# Lista 01 de Sistemas Lineares

**1.4.1** Find the standard matrix of the following linear transformations:

$*$ **(a)** $T(v_1,v_2) = (v_1 + 2v_2, 3v_1 - v_2)$.

**(b)** $T(v_1,v_2) = (v_1 + v_2, 2v_1 - v_2, -v_1 + 3v_2)$.

$*$ **(c)** $T(v_1,v_2,v_3) = (v_1 + v_2, v_1 + v_2 - v_3)$.

**(d)** $T(v_1,v_2,v_3) = (v_2, 2v_1 + v_3, v_2 - v_3)$.

**1.4.2** Find the standard matrices of the linear transformations $T$ that act as follows:

$*$ **(a)** $T(1,0) = (3,-1)$ and $T(0,1) = (1,2)$.

**(b)** $T(1,0) = (1,3)$ and $T(1,1) = (3,7)$.

$*$ **(c)** $T(1,0) = (-1,0,1)$ and $T(0,1) = (2,3,0)$.

**(d)** $T(1,0,0) = (2,1)$, $T(0,1,0) = (-1,1)$, and $T(0,0,1) = (0,3)$.

$*$**(e)** $T(1,0,0) = (1,2,3)$, $T(1,1,0) = (0,1,2)$, and $T(1,1,1) = (0,0,1)$.

**1.4.6** Find the standard matrix of the composite linear transformation $S \circ T$, when $S$ and $T$ are defined as follows:

$*$**(a)** $S(v_1,v_2) = (2v_2, v_1 + v_2)$,
$T(v_1,v_2) = (v_1 + 2v_2, 3v_1 - v_2)$.

**(b)** $S(v_1,v_2) = (v_1 - 2v_2, 3v_1 + v_2)$,
$T(v_1,v_2,v_3) = (v_1 + v_2, v_1 + v_2 - v_3)$.

$*$**(c)** $S(v_1,v_2,v_3) = (v_1, v_1 + v_2, v_1 + v_2 + v_3)$,
$T(v_1,v_2) = (v_1 + v_2, 2v_1 - v_2, -v_1 + 3v_2)$.

**1.4.4** Determine which of the following functions $T : \mathbb{R}^2 \to \mathbb{R}^2$ are and are not linear transformations. If $T$ is a linear transformation, find its standard matrix. If it is not a linear transformation, justify your answer (i.e., show a property of linear transformations that it fails).

$*$**(a)** $T(v_1,v_2) = (v_1^2, v_2)$.

**(b)** $T(v_1,v_2) = (v_1 + 2v_2, v_2 - v_1)$.

$*$**(c)** $T(v_1,v_2) = (\sin(v_1) + v_2, v_1 - \cos(v_2))$.

**2.1.2** Use Gauss–Jordan elimination to compute the reduced row echelon form of each of the following matrices.

*(a) $\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$

(b) $\begin{bmatrix} 2 & 3 & -1 \\ 4 & 1 & 3 \end{bmatrix}$

*(c) $\begin{bmatrix} 1 & 2 \\ 3 & 6 \\ -1 & -2 \end{bmatrix}$

(d) $\begin{bmatrix} 3 & 9 & 3 \\ 1 & 3 & 0 \\ 1 & 3 & 2 \end{bmatrix}$

*(e) $\begin{bmatrix} -1 & 2 & 2 \\ 4 & -1 & 1 \\ 2 & 4 & 2 \end{bmatrix}$

(f) $\begin{bmatrix} -1 & 2 & 7 & 2 \\ 4 & -1 & -7 & 1 \\ 2 & 4 & 10 & 2 \end{bmatrix}$

**2.1.6** Use computer software to find all solutions of the following systems of linear equations.

*(a)
$$\begin{aligned} 6v + 5w + 3x - 2y - 2z &= 1 \\ 3v - w + x + 5y + 4z &= 2 \\ 3v + 4w + x + 3y + 4z &= 3 \\ 2v + 6w + x - 2y - z &= 4 \end{aligned}$$

(b)
$$\begin{aligned} v - w + 2x + 6y + 6z &= 3 \\ 4v + 3w - y + 4z &= 0 \\ 5v + w - 2x - 2y - 2z &= 2 \\ w - 2x + 3y - 2z &= -1 \\ v + 5w - x + 5z &= 3 \end{aligned}$$

