Fundamentals of Statistical Modeling (VT21)

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Lab 1

Load the dataset and the `mlci` command

```
. version 14
. use https://raw.githubusercontent.com/anddis/fsm/master/data/lab1.dta, clear
. run https://raw.githubusercontent.com/anddis/fsm/master/do/mlci.do
```

Exercise 0 (together to get started)

We assume that $f(y_n)$ is normal. Estimate the parameters μ and σ.

```
. local f = "exp(-(y_n-{mu})^2/(2*{sigma}^2))/sqrt(2*_pi*{sigma}^2)"
. mlexp (ln(`f´))
initial:       log likelihood =     -<inf>  (could not be evaluated)
feasible:      log likelihood = -43652.817
rescale:       log likelihood = -1800.6375
rescale eq:    log likelihood = -1328.2447
Iteration 0:   log likelihood = -1328.2447  (not concave)
Iteration 1:   log likelihood = -1120.8309
Iteration 2:   log likelihood = -1064.5796
Iteration 3:   log likelihood = -1060.4702
Iteration 4:   log likelihood = -1059.3325
Iteration 5:   log likelihood = -1059.3319
Iteration 6:   log likelihood = -1059.3319
Maximum likelihood estimation
Log likelihood = -1059.3319                       Number of obs     =        300
------------------------------------------------------------------------------
             |      Coef.   Std. Err.      z    P>|z|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
         /mu |   178.4911   .4772459   374.00   0.000     177.5557    179.4265
      /sigma |   8.266142   .3374638    24.49   0.000     7.604725    8.927559
------------------------------------------------------------------------------
.
. di "The MLE for mu is: "_b[/mu]
The MLE for mu is: 178.49108
. di "The MLE for sigma is: "_b[/sigma]
The MLE for sigma is: 8.2661417
```

Exercise 1

Some of the distributions we'll work with in the course/labs are:

- Normal: `https://en.wikipedia.org/wiki/Normal_distribution`
- Skew-normal: `https://en.wikipedia.org/wiki/Skew_normal_distribution`
- Gamma: `https://en.wikipedia.org/wiki/Gamma_distribution`
- Beta: `https://en.wikipedia.org/wiki/Beta_distribution`
- Exponential: `https://en.wikipedia.org/wiki/Exponential_distribution`
- Chi-squared: `https://en.wikipedia.org/wiki/Chi-squared_distribution`
- Weibull: `https://en.wikipedia.org/wiki/Weibull_distribution`

- Bernoulli: https://en.wikipedia.org/wiki/Bernoulli_distribution
- Negative binomial: https://en.wikipedia.org/wiki/Negative_binomial_distribution

Get familiar with the distributions above.

- Are they useful to model continuous or discrete variables?
- What's their support?
- By how many parameters are they parametrized?
- What does their shape look like?

Exercise 2

We assume that $f(y_n)$ is normal. Estimate the parameters μ and σ. Use the `normalden()` function.

```
. local f = "normalden(y_n, {mu}, {sigma})"
. mlexp (ln(`f´))
initial:       log likelihood =     -<inf>  (could not be evaluated)
feasible:      log likelihood = -43652.817
rescale:       log likelihood = -1800.6375
rescale eq:    log likelihood = -1328.2447
Iteration 0:   log likelihood = -1328.2447  (not concave)
Iteration 1:   log likelihood = -1120.8299
Iteration 2:   log likelihood = -1064.4927
Iteration 3:   log likelihood = -1060.4292
Iteration 4:   log likelihood = -1059.3324
Iteration 5:   log likelihood = -1059.3319
Iteration 6:   log likelihood = -1059.3319
Maximum likelihood estimation
Log likelihood = -1059.3319                     Number of obs     =        300
```

	Coef.	Std. Err.	z	P>|z|	[95% Conf.	Interval]
/mu	178.4911	.4772459	374.00	0.000	177.5557	179.4265
/sigma	8.266142	.3374638	24.49	0.000	7.604725	8.927559

