

Assignment 1:

Comparison of Population Growth between Uganda and France

In order to compare and contrast the population trajectories between a developing nation and a developed nation, and explore the explanatory variables affecting them, single and multiple linear regression was utilized. For both Uganda, a developing nation, and France, a developed nation, a simple linear model was created to explore the per capita growth rates (dN/Ndt) as a function of population size (N_t) and a multiple linear regression model was created to explore significant explanatory variables affecting the per capita growth rates (dN/Ndt) for each nation.

Uganda

Figure 2 shows the per capita growth rate from 1961 to 2006. After exploring multiple lines of fit, a single linear model seemed most appropriate given that the overall trend is growth. According to this model of current growth, Uganda will not reach the carrying capacity (K) because the per capita growth rate is increasing at the same time total population increases. The diagnostic plot for the model shows that assumptions are met, though the scale-location plot indicates that the residuals are not spread totally equally along the ranges of predictors. Though the model shows the per capita growth rate (dN/Ndt) is significantly predicted by population size (N_t), with a low R^2 of 0.22 and an large variability in dN/Ndt , it suggests non-linear models are worth exploring. Looking at the multiple regression analysis it is interesting to note that as food per capita increases, per dN/Ndt decreases. The diagnostic plots for the multiple regression model show an adequate fit.

France

Unlike Uganda, France is a developed nation, thus we expect to see major differences in the per capita growth rate. While a single linear model was utilized for Uganda, two linear models were utilized for France (*Figure 6*). This is due to the fact that France's per capita growth rate decreased dramatically when the population reached approximately 54 million in 1975. France

should reach the carrying capacity (K) when the population reaches approximately 75 million people. The second linear model (blue) shows that K is much larger than 75 million, but it is more appropriate to base K off the growth trend for the last 10 years of data as it fits current growth patterns better. The diagnostic plot for the model shows that a linear model is questionable –the Residuals vs Fitted plot is not convincing and there is an outlier that is beyond Cook's distance. For the first linear model, 1961 to 1975, the Residuals vs Fitted plot indicates that there may be a non-linear relationship and the Residuals vs Leverage shows that there is an outlier observation (*Figure 7*). For the second linear model, 1976 to 2006, the plots look much better than the previous model, likely due to less variability in dN/Ndt during this time period (*Figure 8*). However, an outlier observation in 1989 is likely resulting in the very low R^2 value of 0.03. Looking at the multiple regression, food per capita does not significantly predict dN/Ndt , unlike Uganda.

Conclusions

Uganda shows a strong Allee effect, as seen in Figure 2, in that there is an increase in per capita growth as the overall population increases, after an initial period of decline. This could be due to the increased urbanization of Uganda, in which less resources are needed for each individual, thus increasing density makes it more efficient for more individuals to live. France shows a sharp decline in per capita growth both overtime and as the overall population increases. This is expected of a developed nation that has already urbanized and is utilizing much of its land and resources at near full capacity. Low R^2 values for these models suggest that non-linear models may be a better option for exploring population growth relationships. Lastly, the multiple regression analysis shows that in both nations increased education decreases the per capita growth rate.

