Sensitivity analysis of demographic models

ESM 211

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1 Sensitivity and elasticity analysis in R

Make sure you have the sea turtle matrix loaded:

```
class_names <- c("Egg", "Sm Juv", "Lg Juv", "Subadult", "Adult")
A <- matrix(c(0,     0,     0,     4.665, 61.896,
              0.675, 0.703, 0,     0,      0,
              0,     0.047, 0.657, 0,      0,
              0,     0,     0.019, 0.682,  0,
              0,     0,     0,     0.061,  0.809),
            nrow = 5, ncol = 5, byrow = TRUE, dimnames = list(class_names, class_names))
```

Like the asymptotic growth rate and the stable stage structure, the sensitivities and elasticities can be calculated from the eigenvalues and eigenvectors of the matrix. The **primer** library (which accompanies the book *A Primer of Ecology in R*) bundles the calculations together to produce the biologically relevant output:

```
library(primer) # You may need to install this first with install.packages("primer")
DemoInfo(A)
```

```
## $lambda
## [1] 0.9515489
##
## $SSD
## [1] 0.238508404 0.647732505 0.103356123 0.007285382 0.003117586
##
## $RV
## [1]   1.000000   1.409702   7.454890 115.569961 434.209028
##
## $Sensitivities
##             [,1]       [,2]        [,3]        [,4]         [,5]
## [1,]  0.05792157  0.1573013  0.02509995 0.001769249 0.0007571032
## [2,]  0.08165216  0.2217480  0.03538345 0.002494114 0.0010672899
## [3,]  0.43179894  1.1726640  0.18711737 0.013189557 0.0056441208
## [4,]  6.69399385 18.1793066  2.90080031 0.204472046 0.0874983860
## [5,] 25.15006963 68.3016503 10.89862514 0.768223917 0.3287410397
##
## $Elasticities
##                 Egg     Sm Juv     Lg Juv    Subadult      Adult
## Egg      0.00000000 0.00000000 0.00000000 0.008673803 0.04924777
## Sm Juv   0.05792157 0.16382640 0.00000000 0.000000000 0.00000000
## Lg Juv   0.00000000 0.05792157 0.12919579 0.000000000 0.00000000
## Subadult 0.00000000 0.00000000 0.05792157 0.146550473 0.00000000
## Adult    0.00000000 0.00000000 0.00000000 0.049247770 0.27949327
##
## $PPM
##            Egg Sm Juv Lg Juv Subadult  Adult
## Egg      0.000  0.000  0.000    4.665 61.896
## Sm Juv   0.675  0.703  0.000    0.000  0.000
```

These add to one.

If I increase [5,2] by one unit then I increase lambda by 68. But this is impossible because this would mean jumping from juvenile to adult?

If I increase adult survival by 100% increase lambda by 27.0%

This is just linear algebra, it does not take into account actual biological barriers.

```
## Lg Juv   0.000  0.047  0.657   0.000  0.000
## Subadult 0.000  0.000  0.019   0.682  0.000
## Adult    0.000  0.000  0.000   0.061  0.809
```

1.1 Exercises

1. Referring to the help page and section 2.2 of the Stevens chapter, make sure you understand what each of the outpurs of `DemoInfo` represents. The "RV" (reproductive value) is the only bit we haven't covered in lecture.
2. Looking at the sensitivity and elasticity matrices, what can you conlude about which matrix elements would likely have the biggest impact on λ if they were changed?
3. Compare the elasticity matrix with Fig. 1 in Crowder et al. (1994). Do you understand where the values in the figure come from?
4. Look at the sensitivity matrix produced by `DemoInfo`. What does the sensitivity for element $a_{5,1}$ represent? Does it make sense to have a non-zero value here? Why or why not?