Estimate again the parameters μ and σ, but this time constrain the parameter σ to be positive by replacing $\sigma = \exp(\theta)$. You'll get now an estimate of θ. Recover the MLE of σ. (Note that the MLE $\hat{\sigma}$ hasn't changed – see slide 33 – but this "trick" improves computational stability).

```
. local sigma = "exp({theta})"
. local f = "normalden(y_n, {mu}, `sigma´)"
. mlexp(ln(`f´))
initial:       log likelihood =     -<inf>  (could not be evaluated)
feasible:      log likelihood = -32398.765
rescale:       log likelihood = -1981.1218
rescale eq:    log likelihood = -1440.3171
Iteration 0:   log likelihood = -1440.3171  (not concave)
Iteration 1:   log likelihood = -1112.6119
Iteration 2:   log likelihood = -1085.7986
Iteration 3:   log likelihood = -1059.4172
Iteration 4:   log likelihood =  -1059.332
Iteration 5:   log likelihood = -1059.3319
Maximum likelihood estimation
Log likelihood = -1059.3319                     Number of obs     =        300
```

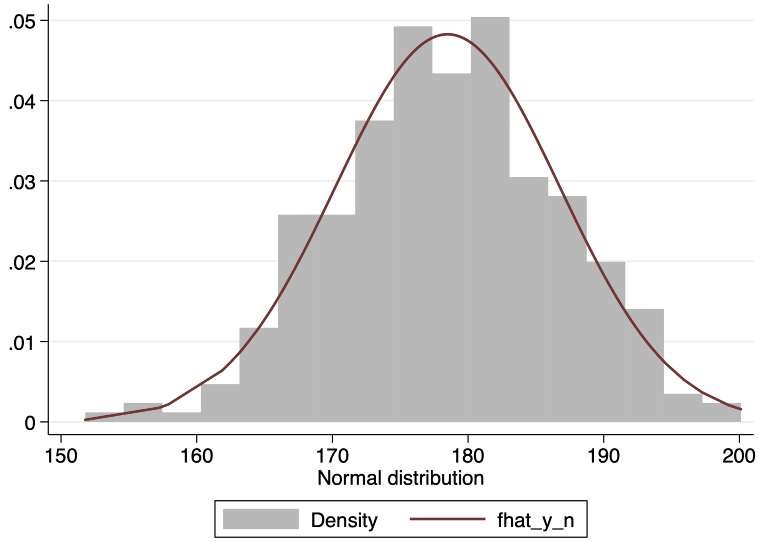
	Coef.	Std. Err.	z	P>|z|	[95% Conf.	Interval]
/mu	178.4911	.4772459	374.00	0.000	177.5557	179.4265
/theta	2.112168	.0408248	51.74	0.000	2.032153	2.192183

```
.
. di exp(_b[/theta])
8.2661406
```

Plot the estimated density $\hat{f}(y_n)$ over the sample histogram.

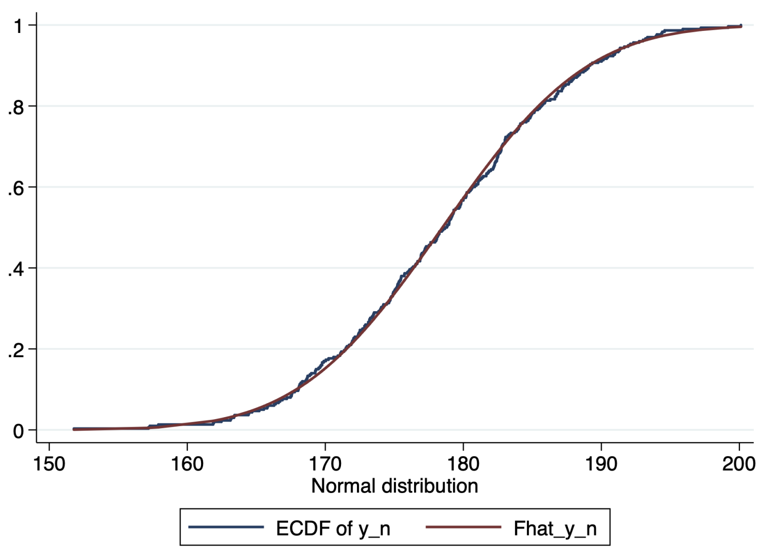
```
. gen fhat_y_n = normalden(y_n, _b[/mu], exp(_b[/theta]))
```

```
. tw (hist y_n) (line fhat_y_n y_n, sort), name(y_n, replace)

. graph export y_n.png, replace
(file y_n.png written in PNG format)
```



