neg}) = n(\text{non neg})$
$+x \cdot +y = +xy$ $-x \cdot -y = +xy$

Therefore, the product of two of the positive numbers or two of the negative numbers will result in a nonnegative number.
nonconstructive

20| $\left(\frac{x-1}{x}\right)^2 \geq 0$

$\sqrt{\left(\frac{x-1}{x}\right)^2} \geq \sqrt{0}$

$\frac{(x+1)\cdot(x-1)}{x} \geq 0 \cdot (x+1)$

$x \cdot \frac{(x+1)(x-1)}{x} \geq 0 \cdot x$

$-1 \cdot (x^2-1) \geq 0 \cdot -1$

$-x^2+1 \geq 0$

$+2x^2 \quad +2x^2$

$\frac{x^2+1}{x^2} \geq \frac{2x^2}{x^2}$

$\boxed{\frac{x^2+1}{x^2} \geq 2}$

26| $(50-n)^2 = 2500-100n+n^2$
$(50+n)^2 = 2500+100n+n^2$

00, 01, 04, 09, 16, 25, 36, 49, 64, 81, 21, 44, 69, 96, 56, 89, 24, 61, 41, 84, 29, 76

The final two decimal digits of a perfect square are ...

cases $|0-24^2|$ because repetition after every 25 numbers.

34) $x = \frac{a}{b}$, rational $z = \frac{a\sqrt{2}}{b}$, irrational $y = \frac{a+\frac{1}{2}\sqrt{2}}{b}$, irrational

$x < y < z$, Therefore, between every rational and irrational number there is an irrational number.

40) 64 (squares) - 4 (squares) = 60 (squares) [30 white + 30 black]

[2 white and 2 black]

1 Domino per 1 black + 1 white (Must have same number of white and black squares)

P = Equal number of white and black squares

Q = can be tiled with dominoes

$P \rightarrow Q$

P

Q

Therefore, the checkerboard can be tiled, because there are an equal number of white and black squares.

Review

4) a) converse = If I drive to work, then it is raining today.

inverse = If it doesn't rain today, then I won't drive to work.

contrapositive = If I don't drive to work, then it didn't rain today.

6) inverse of inverse = inverse of converse inverse of contrapositive

$p \rightarrow q$ $p \rightarrow q$ $p \rightarrow q$

$\neg p \rightarrow \neg q$ $q \rightarrow p$ $\neg q \rightarrow \neg p$

$\boxed{p \rightarrow q}$ = statement $\boxed{\neg q \rightarrow \neg p}$ = contrapositive $\boxed{q \rightarrow p}$ = converse

14) a) $\exists x P(x)$ b) $\exists x \neg P(x)$ c) $\forall y Q(y)$

d) $\forall x \forall y P(x) Q(y)$ e) $\exists y \forall x Q(y) \neg P(x)$

18) $\forall y \exists x\, T(G(x>3, y))$ 30) Existential instantiation, There exists an element x and an element y for P(x) and P(y) because we know that $\exists x,y\, P(x,y)$ is true.

34) $n=0$ $0^2 \leq 0 < (0+1)^2$
$m=0$ $0 \leq 0 < 1$

2.1

2a) $\{x \mid x$ is a nonnegative multiple of $3\}$

b) $\{x \in \mathbb{Z} \mid -3 \leq x \leq 3\}$

c) $\{x \mid x$ is a letter and $m \leq x \leq p\}$

4) $B \subset A$, $C \subset A$, $C \subset D$,

8) a) True b) True c) False d) True e) True f) True g) True

16) $A = \{1,2,3\}$ $B = \{3,1,2\}$
$A = \{1,4\}$ $B = \{1,1,1,4\}$

18) a) 0 b) 1 c) 2 d) 3

20) Yes, because if A and B have all the same subsets then they must be equal to eachother.

22) a) No b) Yes $= P(\{a\})$ c) No d) Yes $= P(\{a,b\})$

30) $A = \{1,2\}$ $A \times B = \{(1,a),(1,b),(2,a),(2,b)\}$
$B = \{a,b\}$ $B \times A = \{(a,1),(a,2),(b,1),(b,2)\}$

So, $A \times B \neq B \times A$ unless $A \neq B$.