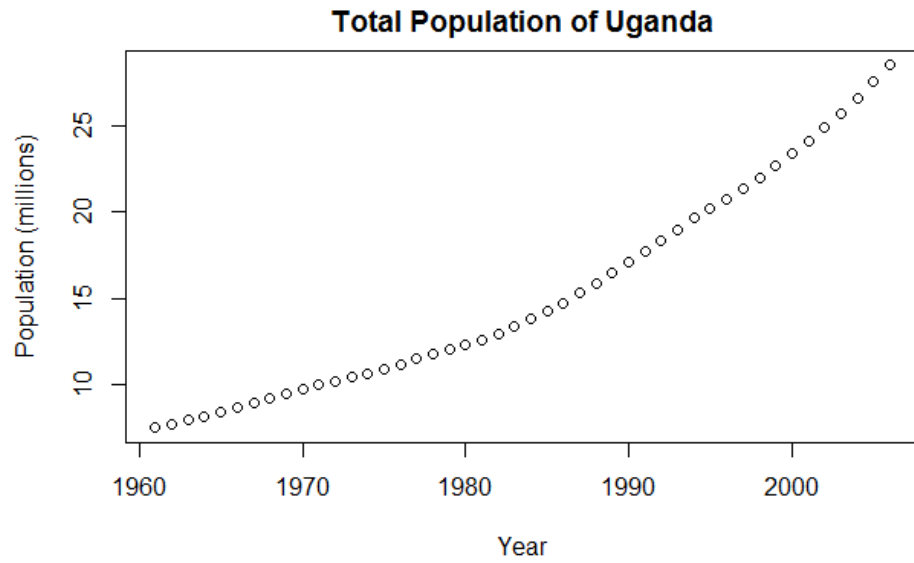


Figure 1: **Population Growth of Uganda, 1961 - 2006**

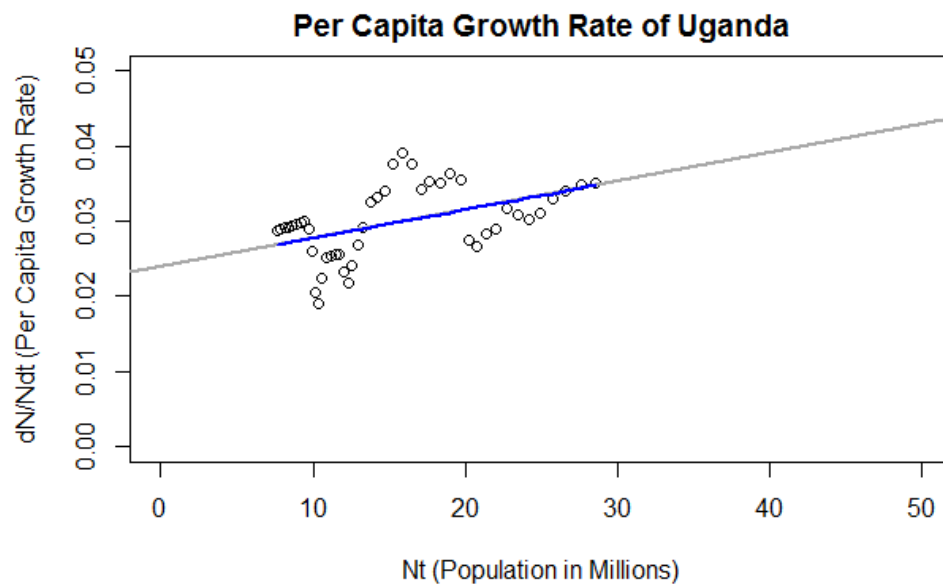


Figure 2: **Population Growth in Uganda 1961 to 2006.** Per capita growth rate (dN/Ndt) is significantly predicted by population size (N_t) ($n = 46$) ($F(1,43) = 13.24$, $p < 0.001$, $R^2 = 0.22$).