Plot the model-based estimated CDF $\hat{F}(y_n)$ (see slide 38) over the empirical CDF (see slide 20). The function `normal()` returns the CDF of a standard normal distribution.

```
. gen Fhat_y_n = normal((y_n - _b[/mu])/exp(_b[/theta]))

. cumul y_n, gen(sampleF_y_n)

. 
. // This is what -cumul- does under the hood
. // sort y_n
. // gen sampleF_y_n = sum(_cons)
. // su sampleF_y_n
. // replace sampleF_y_n = sampleF_y_n / r(max)
. 
. tw (line sampleF_y_n Fhat_y_n y_n, connect(J l) sort), name(Fy_n, replace)

. graph export Fy_n.png, replace
(file Fy_n.png written in PNG format)
```



Exercise 3

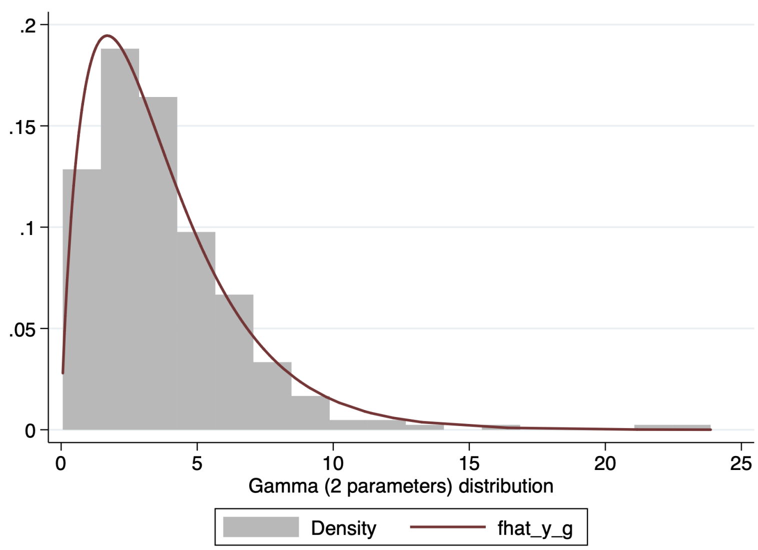
Assume that $f(y_g)$ is (2-parameter) gamma. Estimate the parameters α and β using the `gammaden()` function. Fix the location parameter g (the third argument of the `gammaden()` function), equal to 0.

```
. local f = "gammaden({alpha}, {beta}, 0, y_g)"
. mlexp(ln(`f´))
initial:       log likelihood =     -<inf>  (could not be evaluated)
feasible:      log likelihood = -2465.2095
rescale:       log likelihood = -670.37304
rescale eq:    log likelihood = -670.37304
Iteration 0:   log likelihood = -670.37304
Iteration 1:   log likelihood = -668.66319
Iteration 2:   log likelihood = -668.35073
Iteration 3:   log likelihood = -668.34648
Iteration 4:   log likelihood = -668.34648
Maximum likelihood estimation
Log likelihood = -668.34648                     Number of obs     =        300
```

	Coef.	Std. Err.	z	P>|z|	[95% Conf.	Interval]
/alpha	1.830878	.138033	13.26	0.000	1.560338	2.101418
/beta	2.042909	.1769745	11.54	0.000	1.696046	2.389773

Plot the estimated density $\hat{f}(y_g)$ over the sample histogram.

```
. gen fhat_y_g = gammaden(_b[/alpha], _b[/beta], 0, y_g)
. tw (hist y_g) (line fhat_y_g y_g, sort), name(y_g, replace)
. graph export y_g.png, replace
(file y_g.png written in PNG format)
```



Exercise 4

We assume that $f(y_b)$ is beta. Estimate the parameters α and β using the `betaden()` function.

