1 Analytic expressions

The partition function is given by:

$$Z = \sum_{i=1}^{M} e^{-\beta E_i}$$

where M is the number of microstates, $\beta = 1/kT$ and E_i is the energy of the system in configuration i. The mean energy is given by:

$$\begin{aligned}
\langle E \rangle &= \frac{1}{Z} \sum_{i=1}^{M} E_i e^{-\beta E_i} \\
&= \frac{1}{e^{8\beta J}} \left(-16J e^{8\beta J} + 16J e^{-8\beta J} \right) \\
&= \frac{1}{e^{8\beta J}} \left(-16J \left(e^{8\beta J} - e^{-8\beta J} \right) \right) \\
&= \frac{1}{e^{8\beta J}} \left(-32J \sinh 8\beta J \right)
\end{aligned}$$

The susceptibility is given by:

$$\chi = \frac{1}{k_B T} \left(\sigma_M^2 \right)$$

where the variance is given by:

$$\begin{aligned}
\sigma_M^2 &= \langle M^2 \rangle - \langle M \rangle^2 \\
&= \frac{1}{Z} \sum_{i=1}^{M} M_i^2 e^{-\beta E_i} - \left(\frac{1}{Z} \sum_{i=1}^{M} M_i e^{-\beta E_i} \right)^2
\end{aligned}$$

The heat capacity is given by:

$$C_V = \frac{1}{k_B T^2} \left(\angle E^2 \rangle - \langle E \rangle^2 \right)^2$$

The energy of a specific configuration is:

$$E_i = -J \sum_{<kl>}^{N} s_k s_l$$