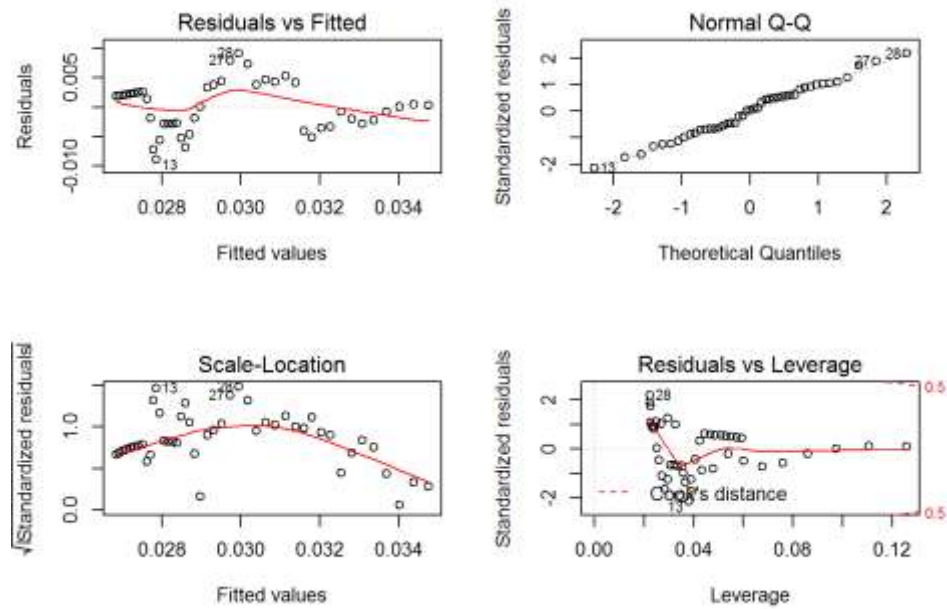


Figure 3: **Diagnostic plots for Uganda dN/Ndt model**

Table 1: **Uganda Population Growth.** The model ($dN/Ndt = -0.023 \sim \text{Food per Capita} + 0.012 * \text{Years of Education} + -0.002 \text{ Population Size}$) explains a significant amount of variance in per capita population growth (dN/Ndt) ($F(3, 38) = 7.722, p < 0.001, R^2 = 0.33$).

<u>Dependent variable:</u>	
	dN/Ndt
Food per Capita	-0.023 ^{***} (0.008)
Years of Education	0.012 ^{**} (0.004)
Population	-0.002 ^{**} (0.001)
Constant	0.278 ^{***} (0.083)
Observations	42
R^2	0.379
Adjusted R^2	0.330
Residual Std. Error	0.004 (df = 38)
F Statistic	7.722 ^{***} (df = 3; 38)
Note:	$p < 0.1$; $p < 0.05$; $p < 0.01$