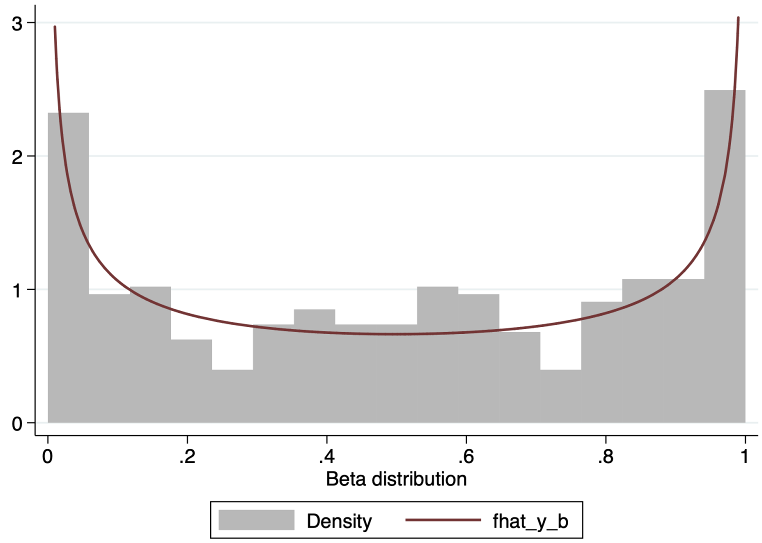
```
. local f = "betaden({alpha}, {beta}, y_b)"
. mlexp(ln(`f´))
initial:       log likelihood =     -<inf>  (could not be evaluated)
feasible:      log likelihood =  57.883521
rescale:       log likelihood =  57.883521
rescale eq:    log likelihood =  57.883521
Iteration 0:   log likelihood =  57.883521
Iteration 1:   log likelihood =  58.345242
Iteration 2:   log likelihood =  58.346286
```

```
Iteration 3:   log likelihood =  58.346286

Maximum likelihood estimation

Log likelihood =  58.346286                     Number of obs     =        300
```

	Coef.	Std. Err.	z	P>|z|	[95% Conf.	Interval]
/alpha	.5346582	.0394503	13.55	0.000	.457337	.6119794
/beta	.5286525	.0388869	13.59	0.000	.4524355	.6048694

Plot the estimated density $\hat{f}(y_b)$ over the sample histogram. (OBS: We need to be careful when plotting the estimated density function close to the boundaries of the distribution's support).

```
. gen fhat_y_b = betaden(_b[/alpha], _b[/beta], y_b)

. tw (hist y_b) (line fhat_y_b y_b if inrange(y_b, .01, .99), sort), name(y_b, replace)

. graph export y_b.png, replace
(file y_b.png written in PNG format)
```



Exercise 5

We assume that $f(y_e)$ is exponential. Estimate the parameter λ. You'll need to code the density of an exponential distribution yourself: $f(y;\lambda) = \lambda \exp(\lambda y)$

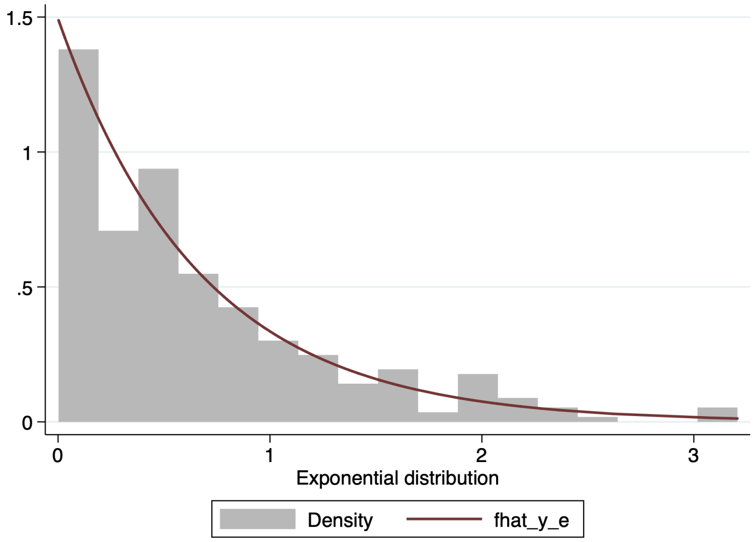
```
. local f = "{lambda}*exp(-y_e * {lambda})"

. mlexp(ln(`f´))

initial:       log likelihood =     -<inf>  (could not be evaluated)
feasible:      log likelihood =  -308.3512
rescale:       log likelihood = -193.68405
Iteration 0:   log likelihood = -193.68405
Iteration 1:   log likelihood = -181.67541
Iteration 2:   log likelihood = -179.57917
Iteration 3:   log likelihood = -179.57914
Iteration 4:   log likelihood = -179.57914

Maximum likelihood estimation

Log likelihood = -179.57914                     Number of obs     =        300
```

	Coef.	Std. Err.	z	P>|z|	[95% Conf.	Interval]
/lambda	1.493919	.0862515	17.32	0.000	1.324869	1.662969

