Qn 1

Let X and Y be binomial random variables, representing the number of shots by Andrew & Baylen respectively after 3 rounds.
Let $i = \{0, 1, 2, 3\}$ represent the round number.

$Pr(X=i) = \binom{3}{i}(0.7)^i(1-0.7)^{3-i}$
$Pr(Y=i) = \binom{3}{i}(0.5)^i(1-0.5)^{3-i}$

$Pr(X=0) = 0.027$, $P(X=1) = 0.189$, $P(X=2) = 0.441$, $P(X=3) = 0.343$
$Pr(Y=0) = 0.125$, $P(Y=1) = 0.375$, $P(Y=2) = 0.375$, $P(Y=3) = 0.125$

(i) $Pr(X=Y) = (0.027 \times 0.125) + (0.189 \times 0.375) + (0.441 \times 0.375) + (0.343 \times 0.125)$
$= 0.2825$ //

(ii) $Pr(X>Y) = 0.343(1-0.125) + 0.441(0.125+0.375) + 0.189(0.125)$
$= 0.544$ //

Qn 2

(i) Let X be a binomial RV whose pmf is $p(x; n, p)$ where $\underset{\text{dice}}{n=2}$ and $\underset{\text{of getting sum}=5\text{ when roll 2 dice}}{p = \frac{4}{6\times6}} = \frac{1}{9}$

$\underbrace{Pr(X=2)}_{\text{can only win if both tries succeed}} = \binom{2}{2}\left(\frac{1}{9}\right)^2\left(1-\frac{1}{9}\right)^0 = \frac{1}{81}$ //

(ii) Place both balls in box labelled 7. (best strategy).
Let Y be a binomial RV whose pmf is $p(y; n, p)$, where $n=2$ and $p = \frac{6}{36} = \frac{1}{6}$
$Pr(Y=2) = \binom{2}{2}\left(\frac{1}{6}\right)^2\left(1-\frac{1}{6}\right)^0 = \frac{1}{36}$ //

7: 1 2 3 4 5 6
1 2 3 4 5 6

8: 1 2 3 4 5 6
1 2 3 4 5 6

$p = \frac{6}{36}$

Qn J

Let A_w and A_b represent the event that a white & black ball is selected respectively from box A, and B_w and B_b represent likewise for box B.

(i) $Pr(B_b | A_w) = Pr(B_b)$ (the events are independent)

$= \frac{6}{9} = 0.667$ //

(ii) Let X represent the event that the other ball is black, and Y represent the event that one of the balls is white.

$$\underbrace{Pr(X|Y)}_{\text{independent}} = \frac{Pr(X)}{Pr(Y)} = \frac{\overset{A_w}{\frac{5}{9}} \times \overset{B_b}{\frac{6}{9}} + \overset{A_b}{\frac{4}{9}} \times \overset{B_w}{\frac{3}{9}}}{\underbrace{\frac{5}{9} + \frac{3}{9} - \frac{5}{9} \times \frac{3}{9}}_{A_w \cup B_w}}$$

$= \frac{14}{19} = 0.737$ //

(iii) Let X represent the event that the transferred ball from A is black.
Let Y represent the event that the ball selected from B afterwards is black

$$Pr(X|Y) = \frac{Pr(X \cap Y)}{Pr(Y)} = \frac{\overset{A_b}{\frac{4}{9}} \times \frac{7}{10}}{\underbrace{\frac{4}{9} \times \frac{7}{10}}_{A_b \text{ selected}} + \underbrace{\frac{5}{9} \times \frac{6}{10}}_{A_b \text{ not selected}}}$$

$= 0.483$ //

Qn 4

let C and C^c represent the event that the patient ~~is~~ has critical condition and doesn't have critical conditions respectively.

let S and S^c represent the events that the patient is a smoker and non-smoker respectively.

Given:

$Pr(C|S) = 2\,Pr(C|S^c) \Rightarrow Pr(C|S^c) = \frac{1}{2} Pr(C|S)$

$Pr(S) = 0.32$

$$Pr(S|C) = \frac{Pr(C|S)\,Pr(S)}{Pr(C|S)\,Pr(S) + Pr(C|S^c)\,Pr(S^c)}$$ (posterior probability)

$$= \frac{\cancel{Pr(C|S)}\,Pr(S)}{\cancel{Pr(C|S)}\,Pr(S) + \frac{1}{2}\cancel{Pr(C|S)}\,Pr(S^c)}$$

$$= \frac{Pr(S)}{Pr(S) + \frac{1}{2}Pr(S^c)}$$

$$= \frac{0.32}{0.32 + \frac{1}{2}(1-0.32)}$$

$$= \frac{16}{33}$$ //

Qn 5

(i) $\int_{-\infty}^{\infty} f(x)\,dx = \int_{-2}^{0} \frac{k}{x^2+2x+2}\,dx = 1$

let $u = x+1$, $\frac{du}{dx} = 1$, $dx = du$ $\qquad x = u-1$

$\int \frac{k}{(u-1)^2+2(u-1)+2}\,du = \int \frac{k}{u^2-2u+1+2u-2+2}\,du$

$= \int \frac{k}{u^2+1}\,du$

$= k[\tan^{-1}(u)] + c$

$\therefore \int_{-2}^{0} \frac{k}{x^2+2x+2}\,dx = k[\tan^{-1}(x+1)]_{-2}^{0}$

$= k(0.785 - (-0.785))$

$= 1.5708k$

$= 1$

$\therefore k = 0.6366$ //

(ii) $Pr(X \leq -\frac{1}{\sqrt{2}}) = 0.6366 \int_{-2}^{-\frac{1}{\sqrt{2}}} \frac{1}{x^2+2x+2}\,dx$

$= 0.6366 \left[\tan^{-1}(x+1)\right]_{-2}^{-\frac{1}{\sqrt{2}}}$

$= 0.6366(0.285 - (-0.785))$

$= 0.681$ //