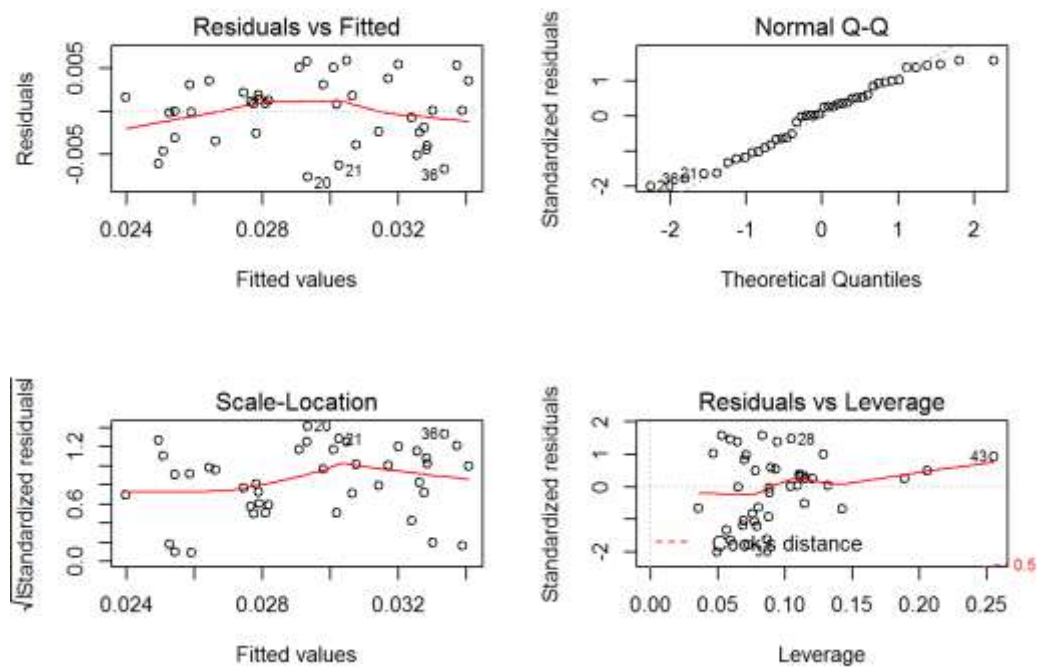


Figure 4: *Diagnostic Plots for Uganda Multiple Regression Model*

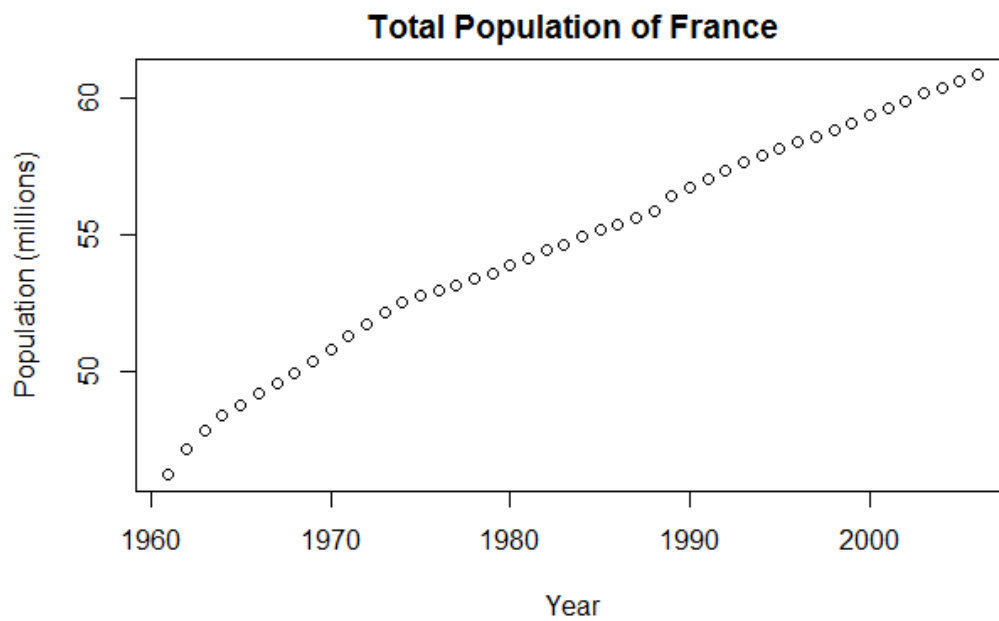


Figure 5: *Population Growth of France, 1961 – 2006*

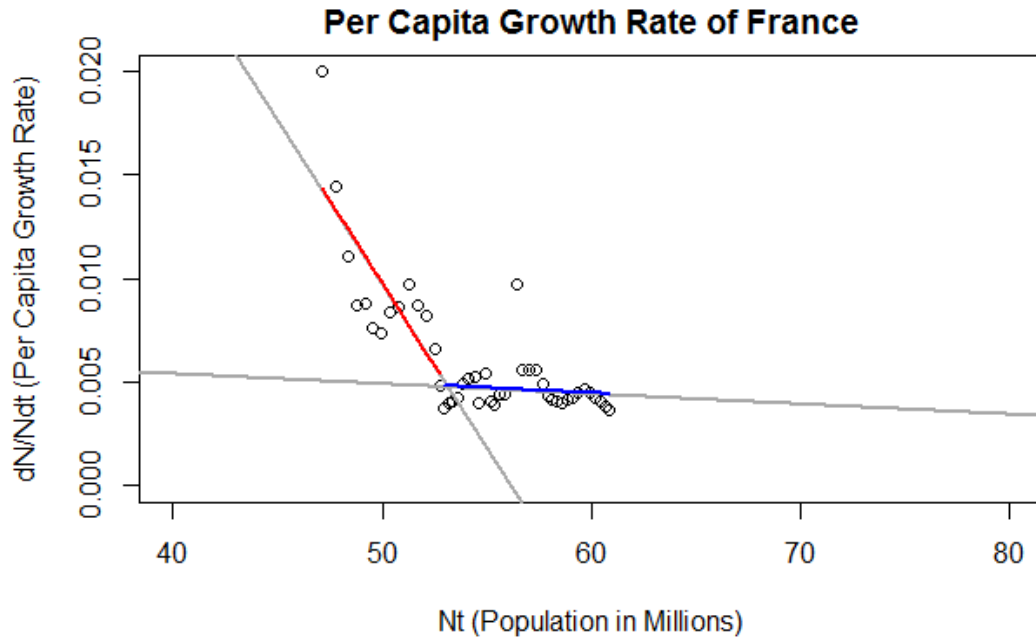


Figure 6: **Per Capita Growth Rate of France.** Red model - Per capita growth rate (dN/Ndt) is significantly predicted by