**2.3.1** Determine which of the following sets are and are not subspaces.

*(g) $\{(x,y) \in \mathbb{R}^2 : x + 2y = 0\}$

(h) $\{(x,y) \in \mathbb{R}^2 : x + y \geq 0\}$

*(i) $\{(x,y) \in \mathbb{R}^2 : xy \geq 0\}$

(j) $\{(x,y,z) \in \mathbb{R}^3 : xy + yz = 0\}$

**2.3.6** Determine which of the following sets are and are not linearly independent.

*(a) $\{(1,2),(3,4)\}$
(b) $\{(1,0,1),(1,1,1)\}$
*(c) $\{(1,0,-1),(1,1,1),(1,2,-1)\}$
(d) $\{(1,2,3),(4,5,6),(7,8,9)\}$
*(e) $\{(1,1),(2,1),(3,-2)\}$
(f) $\{(2,1,0),(0,0,0),(1,1,2)\}$
*(g) $\{(1,2,4,1),(2,4,-1,3),(-1,1,1,-1)\}$

**2.3.7** Use computer software to determine which of the following sets of vectors span all of $\mathbb{R}^4$.

*(a) $\{(1,2,3,4),(3,1,4,2),(2,4,1,3),(4,3,2,1)\}$
(b) $\{(4,2,5,2),(3,1,2,4),(1,4,2,3),(3,1,4,2)\}$
*(c) $\{(4,4,4,3),(3,3,-1,1),(-1,2,1,2),$
$(1,0,1,-1),(3,3,2,2)\}$

**2.3.17** Use computer software to determine whether or not $\mathbf{v} = (1,2,3,4,5)$ is in the range of the given matrix.

*(a) $\begin{bmatrix} 3 & -1 & 1 & 0 & -1 \\ 2 & 1 & -1 & 4 & 0 \\ 1 & 1 & -1 & 1 & 2 \\ 2 & 1 & 2 & 0 & 0 \\ -1 & 3 & -1 & 3 & 2 \end{bmatrix}$

(b) $\begin{bmatrix} 4 & 1 & 2 & 3 & 2 & 2 & -1 \\ 0 & -1 & 4 & 2 & 0 & 4 & 2 \\ -1 & 2 & 0 & 2 & 4 & 3 & 3 \\ 3 & 1 & -1 & 0 & 2 & 2 & 3 \\ -1 & 0 & -1 & 4 & 3 & 0 & 3 \end{bmatrix}$

**2.3.10** For what values of $k$ is the following set of vectors linearly independent?

$$\{(1,2,3),(-1,k,1),(1,1,0)\}$$

**2.4.5** For each of the following matrices $A$, find bases for each of range$(A)$, null$(A)$, range$(A^T)$, and null$(A^T)$.

$*$(g) $\begin{bmatrix} 1 & 2 & 0 & 3 & 0 \\ 0 & 0 & 1 & -1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}$ (h) $\begin{bmatrix} 0 & 0 & 1 & 1 \\ 0 & 1 & 1 & 0 \\ 1 & 1 & 0 & 0 \end{bmatrix}$

$*$(i) $\begin{bmatrix} 0 & -4 & 0 & 2 & 1 \\ -1 & 2 & 1 & 2 & 1 \\ -2 & 0 & 2 & 6 & 3 \end{bmatrix}$

$**$ **2.D.9** Let

$$A = \begin{bmatrix} 2 & -1 & 0 & 0 & 0 & 0 \\ -1 & 3 & -1 & 0 & 1 & 0 \\ 0 & -1 & 4 & -1 & 0 & 0 \\ 0 & 0 & -1 & 4 & -1 & 0 \\ 0 & 0 & 0 & -1 & 3 & -1 \\ 0 & 1 & 0 & 0 & -1 & 2 \end{bmatrix}.$$

(a) Use computer software to compute a unit LU decomposition of $A$.

**Exemplo 6:** *A transformação linear* $T : \mathbb{R}^3 \longrightarrow P_2(\mathbb{R})$ *definida por:*

$$T(a,b,c) = (a-b) + (c-a)x + (b+c)x^2$$

*é um isomorfismo de* $\mathbb{R}^3$ *em* $P_2(\mathbb{R})$.

Mostre que a afirmação do exemplo 6 é verdadeira;