Assignment 8

$\boxed{4.4}$

6) $gcd(12,17) = gcd(17 \bmod 12, 12) = gcd(5,12)$
$gcd(5,12) = gcd(12 \bmod 5, 5) = gcd(7,5)$
$gcd(7,5) = gcd(5 \bmod 7, 7) = gcd(2,7)$
$gcd(2,7) = gcd(7 \bmod 2, 2) = gcd(5,2)$
$gcd(5,2) = gcd(2 \bmod 5, 5) = gcd(3,5)$
$gcd(3,5) = gcd(5 \bmod 3, 3) = gcd(2,3)$
$gcd(2,3) = gcd(3 \bmod 2, 2) = gcd(1,2)$
$gcd(1,2) = gcd(2 \bmod 1, 1) = gcd(0,1)$
$gcd(0,1) = \boxed{gcd(12,17) = 1}$

8) procedure sum(n: positive integeger)
if n=1 then sum(n):=1
else sum(n):= sum(n-1)

10) procedure maximum($a_1, \ldots, a_n$: integers)
if n=1 then maximum($a_1, \ldots, a_n$) = a_1
else maximum($a_1, \ldots, a_n$) := max(maximum($a_1, \ldots, a_{n-1}$), a_n)

$\boxed{5.1}$

2) $27 \cdot 37 = \boxed{999 \text{ offices}}$

4) $12 \cdot 2 \cdot 3 = \boxed{72 \text{ types}}$

6) $6 \cdot 4 = \boxed{24 \text{ major routes}}$

22) a) $9999 - 1000 + 1 = 9000 / 9 = \boxed{1000}$
b) $9000/2 = \boxed{4500}$
c) $9000 - 9 = \boxed{8991}$
d) $9000/3 = 3000 \quad 9000 - 3000 = \boxed{6000}$
e) $9000/5 = 1800 \quad 9000/7 = 1285 \quad 9000/35 = 257 \quad 1800 + 1285 - 257 = \boxed{2828}$

f) $9000 - 2828 = \boxed{6172}$

g) $1800 - 257 = \boxed{1543}$

h) $\boxed{257}$

26) 9 - digits $\quad 9^3 + 26^3 = \boxed{18305 \text{ license plates}}$

26 - letters

30) a) 26^8

b) $26 \cdot 25 \cdot 24 \cdot 23 \cdot 22 \cdot 21 \cdot 20 \cdot 19$

c) $X \cdot 26^7 = \boxed{26^7}$

d) $X \cdot \boxed{25 \cdot 24 \cdot 23 \cdot 22 \cdot 21 \cdot 20 \cdot 19}$

e) $X \cdot 26^6 \cdot X = \boxed{26^6}$

f) $BO \cdot 26^6 = \boxed{26^6}$

g) $BO \cdot 26^4 \cdot BO = \boxed{26^4}$

h) $BO \cdot 26^6 + 26^6 \cdot BO - BO \cdot 26^4 \cdot BO = 26^6 + 26^6 - 26^4$

$\boxed{5.2}$

2) 26 letters

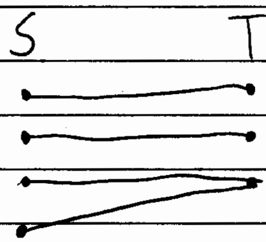
26 last names starting with a different letter
4 must be distributed (but will share a letter)
So, the pidgeon hole principle shows that atleast two students have last names that begin with the same letter.

26 unique
4 unaccounted → So, atleast two students last names will start with the same letter if not more, because 4 students will share a ~~letter with the other 26~~ students.

b) Remainder after division by $d = 0 \le r \le d-1$
$[0, 1, 2, \ldots, d-1]$

So, there are only d possible remainders when an integer is divided by d. By the pigeonhole principle, atleast two of any collection of $d+1$ integers must have the same remainder when divided by d.

8) $S \to T$
$|S| > |T|$



f is not one-to-one, because S maps to T and $|S| > |T|$. So, S must have atleast 1 element that shares a spot on T with another element from S.

16) $(1, 15)$
$(3, 13)$
$(5, 11)$
$(7, 9)$ $\Big\}$ 16

$\{1, 3, 5, 7\}$ No pair adds up to 16

So, if 5 numbers are selected then at least one pair is guaranteed to add up to 16.