population size (N_t) ($n = 11$) ($F(1,12) = 16.54$, $p < 0.001$, $R^2 = 0.56$). Blue model - Per capita growth rate (dN/Ndt) is significantly predicted by population size (N_t) ($n = 27$) ($F(1,28) = 0.7934$, $p < 0.001$, $R^2 = 0.03$).

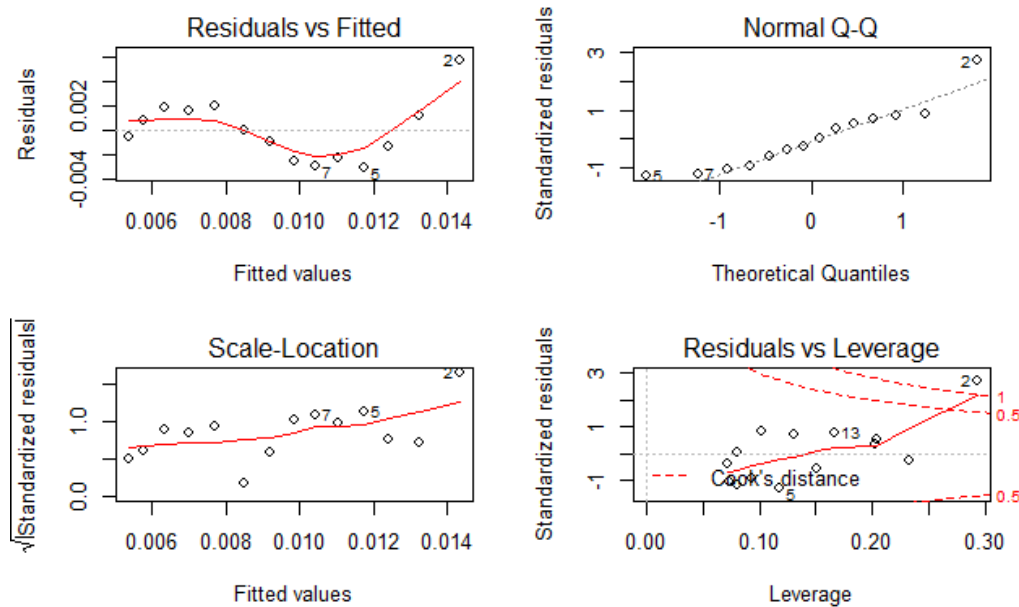


Figure 7: **Diagnostics for Linear Model of French Growth Rate from 1961 to 1975 (Red)**

