Homework 5

Due June 19, 2017 in class

Please show all work for full credit. Print and staple your assignment together and submit by end of class the due date. If you cannot attend class on the due date, please arrange to submit your homework prior to the due date.

1. The mileage to first failure for a model of military personnel carrier can be modeled as exponential with mean $1,000$ miles.

   a) Evaluate the probability that a vehicle of this type gives less than 500 miles of service before first failure. Evaluate the probability that a vehicle of this type gives less than 2000 miles of service before first failure.
   b) Find the .05 quantile of the distribution of mileage to first failure. Then find the .9 quantile of the distribution.

2. [Ch. 5.2, Exercise 2, pg. 263] Suppose that Z is a standard normal random variable. Evaluate the following probabilities involving Z:

   a) $P[Z < -.62]$
   b) $P[Z > 1.06]$
   c) $P[-.37 < Z < .51]$
   d) $P[|Z| \leq .47]$
   e) $P[|Z| > .93]$
   f) $P[-3.0 < Z < 3.0]$

   Now find numbers # such that the following statements involving Z are true:

   g) $P[Z \leq \#] = .90$
   h) $P[|Z| \leq \#] = .90$
   i) $P[|Z| > \#] = .03$

3. [Ch. 5.2, Exercise 3, pg. 263] Suppose that X is a normal random variable with mean 43 and standard deviation 3.6. Evaluate the following probabilities involving X:

   a) $P[X < 45.2]$
   b) $P[X \leq 41.7]$
   c) $P[43.8 < X \leq 47.0]$
   d) $P[|X - 43| \leq 2]$
   e) $P[|X - 43| > 1.7]$

   Now find numbers # such that the following statements involving X are true:

   f) $P[X < \#] = .95$
   g) $P[X \geq \#] = .30$
   h) $P[|X - 43| > \#] = .05$

4. [Ch. 5, Exercise 7, pg. 323] In a grinding operation, there is an upper specification of 3.150 in. on a dimensions of a certain part after grinding. Suppose that the standard deviation of this normally distributed dimension for parts of this type ground to any particular mean dimension μ is $\sigma = 0.002$ in. Suppose further that you desire to have no more than 3% of the parts fail to meet specifications. What is the maximum (minimum machining cost) μ that can be used if this 3% requirement is to be met?

5. [Ch 5, Exercise 42, pg. 332] Suppose that engineering specifications on the shelf depth of a certain slug to be turned on a CNC lathe are from 0.0275 in. to 0.0278 in. and that values of this dimension produced on the lathe can be described using a normal distribution with mean μ and standard deviation σ.

a) If $\mu = 0.0276$ and $\sigma = 0.0001$, about what fraction of shelf depths are in specifications?
b) What machine precision (as measured by σ) would be required in order to produce about 98% of shelf depths within engineering specifications (assuming that μ is at the midpoint of the specifications)?

6. [Ch 5.5, Exercise 3, pg. 322] Consider again the random number generator from Homework 4, Number 7. Suppose that it is used to generate 25 random numbers and that these may reasonable be thought of as independent random variables with common individual (marginal) distribution

$$f(x) = \begin{cases} k(5-x) & 0 < x < 1 \\ 0 & \text{otherwise} \end{cases}.$$

Let $\overline{X}$ be the sample mean of these 25 values.

a) What are the mean and standard deviation of the random variable $\overline{X}$?
b) What is the approximate probability distribution of $\overline{X}$?
c) Approximate the probability that $\overline{X}$ exceeds 0.5.
d) Approximate the probability that $\overline{X}$ takes a value within 0.02 of its mean.
e) Redo parts a) through d) using a sample size of 100 instead of 25.

7. [Ch 5, Exercise 10, pg. 324] Suppose that the thickness of sheets of a certain weight of book paper have mean 0.1 mm and a standard deviation of 0.003 mm. A particular textbook will be printed on 370 sheets of this paper. Find sensible values for the mean and standard deviation of the thicknesses of copies of the text (excluding the books' cover).

8. [Ch 5, Exercise 20, pg. 326] Suppose that the raw daily oxygen purities delivered by an air-products supplier have a standard deviation $\sigma \approx .1$ (percent), and it is plausible to think of daily purities as independent random variables. Approximate the probability that the sample mean $\overline{X}$ of $n = 25$ delivered purities falls within 0.03 (percent) of the raw daily purity mean, μ.