2 Sensitivity and elasticity of λ to vital rates

The above analyses evaluate the sensitiviities and elasticities of λ to matrix elements. To get the effect for a vital rate v you need to sum over all the matrix elements that v affects, weighted by the strength of the effect:

$$
\begin{aligned}
S_v = \frac{\partial \lambda}{\partial v} &= \sum_{i=1}^{n} \sum_{j=1}^{n} \frac{\partial \lambda}{\partial a_{i,j}} \frac{\partial a_{i,j}}{\partial v} \\
&= \sum_{i=1}^{n} \sum_{j=1}^{n} S_{i,j} \frac{\partial a_{i,j}}{\partial v} \\
E_v = \frac{v}{\lambda} \frac{\partial \lambda}{\partial v} &= \sum_{i=1}^{n} \sum_{j=1}^{n} \left[\frac{a_{i,j}}{\lambda} \frac{\partial \lambda}{\partial a_{i,j}} \right] \left[\frac{v}{a_{i,j}} \frac{\partial a_{i,j}}{\partial v} \right] \\
&= \sum_{i=1}^{n} \sum_{j=1}^{n} E_{i,j} \frac{v}{a_{i,j}} \frac{\partial a_{i,j}}{\partial v} \\
&= \frac{v}{\lambda} \sum_{i=1}^{n} \sum_{j=1}^{n} S_{i,j} \frac{\partial a_{i,j}}{\partial v}
\end{aligned}
$$

If you have a single, relatively simple matrix then you can do this by hand; but a more robust and scalable solution is available from the **popbio** package where you can write out the matrix symbolically.

For the turtle model, we write the matrix as

```
A.vr <- expression(0,  0,          0,          p4*g4*f,    p5*f,
                   p1, p2*(1-g2), 0,          0,          0,
                   0,  p2*g2,     p3*(1-g3), 0,          0,
                   0,  0,          p3*g3,     p4*(1-g4), 0,
                   0,  0,          0,          p4*g4,     p5)
```

We then set the values of the vital rates. The survivals and fecundity come from Table 1 of Crowder et al. (1994). The growth terms emerged from a complex calculation (described in the paper), but we can reconstruct them from the matrix by noticing that $g_i = a_{i+1,i}/p_i$ (ensure that you understand this). Thus we can do:

```r
p <- c(0.6747, 0.75, 0.6758, 0.7425, 0.8091) # Survivals
vr.vals <- list(p1 = p[1], p2 = p[2], p3 = p[3], p4 = p[4], p5 = p[5],
                g2 = A[3,2]/p[2], g3 = A[4,3]/p[3], g4 = A[5,4]/p[4],
                f = 76.5)
```

Finally, apply the `vitalsens()` function:

```r
library(popbio)
vitalsens(A.vr, vr.vals)
```

```
##       estimate sensitivity elasticity
## p1   0.67470000 0.081656919 0.05790702
## p2   0.75000000 0.281337456 0.22177691
## p3   0.67580000 0.263266192 0.18699965
## p4   0.74250000 0.261543492 0.20411169
## p5   0.80910000 0.387111746 0.32920473
## g2   0.06266667 0.712825665 0.04695128
## g3   0.02811483 1.833229495 0.05417261
## g4   0.08215488 0.519057526 0.04482047
## f   76.50000000 0.000720182 0.05790702
```

If you increase f by 1, increase lambda by 0.00072.

Crowder has an elasticity of lambda compared to these vital rates.

2.1 Exercises

5. Compare the elasticities to survival with those plotted in Fig. 2 of Crowder et al. (1994). Which ones seem to be a good match?
6. What distinguishes the elasticities that match from those that don't?
   1. If you are feeling mathematically ambitious, read Appendix 1 of the paper and try to understand what is missing from your analysis
7. How would you re-parameterize the matrix so that you can look at the effects of proportional changes in *mortality* (rather than survival), as plotted in Fig. 4 of the paper?

We have sensitivities and elasticities, which should you use? It depends...

Sensitivity is absolute changes in vital rates.
Elasticity is a relational change in vital rates.