Plot the estimated density $\hat{f}(y_e)$ over the sample histogram.

```
. gen fhat_y_e = _b[/lambda] * exp(-y_e * _b[/lambda])
```

```
. tw (hist y_e) (line fhat_y_e y_e, sort), name(y_e, replace)
. graph export y_e.png, replace
(file y_e.png written in PNG format)
```



Exercise 6

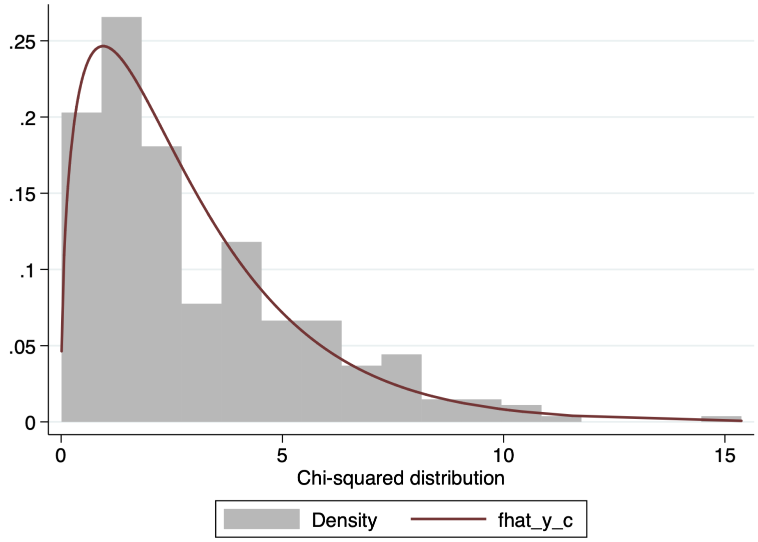
We assume that $f(y_c)$ is chi-squared. Estimate the parameter k using the `chi2den()` function.

```
. local f = "chi2den({k}, y_c)"
. mlexp(ln(`f´))
initial:       log likelihood =     -<inf>  (could not be evaluated)
feasible:      log likelihood = -1053.9877
rescale:       log likelihood = -660.78037
Iteration 0:   log likelihood = -660.78037
Iteration 1:   log likelihood = -628.29724
Iteration 2:   log likelihood = -626.40531
Iteration 3:   log likelihood = -626.40048
Iteration 4:   log likelihood = -626.40048
Maximum likelihood estimation
Log likelihood = -626.40048                     Number of obs     =        300
```

	Coef.	Std. Err.	z	P>|z|	[95% Conf. Interval]	
/k	2.950201	.1181046	24.98	0.000	2.718721	3.181682

Plot the estimated density $\hat{f}(y_c)$ over the sample histogram.

```
. gen fhat_y_c = chi2den(_b[/k], y_c)
. tw (hist y_c) (line fhat_y_c y_c, sort), name(y_c, replace)
. graph export y_c.png, replace
(file y_c.png written in PNG format)
```



Exercise 7

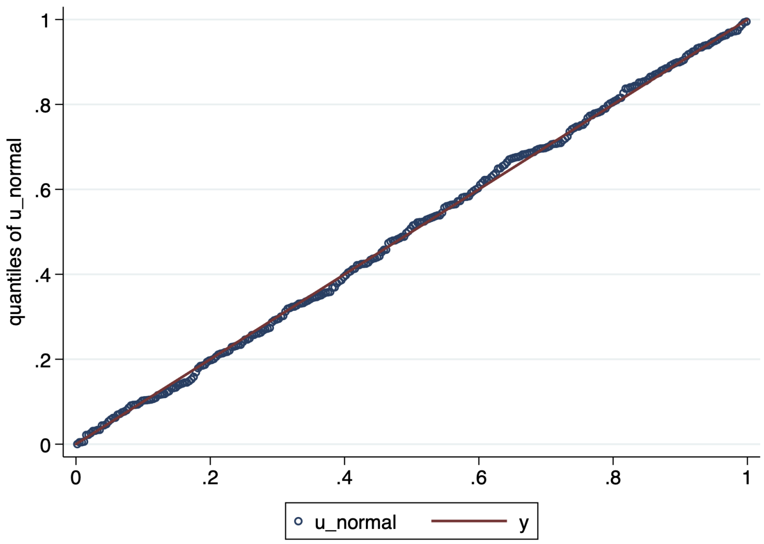
Install the qplot command (you need to be connected to the Internet)

```
. net sj 16-3 gr42_7
--------------------------------------------------------------------------------
package gr42_7 from http://www.stata-journal.com/software/sj16-3
--------------------------------------------------------------------------------

TITLE
      SJ16-3 gr42_7. Update: Quantile plots

DESCRIPTION/AUTHOR(S)
      Update: Quantile plots
      by Nicholas J. Cox, Durham University,
           Department of Geography, Durham, UK
      Support:  n.j.cox@durham.ac.uk
      After installation, type help qplot

INSTALLATION FILES                              (type net install gr42_7)
      gr42_7/qplot.ado
      gr42_7/qplot.sthlp
--------------------------------------------------------------------------------

. net install gr42_7
checking gr42_7 consistency and verifying not already installed...
all files already exist and are up to date.
```

