

Bayesian Estimation of the Spectral Density

Project Proposal

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Main Goal

To estimate the spectral density of a stationary time series using Bayesian methods.

Details

Result. Let $f(\omega_r) \neq 0, 1 \leq r \leq k$ where f denotes the spectral density of a stationary time series $\{X_t\}$. Then when $n \rightarrow \infty$ the joint distribution of the periodogram at ω_r , $I_n(\omega_r)$, tends to that of k mutually independent random variables distributed as $\text{Exponential}(2\pi f(\omega_r))$ for $0 < \omega_r < \pi$ [1].

We would like to use the asymptotic distributional properties of a periodogram to obtain estimates of the spectral density at ω_r .

$$\begin{aligned} I_n(\omega_r) | f(\omega_r) &\overset{\cdot}{\sim} \text{indep Exp}(2\pi f(\omega_r)) \\ f(\omega_r) &\sim \pi(\theta) \\ f(\omega_r) | I_n(\omega_r) &\propto 2\pi f(\omega_r) e^{-2\pi f(\omega_r)y} \pi(\theta) \end{aligned}$$

In order to implement this method, two fundamental sets of questions need to be explored. These involve

1. Prior distributions on ω_r .
2. The asymptotic property relies on fixed frequencies.

Prior Distributions

A natural choice of prior for this analysis would be a [conjugate Gamma distribution](#). We would also like to explore the possibility of using an improper prior.

Fixed Frequencies

It is common practice when dealing with the periodogram to work with the Fourier frequencies \mathcal{F}_n of which there are n . However, the asymptotic result [holds for](#) a fixed number of frequencies k . We would like to explore at what ratio $\frac{k}{n}$ do the approximate independence and exponential distribution [assumptions in the result](#) break down in the periodogram values.

The goal would be to develop a procedure to find the largest acceptable k that still maintains [the asymptotic result](#). This procedure would be completed before the Bayesian estimation of the spectral density.

In order to complete this step, [simulations from known spectral densities will be used to estimate](#) periodograms created from evenly spaced frequencies chopping up $(0, \pi)$ in decreasing density starting at n (Fourier frequencies). [Monte Carlo approximations of first and second moments and a \$\chi^2\$ goodness-of-fit test will be used to assess if the asymptotic result holds.](#)

Application

We will use our Bayesian estimation method of the spectral density on a real dataset [of remotely sensed phenology data from India](#). The data consists of 2500 locations each with a time-series of weekly measurements of chlorophyll content over five years. A sample of locations will be used to [implement the method](#).

References

- [1] A. M. Walker. “Some asymptotic results for the periodogram of a stationary time series”. In: *Journal of the Australian Mathematical Society* 5 (01 Feb. 1965), pp. 107–128. ISSN: 1446-8107. DOI: 10.1017/S1446788700025921. URL: http://journals.cambridge.org/article_S1446788700025921.