Due Friday May 5th.

For reference, see the notes Paul gave in the discussion. His notes are the same as what we learned in class, but presented in a slightly different way. This is the first homework that Paul has designed. Please direct questions to him, as creating and answering homework questions is an important part of his training as a professor.

Homework 5

Consider a bus route that is 8.3 km in length and has a headway of 10 minutes. Including the termini, this route has 24 equally spaced stops, for a stop spacing of approximately 361 m.

Travel time from one end of the route to the other is 25 minutes. Use this as the *scheduled* component of travel time.

The *unscheduled* component of travel time has two terms. One is a random term $\gamma_{n,s}$, which follows an exponential distribution with $\lambda = 6.9$ [min^{-1}/stop]. It refers to random time added or lost on the way from one top to another (perhaps due to traffic or extra boarding). The other unscheduled term represents the effect that irregular headways have on boarding time. Deviations from the scheduled headway, h, speed up or slow down the bus at rate β. So if the actual headway at a stop is a minute larger than the scheduled headway, the bus gets delayed by $\beta = 0.1$ minutes. Likewise, when the actual headway is a minute smaller than the scheduled headway, the bus gets gains $\beta = 0.1$ min en route to the next stop (unless there is control).

(Note: In class we derived β from the arrival rate and the boarding time per pax, but here we are just subsuming the product of boarding time and passenger demand into a single parameter, for simplicity, and to avoid ridiculous situations with .25 extra passengers.)

a) Write the dynamic equation for the actual arrival time of bus n at stop $s + 1$.

b) What is the commercial speed in the uncontrolled case? Use the mean travel time as the schedule.

Now suppose that bus 0 is exactly on schedule, and bus 1 is dispatched on time from the beginning of the route but experiences a delay of 30 seconds at the first stop. (For your information, 30 seconds is approximately the amount of time it takes for a cyclist to load their bike onto the rack at the front of the bus, or for a relatively fast wheelchair boarding.)

c) How long does it take for bus 1 and bus 2 to bunch?

As you have learned, uncontrolled bus systems are inherently unstable. Therefore, this transit agency wants to implement conventional schedule control, adding slack time such that buses are on schedule 95% of the time.

d) How much slack time per stop is needed to achieve a 95% on time rating? You can assume that every stop is a control point.

e) What is the commercial speed now, with the slack from part (d) added to the schedule?