Let's go back to the normal distributed variable (Exercise 1). Assume that $f(y_n)$ is normal and estimate the parameters μ and σ. Generate the transform $u = \hat{F}(y_n)$ as seen during the lecture this morning. Draw the estimated quantile plot using the qplot command.

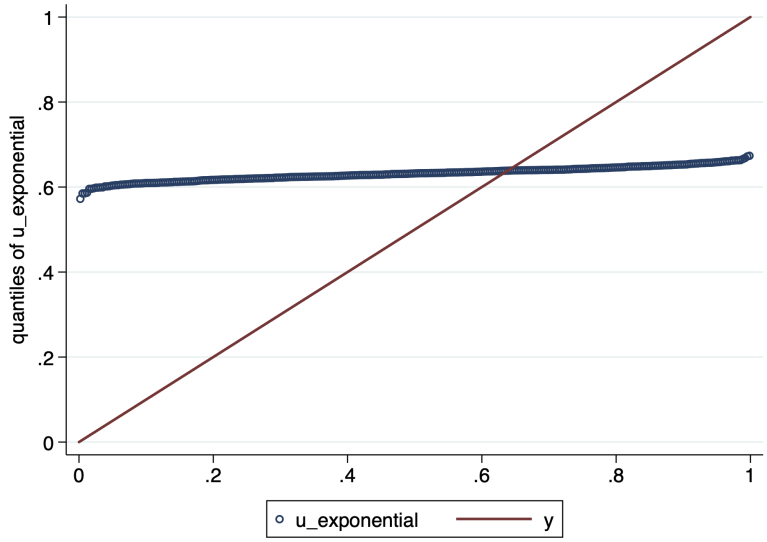
```
. local f = "normalden(y_n, {mu}, exp({theta}))"

. mlexp(ln(`f´))

initial:       log likelihood =     -<inf>  (could not be evaluated)
feasible:      log likelihood = -32398.765
rescale:       log likelihood = -1981.1218
rescale eq:    log likelihood = -1440.3171
Iteration 0:   log likelihood = -1440.3171  (not concave)
Iteration 1:   log likelihood = -1112.6119
Iteration 2:   log likelihood = -1085.7986
Iteration 3:   log likelihood = -1059.4172
Iteration 4:   log likelihood =  -1059.332
Iteration 5:   log likelihood = -1059.3319

Maximum likelihood estimation

Log likelihood = -1059.3319                     Number of obs     =        300

------------------------------------------------------------------------------
             |      Coef.   Std. Err.      z    P>|z|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
```

```
        /mu |   178.4911   .4772459   374.00   0.000     177.5557    179.4265
     /theta |   2.112168   .0408248    51.74   0.000     2.032153    2.192183
------------------------------------------------------------------------------

. mlci exp /theta
 8.266141   95% CI: 7.630494, 8.954739

.
. gen u_normal = normal((y_n-_b[/mu])/exp(_b[/theta]))

. qplot u_normal, addplot(function y = x) name(p1, replace)

. graph export p1.png, replace
(file p1.png written in PNG format)
```



