Bayesian Estimation of the Spectral Density

Project Proposal

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Main Goal

To estimate the spectral density of a stationary time series using Bayesian methods.

Details

Result. *Let $f(\omega_r) \neq 0, 1 \leq r \leq k$ where f denotes the spectral density of a stationary time series $\{X_t\}$. Then when $n \to \infty$ the joint distribution of the periodogram at ω_r, $I_n(\omega_r)$, tends to that of k mutually indepependent random variables distributed as Exponential$(2\pi f(\omega_r))$ for $0 < \omega_r < \pi$ [1].*

We would like to use the asymptotic distributional properties of a periodogram to obtain estimates of the spectral density at ω_r.

$$
\begin{aligned}
I_n(\omega_r)|f(\omega_r) &\dot{\sim} \text{ indep Exp}(2\pi f(\omega_r)) \\
f(\omega_r) &\sim \pi(\theta) \\
f(\omega_r)|I_n(\omega_r) &\propto 2\pi f(\omega_r) e^{-2\pi f(\omega_r) y} \pi(\theta)
\end{aligned}
$$

In order to implement this method, two fundamental sets of questions need to be explored. These involve

1. Prior distributions on ω_r
2. The asymptotic property relies on fixed frequencies.

Prior Distributions

A natural choice of prior for this analysis would be a conjugate Gamma distribution. We would also like to explore the possibility of using an improper prior.

Fixed Frequencies

It is common practice when dealing with the periodogram to work with the Fourier frequencies $\mathcal{F}_n$ of which there are n. However, the asymptotic result holds for a fixed number of frequencies k. We would like to explore at what ratio $\frac{k}{n}$ do the approximate independence and exponential distribution assumptions in the result break down in the periodogram values.

The goal would be to develop a procedure to find the largest acceptable k that still maintains the asymptotic result. This procedure would be completed before the Bayesian estimation of the spectral density.

In order to complete this step, simulations from known spectral densities will be used to estimate periodograms created from evenly spaced frequencies chopping up $(0, \pi)$ in decreasing density starting at n (Fourier frequencies). Monte Carlo approximations of first and second moments and a χ^2 goodness-of-fit test will be used to assess if the asymptotic result holds.

Application

We will use our Bayesian estimation method of the spectral density on a real dataset Maggie, talk about your data here.

References

[1] A. M. Walker. "Some asymptotic results for the periodogram of a stationary time series". In: *Journal of the Australian Mathematical Society* 5 (01 Feb. 1965), pp. 107–128. ISSN: 1446-8107. DOI: `10.1017/S1446788700025921`. URL: `http://journals.cambridge.org/article_S1446788700025921`.