Asn 7

1) a) $11 = 1 \cdot 10 + 1$

$\boxed{1 = 1 \cdot 11 + (-1) \cdot 10}$

b) $44 = 2 \cdot 21 + 2$

$21 = 10 \cdot 2 + 1$

$2 = 2 \cdot 1$

$\boxed{1 = 21(21) + (-10 \cdot 44)}$

c) $48 = 1 \cdot 36 + 12$

$36 = 3 \cdot 12$

$\boxed{12 = +1 \cdot 48 + (-1) \cdot 36}$

d) $55 = 1 \cdot 34 + 21$

$34 = 1 \cdot 21 + 13$

$21 = 1 \cdot 13 + 8$

$13 = 1 \cdot 8 + 5$

$8 = 1 \cdot 5 + 3$

$5 = 1 \cdot 3 + 2$

$3 = 1 \cdot 2 + 1$

$\boxed{1 = 13 \cdot 55 + (-21) \cdot 34}$

e) $213 = 1 \cdot 117 + 96$

$117 = 1 \cdot 96 + 21$

$96 = 4 \cdot 21 + 12$

$21 = 1 \cdot 12 + 9$

$12 = 1 \cdot 9 + 3$

$9 = 3 \cdot 3$

$\boxed{3 = 11 \cdot 213 + (-20) \cdot 117}$

f) $223 = 1 \cdot 223 + 1 \cdot 0$

g) $2347 = 19 \cdot 123 + 10$

$123 = 12 \cdot 10 + 3$

$10 = 3 \cdot 3 + 1$

$3 = 2 \cdot 1 + 1$

$\boxed{1 = 37 \cdot 2347 + (-706) \cdot 123}$

h) $4666 = 1 \cdot 3454 + 1212$

$3454 = 2 \cdot 1212 + 1030$

$1212 = 1 \cdot 1030 + 182$

$1030 = 5 \cdot 182 + 120$

$182 = 1 \cdot 120 + 62$

$120 = 1 \cdot 62 + 58$

$62 = 1 \cdot 58 + 4$

$58 = 14 \cdot 4 + 2$

$4 = 2 \cdot 2$

$\boxed{2 = 1128 \cdot 3454 + (-835) \cdot 4666}$

i) $1 = 2468 \cdot 9999 + (-2221) \cdot 11111$

i) $11111 = 1 \cdot 9999 + 1112$
$9999 = 8 \cdot 1112 + 1103$
$1112 = 1 \cdot 1103 + 9$
$1103 = 122 \cdot 9 + 5$
$9 = 1 \cdot 5 + 4$
$5 = 1 \cdot 4 + 1$
$4 = 4 \cdot 1$

$\boxed{1 = 2468 \cdot 9999 + (-2221) \cdot 11111}$

12) $17 = 8 \cdot 2 + 1$
$-8 \cdot 2 + 1 \cdot 17 = 1$

$-8 \cdot 2x \equiv -8 \cdot 7 \pmod{17}$
$-16 \equiv 1 \pmod{17}$, $-56 \equiv 12 \pmod{17}$

$x \equiv -56 \equiv 12 \pmod{17}$
$2x \equiv 2 \cdot -56 \equiv 7 \pmod{17}$

$\boxed{x \equiv 12 \pmod{17}}$

18) $x \equiv 2 \pmod 3$
$x \equiv 1 \pmod 4$
$x \equiv 3 \pmod 5$

$x = 3k + 2$
$3k \equiv -1 \pmod 4$
$k \equiv 1 \bmod 4$
$k = 4L + 1$
$x = 3(4L+1) + 2$
$x = 12L + 5$

$4 = 1 \cdot 3 + 1$
$-1(3) + 1 \cdot 4 = 1$

$12L \equiv -2 \pmod 5$
$L \equiv 4 \bmod 5$
$L = 5m + 4$
$x = 12(5m+4) + 5$
$x = 60m + 53$