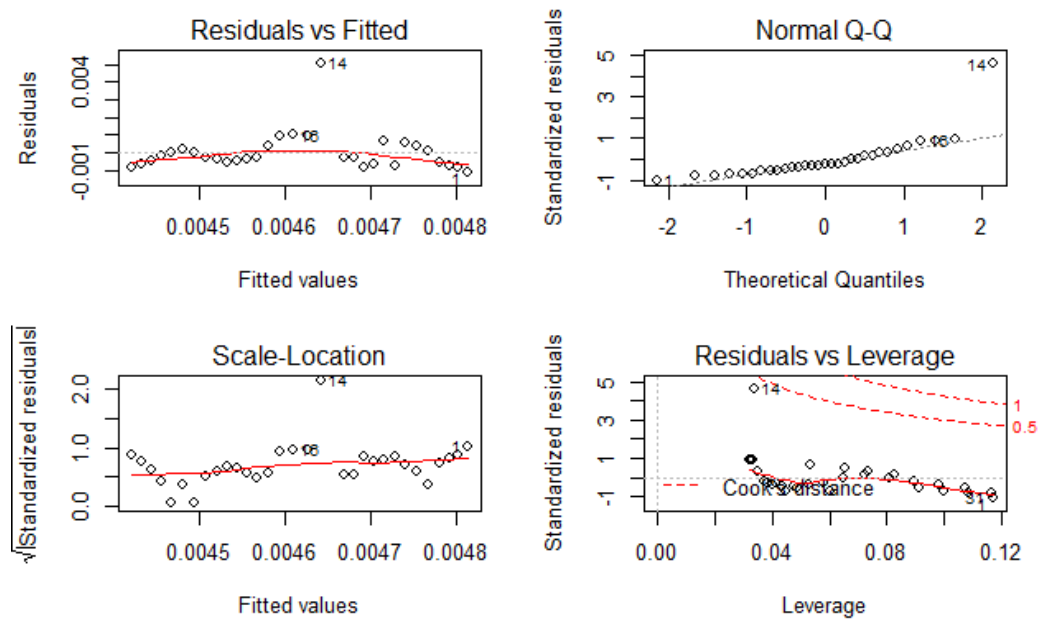


Figure 8: **Diagnostics for Linear Model of French Growth Rate from 1976 to 2006 (Blue)**

Table 2: **France Population Growth.** The model ($dN/Ndt = -0.006 \sim \text{Food per Capita} + 0.003 * \text{Years of Education} + -0.002 \text{ Population Size}$) explains a significant amount of variance in per capita population growth (dN/Ndt) ($F(3, 38) = 35.983, p < 0.001, R^2 = 0.74$).

<i>Dependent variable:</i>	
	dNNdt
Food per Capita	-0.006 (0.008)
Years of Education	0.003 ^{***} (0.001)
Population	-0.002 ^{***} (0.001)
Constant	0.172 [*] (0.092)
Observations	42
R ²	0.740
Adjusted R ²	0.719
Residual Std. Error	0.002 (df = 38)
F Statistic	35.983 ^{***} (df = 3; 38)
Note:	$p < 0.1$; $p < 0.05$; $p < 0.01$

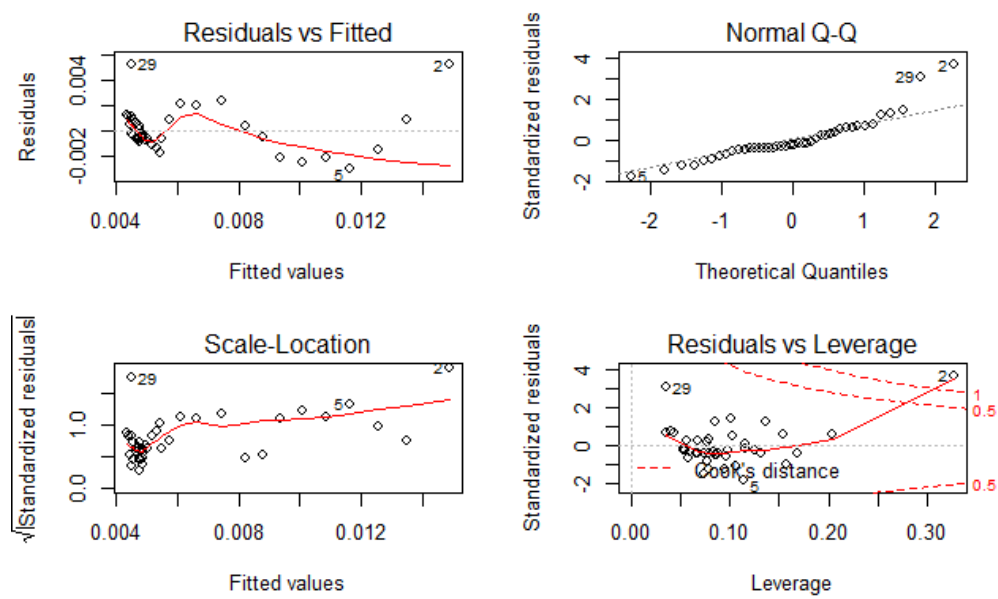


Figure 9: *Diagnostics for Linear Multi-Regression Model of French Growth*