5.2 Continuous random variables

It is often convenient to think of a random variable as having a whole (continuous) interval for its set of possible values.

The devices used to describe continuous probability distributions differ from those that describe discrete probability distributions.

Examples of continuous random variables:

5.2.1 Probability density functions and cumulative distribution functions

A *probability density function (pdf)* is the continuous analogue of a discrete random variable's probability mass function (pmf).

Definition 5.12. A *probability density function (pdf)* for a continuous random variable X is a nonnegative function $f(x)$ with

$$\int_{-\infty}^{\infty} f(x) = 1$$

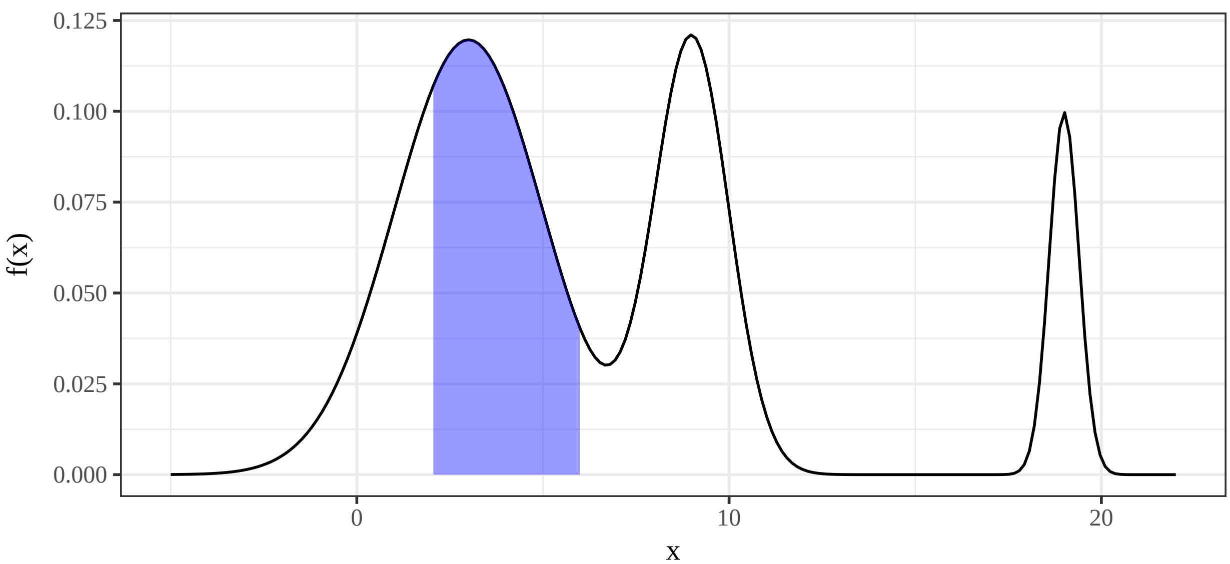
and such that for all $a \leq b$,

$$P[a \leq X \leq b] = \int_{a}^{b} f(x)dx.$$

1.

2.

3.



Example 5.17 (Compass needle)**.** Consider a de-magnetised compass needle mounted at its centre so that it can spin freely. It is spun clockwise and when it comes to rest the angle, θ, from the vertical, is measured. Let

$$Y = \text{the angle measured after each spin in radians}$$

What values can Y take?

What form makes sense for $f(y)$?

If this form is adopted, that what must the pdf be?

Using this pdf, calculate the following probabilities:

1. $P[Y < \frac{\pi}{2}]$

2. $P[\frac{\pi}{2} < Y < 2\pi]$

3. $P[\frac{\pi}{6} < Y < \frac{\pi}{4}]$

4. $P[Y = \frac{\pi}{6}]$

Definition 5.13. The *cumulative distribution function (cdf)* of a continuous random variable X is a function F such that

$$F(x) = P[X \leq x] = \int\limits_{-\infty}^{x} f(t)dt$$

$F(x)$ is obtained from $f(x)$ by integrtion, and applying the fundamental theorem of calculus yields

$$\frac{d}{dx}F(x) = f(x).$$

That is, $f(x)$ is obtained from $F(x)$ by differentiation.

As with discrete random variables, F has the following properties:

1.

2.

3.

Example 5.18 (Compass needle, cont'd)**.** Recall the compass needle example, with

$$f(x) = \begin{cases} \frac{1}{2\pi} & 0 \leq y \leq 2\pi \\ 0 & \text{otherwise} \end{cases}$$

Find the cdf.

For $y < 0$

For $0 \leq y \leq 2\pi$

For $y > 2\pi$

Calculate the following using the cdf:

$F(1.5)$

$P[Y \leq \frac{4\pi}{5}]$

$P[Y > 5]$

$P[\frac{\pi}{3} < Y \leq \frac{\pi}{2}]$

5.2.2 Means and variances for continuous distributions

5.2.3 Quantiles

5.2.4 The Normal distribution

5.2.5 Other special continuous distributions