18) a) 4 male / 4 Female = 8 students
So, the class must have atleast 5 males or 5 females.

b) 2 male / 6 Female = 8 students
So, the class must have atleast 3 males or atleast 7 females. $\rightarrow$

32) Each computer can be connected to 1, 2, 3, 4, or 5 other computers. Since there are 6 computers, there are more computers than possible numbers of connections (unique). So, atleast two computers share the same number of connections.

34) 8 computers x 3 printers = 24 cables
4 computers → 1 printer = 4 cables
So, all the computers can access 3 printers. To get four computers to access the 4th printer, just add 4 more cables.

28 cables, because it guarantees 4 computers can access all 4 printers, and the other 4 computers can access the other 3.

$\boxed{5.3}$

$C(n,r) = \frac{n!}{r!(n-r)!}$

2) $7\cdot6\cdot5\cdot4\cdot3\cdot2\cdot1 = \boxed{5040}$

6) a) $\frac{\cancel{5!}\,5}{1!(\cancel{4!})} = \boxed{5}$

b) $\frac{\cancel{5!}\,5\cdot4\cdot3}{3!(\cancel{2!})} = \frac{5\cdot4\cdot\cancel{3}}{\cancel{3}\cdot2\cdot1} = \boxed{10}$

c) $\frac{\cancel{8!}\,8\cdot7\cdot6\cdot5}{4!(\cancel{4!})} = \frac{8\cdot7\cdot6\cdot5}{4\cdot3\cdot2\cdot1} = \boxed{70}$

d) $\frac{\cancel{8!}}{\cancel{8!}(\cancel{0!})\,1} = \boxed{1}$

e) $\frac{8!}{1\,\cancel{0!}(8!)} = \boxed{1}$

f) $\frac{\cancel{12!}}{6!(\cancel{6!})} = \frac{12\cdot11\cdot10\cdot9\cdot8\cdot7}{6\cdot5\cdot4\cdot3\cdot2\cdot1} = \boxed{924}$

12) a) $C(12,3) = \frac{12!}{3!(9!)} = \frac{12\cdot 11\cdot 10}{3\cdot 2\cdot 1} = \boxed{220}$

b) $C(12,0) = \frac{12!}{0!(12!)} = 1 + C(12,1) = \frac{12!}{1!(11!)} = 12 +$

$C(12,2) = \frac{12!}{2!(10!)} = \frac{12\cdot 11}{2} = 66 + C(12,3) = 220 = \boxed{299}$

c) $C(12,3) = 220$ $C(12,6) = 924$ $C(12,9) = 220$
$C(12,4) = 495$ $C(12,7) = 792$ $C(12,10) = 66$
$C(12,5) = 792$ $C(12,8) = 495$ $C(12,11) = 12$

$C(12,12) = 1$ $\boxed{4017}$

d) $C(12,6) = \boxed{924}$

16) $C(10,1) = 10$ $C(10,7) = 120$ $\boxed{512}$
$C(10,3) = 120$ $C(10,9) = 10$
$C(10,5) = 252$

$\frac{n!}{(n-r)!}$

22) a) A, B, C, ED, F, G, H $= 7! = \boxed{5040}$
b) A, B, CDE, F, G, H $= 6! = \boxed{720}$
c) AB, C, D, E, FGH $= 5! = \boxed{120}$
d) AB, C, DE, F, GH $= 5! = \boxed{120}$
e) CABED, F, G, H $= 4! = \boxed{24}$
f) BCA can't have both $\boxed{0}$
ABF strings in one permutation

24) women $= 10!$
men $= P(11,6) = \frac{11!}{5!}$

$\boxed{10! \cdot \frac{11!}{5!}}$

32) a) total strings $= 26^6$

no $a = 25^6$

$\boxed{26^6 - 25^6}$

b) total $= 26^6$

no $a = 25^6$

no $b = 25^6$

no a and $b = 24^6$

$\boxed{26^6 - 25^6 - 25^6 + 24^6}$

c) $ab = 5$ places

4 spots left $= 24, 23, 22, 21$

$\boxed{5 \cdot 24 \cdot 23 \cdot 22 \cdot 21}$

d) $C(6,2) = 15$

$\boxed{15 \cdot 24 \cdot 23 \cdot 22 \cdot 21}$