Assume now that $f(y_n)$ is exponential and estimate the parameter λ. Generate the transform $u = \hat{F}(y_n)$. Draw the estimated quantile plot using the `qplot` command.

```
. local f = "exp({theta})*exp(-y_n * exp({theta}))"

. mlexp(ln(`f´))

initial:       log likelihood = -53547.324
alternative:   log likelihood = -32628.094
rescale:       log likelihood = -2180.7534
Iteration 0:   log likelihood = -2180.7534
Iteration 1:   log likelihood = -1857.2088
Iteration 2:   log likelihood = -1855.3682
Iteration 3:   log likelihood = -1855.3616
Iteration 4:   log likelihood = -1855.3616

Maximum likelihood estimation

Log likelihood = -1855.3616                     Number of obs     =        300

------------------------------------------------------------------------------
             |      Coef.   Std. Err.      z    P>|z|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
      /theta |  -5.184539    .057735   -89.80   0.000    -5.297697    -5.07138
------------------------------------------------------------------------------

. mlci exp /theta
 .0056025   95% CI: .0050031, .0062738

.
. gen u_exponential = 1-exp(-y_n * exp(_b[/theta]))

. qplot u_exponential, addplot(function y = x) name(p2, replace)

. graph export p2.png, replace
(file p2.png written in PNG format)
```



Comment the two quantile plots.

Exercise 8

Assume that y_{ber} follows a Bernoulli distribution. We want to estimate the probability of "success" ($y_{ber} = 1$). Estimate the probability η while constraining it to be bounded between 0 and 1. First, write down the likelihood by hand. Then, use the `binomialp()` function.

Are the results you obtain identical to those obtained from logistic regression?

```
. local eta = "invlogit({theta})"
. local f = "`eta´^y_ber * (1-`eta´)^(1-y_ber)"
. mlexp (ln(`f´))
initial:       log likelihood = -207.94415
alternative:   log likelihood =  -199.7231
rescale:       log likelihood =  -199.7231
Iteration 0:   log likelihood =  -199.7231
Iteration 1:   log likelihood = -199.70172
Iteration 2:   log likelihood = -199.70172
Maximum likelihood estimation
Log likelihood = -199.70172                     Number of obs     =        300
```

	Coef.	Std. Err.	z	P>|z|	[95% Conf. Interval]	
/theta	.4754237	.1187479	4.00	0.000	.2426821	.7081653

```
. mlci invlogit /theta
 .6166667   95% CI: .5603745, .6699956

.
. local eta = "invlogit({theta})"
. local f = "binomialp(1, y_ber, `eta´)"
. mlexp (ln(`f´))
initial:       log likelihood = -207.94415
alternative:   log likelihood =  -199.7231
rescale:       log likelihood =  -199.7231
Iteration 0:   log likelihood =  -199.7231
Iteration 1:   log likelihood = -199.70172
Iteration 2:   log likelihood = -199.70172
Maximum likelihood estimation
Log likelihood = -199.70172                     Number of obs     =        300
```

	Coef.	Std. Err.	z	P>|z|	[95% Conf. Interval]	

```
      /theta |   .4754237   .1187479     4.00   0.000     .2426821    .7081653
--------------------------------------------------------------------------------

. mlci invlogit /theta
 .6166667   95% CI: .5603745, .6699956

.
. logit y_ber

Iteration 0:   log likelihood = -199.70172
Iteration 1:   log likelihood = -199.70172

Logistic regression                             Number of obs     =        300
                                                LR chi2(0)        =       0.00
                                                Prob > chi2       =          .
Log likelihood = -199.70172                     Pseudo R2         =     0.0000

------------------------------------------------------------------------------
       y_ber |      Coef.   Std. Err.      z    P>|z|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
       _cons |   .4754237   .1187479     4.00   0.000     .2426821    .7081653
------------------------------------------------------------------------------
```

Now simplify the likelihood as shown in the slides. Check, again, that the estimate of η is the same.

```
. local eta = "invlogit({theta})"

. mlexp (y_ber*ln(`eta´)+(1-y_ber)*log(1-`eta´))

initial:       log likelihood = -207.94415
alternative:   log likelihood =  -199.7231
rescale:       log likelihood =  -199.7231
Iteration 0:   log likelihood =  -199.7231
Iteration 1:   log likelihood = -199.70172
Iteration 2:   log likelihood = -199.70172

Maximum likelihood estimation

Log likelihood = -199.70172                     Number of obs     =        300

------------------------------------------------------------------------------
             |      Coef.   Std. Err.      z    P>|z|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
      /theta |   .4754237   .1187479     4.00   0.000     .2426821    .7081653
------------------------------------------------------------------------------
```