$12 = 2 \cdot 5 + 2$
$5 = 2 \cdot 2 + 1$
$5 \cdot 5 + -2(12) = 1$

All integers of the form $53 + 60m$ where m is an integer

2) a) $A+B=\begin{bmatrix} 0 & 3 & 9 \\ 1 & 4 & -1 \\ 2 & -5 & -3 \end{bmatrix}$

b) $A+B=\begin{bmatrix} -4 & 9 & 2 & 10 \\ -4 & -5 & 4 & 0 \end{bmatrix}$

4) a) $AB=\begin{bmatrix} -1 & 1 & 0 \\ 0 & 1 & -1 \\ 1 & -2 & 1 \end{bmatrix}$

b) $AB=\begin{bmatrix} 4 & -1 & -7 & 6 \\ -7 & -5 & 8 & 5 \\ 4 & 0 & 7 & 3 \end{bmatrix}$

c) $AB=\begin{bmatrix} 2 & 0 & -3 & -4 & -1 \\ 24 & -7 & 20 & 29 & 2 \\ -10 & 4 & -17 & -24 & -3 \end{bmatrix}$

6) $\begin{bmatrix} 1 & 3 & 2 \\ 2 & 1 & 1 \\ 4 & 0 & 3 \end{bmatrix} \cdot \begin{bmatrix} a & b & c \\ x & y & z \\ p & q & r \end{bmatrix} = \begin{bmatrix} 7 & 1 & 3 \\ 1 & 0 & 3 \\ -1 & -3 & 7 \end{bmatrix}$

$a+3x+2p=7$
$b+3y+2q=1$
$c+3z+2r=3$
$2a+x+p=1$
$2b+y+q=0$
$2c+z+r=3$
$4a+0x+3p=-1$
$4b+0y+3q=-3$
$4c+0z+3r=7$

$a+3x+2p=7$
$2a+x+p=1$
$4a+3p=-1$

$a=-3x-2p+7$
$2(-3x-2p+7)+x+p=1$
$-6x-4p+14+x+p=1$
$-5x-3p=-13$
$x=-\frac{3}{5}p+\frac{13}{5}$
$4(-3x-2p+7)+3p=-1$
$-12x-8p+28+3p=-1$
$-12x-5p=-29$
$-12(-\frac{3}{5}p+\frac{13}{5})+5p=-29$

$\frac{36}{5}p-\frac{156}{5}-5p=-29$
$\frac{11}{5}p=\frac{11}{5}$
$p=1$

$-12x - 5 = -29$

$x = 2$

$a = -3(2) - 2(1) + 7$

$a = -1$

$p = 1$

$b + 3y + 2q = 1$

$2b + y + q = 0$

$4b + 3q = -3$

$b = -3y - 2q + 1$

$2(-3y - 2q + 1) + y + q = 0$

$-6y - 4q + 2 + y + q = 0$

$\bigstar\ -5y - 3q = -2$

$y = -\frac{3}{5}q + \frac{2}{5}$

$-12y - 5(-1) = -7$

$y = 1$

$4(-3y - 2q + 1) + 3r = -3$

$-12y - 8q + 4 + 3q = -3$

$\bigstar\ -12y - 5q = -7$

$-12\left(-\frac{3}{5}q + \frac{2}{5}\right) - 5q = -7$

$\frac{36}{5}q - \frac{24}{5} - 5q = -7$

$\frac{11}{5}q = -\frac{11}{5}$

$q = -1$

$b = -3(1) - 2(-1) + 1$

$b = 0$

$c + 3z + 2r = 3$

$2c + z + r = 3$

$4c + 3r = 7$

$c = -3z - 2r + 3$

$2(-3z - 2r + 3) + z + r = 3$

$-6z - 4r + 6 + z + r = 3$

$\bigstar\ -5z - 3r = -3$

$z = -\frac{3}{5}r + \frac{3}{5}$

$-5z - 3(1) = -3$

$z = 0$

$4(-3z - 2r + 3) + 3r = 7$

$-12z - 8r + 12 + 3r = 7$

$\bigstar\ -12z - 5r = -5$

$-12\left(-\frac{3}{5}r + \frac{3}{5}\right) - 5r = -5$

$\frac{36}{5}r - \frac{36}{5} - 5r = -5$

$r = 1$

$b = -3(0) - 2(1) + 3$

$c = 1$

$$A = \begin{bmatrix} -1 & 0 & 1 \\ 2 & 1 & 0 \\ 1 & -1 & 1 \end{bmatrix}$$