2.2 2| a| $A \cap B$ b| $A - B$ c| $A \cup B$ d| $\overline{A \cup B}$

4| a| $A \cup B = \{a, b, c, d, e, f, g, h\}$ b| $A \cap B = \{a, b, c, d, e\}$

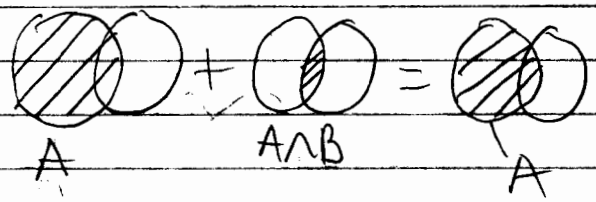
c| $A - B = \emptyset$ d| $B - A = \{f, g, h\}$

12| $A \cup (A \cap B)$

$\{x \mid x \in A \cup (A \cap B)\}$

$\{x \mid x \in A \cup (x \in A \cap B)\}$

$\{x \mid x \in A \cup (x \in A \wedge x \in B)\}$



Since $A \cap B$ is a subset of A, $A \cup (A \cap B) = A$.

16| a| $A \cap B$ $A = \{x \mid x \in A\}$

$\{x \mid x \in A \cap B\}$

$\{x \mid x \in A \wedge x \in B\}$

$A \cap B \subseteq A$

b| $A \cup B$ $A = \{x \mid x \in A\}$

$\{x \mid x \in A \cup B\}$

$\{x \mid x \in A \vee x \in B\}$

$A \subseteq A \cup B$

c| $A - B$ $A = \{x \mid x \in A\}$

$\{x \mid x \in A \wedge x \notin B\}$

$A - B \subseteq A$

d| $A \cap (B - A)$

$\{x \mid x \in A \wedge (x \in B \wedge x \notin A)\}$

$\{x \mid x \in A \wedge x \notin A \wedge x \in A\}$

$A \wedge \bar{A} \wedge B$

$\emptyset \wedge B$

$\emptyset$

$A \cap (B - A) = \emptyset$

e| $A \cup (B - A)$

$\{x \mid x \in A \vee (x \in B \wedge x \notin A)\}$

$A \cup (B \wedge \bar{A})$

$A \cup B \wedge A \cup \bar{A}$

$A \cup B \wedge U$

$A \cup B$

$A \cup (B - A) = A \cup B$

18) a) $A \cup B$ $\quad A \cup B \cup C$

$\{x \mid x \in A \cup B\}$ $\quad \{x \mid x \in A \cup B \cup C\}$

$\{x \mid x \in A \cup x \in B\}$ $\quad \{x \mid x \in A \cup x \in B \cup x \in C\}$

$(A \cup B) \subseteq (A \cup B \cup C)$

b) $A \cap B \cap C$ $\quad A \cap B$

$\{x \mid x \in A \cap B \cap C\}$ $\quad \{x \mid x \in A \cap B\}$

$\{x \mid x \in A \wedge x \in B \wedge x \in C\}$ $\quad \{x \mid x \in A \wedge x \in A\}$

$(A \cap B \cap C) \subseteq (A \cap B)$

c) $(A - B) - C$ $\quad A - C$

$\{x \mid x \in A \wedge x \notin B \wedge x \notin C\}$ $\quad \{x \mid x \in A \wedge x \notin C\}$

$(A - B) - C \subseteq A - C$

d) $(A - C) \cap (C - B)$

$\{x \mid x \in A \wedge x \notin C \wedge x \in C \wedge x \notin B\}$

$A \cap \bar{C} \cap C \cap B$

$A \cap \emptyset \cap B$

$\emptyset \cap B$

$\emptyset$

e) $(B - A) \cup (C - A)$

$\{x \mid (x \in B \wedge x \notin A) \cup (x \in C \wedge x \notin A)\}$

$(B \cap \bar{A}) \cup (C \cap \bar{A})$

$(B \cup C) \cap \bar{A}$

$\{x \mid x \in B \cup x \in C \wedge x \notin A\}$

$(B \cup C) - A$

30) a) $A = \{1, 2, 3\}$

$B = \{5, 6\}$

$C = \{1, 2, 3, 4, 5, 6\}$

$A \cup C = B \cup C$

$\{1, 2, 3, 4, 5, 6\} = \{1, 2, 3, 4, 5, 6\}$

$A \neq B$

No

b) $A = \{1, 2, 3, 4\}$

$B = \{1, 2, 5, 6\}$

$C = \{1, 2, 3\}$

$A \cap C = B \cap C$

$\{1, 2, 3\} = \{1, 2, 3\}$

$A \neq B$

No

4) Yes

36) $A \oplus B$ $\quad (A - B) \cup (B - A)$

$(A \wedge \bar{B}) \vee (B \wedge \bar{A})$ $\quad \{x \mid x \in A \wedge x \notin B \ \cup \ x \in B \wedge x \notin A\}$

$(A \wedge \bar{B}) \vee (B \wedge \bar{A})$

$A \oplus B = (A - B) \cup (B - A)$