Construction of DC-DC Converters Fuzzy Models

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1 Takagi-Sugeno Fuzzy Model

The design procedure begins with representing a given non-linear plant by the Takagi-Sugeno fuzzy model. This model is characterized by fuzzy IF-THEN rules which describe local linear input-output relations of a non-linear system. The TS Fuzzy model expresses the local dynamics of each fuzzy rule using a linear system model, while the global model is achieved by combining these linear system models

The i-th fuzzy rules for Continuous Fuzzy Systems (CFS) are of the following forms:

Model Rule i:

IF $z_1(t)$ is M_{i1} and $\ldots$ and $z_p(t)$ is M_{ip}

$$\textbf{THEN} \begin{cases} \dot{x} = A_i x(t) + B_i u(t) \\ y = C_i x, \end{cases} \quad , \quad i = 1, 2, \ldots, r.$$

Here, M_{ij} is the fuzzy set and r is the number of model rules; $x(t) \in \mathbb{R}^n$ is the state vector, $u(t) \in \mathbb{R}^m$ is the input vector, $y(y) \in \mathbb{R}^q$ is the output vector, $A_i \in \mathbb{R}^{n\times n}$, $B_i \in \mathbb{R}^{n\times m}$ and $C_i \in \mathbb{R}^{q\times n}$; $z_i(t), \ldots, z_p(t)$ are known premise variables which may be functions of the state variables, external disturbances, and/or time.

Given a pair of $(x(t), u(t))$, the final outputs of the CFS are inferred as follows:

$$\dot{x} = \frac{\sum\limits_{i=1}^{r} w_i(z(t))\{A_i x(t) + B_i u(t)\}}{\sum\limits_{i=1}^{r} w_i(z(t))} = \sum_{i=1}^{r} h_i(z(t))\{A_i x(t) + B_i u(t)\} \tag{1}$$

$$y(t) = \frac{\sum\limits_{i=1}^{r} w_i(z(t)) C_i x(t)}{\sum\limits_{i=1}^{r} w_i(z(t))} = \sum_{i=1}^{r} h_i(z(t)) C_i x(t) \tag{2}$$

where $z(t) = [z_1(t), z_2(t), \ldots, z_p(t)]$,

$$w_i(z(t)) = \prod_{j=1}^{p} M_{ij}(z_j(t)) \qquad \text{and} \qquad h_i(z(t)) = \frac{w_i(z(t))}{\sum_{j=1}^{r} w_i(z(t))}, \tag{3}$$

for all time t. The term $M_{ij}(z(t))$ is the grade of membership of $z_j(t)$ in M_{ij}. Since

$$\sum_{i=1}^{r} w_i(z(t)) > 0, \quad w_i(z(t)) \geq 0, \quad i = 1, 2, \ldots, r, \tag{4}$$

we have

$$\sum_{i=1}^{r} h_i(z(t)) > 0, \quad h_i(z(t)) \geq 0, \quad i = 1, 2, \ldots, r. \tag{5}$$

2 Buck Converter

The buck converter circuit is illustrated in Figure 1. In this circuit, $v_{\text{in}}(t)$ represents the input voltage, and D denotes the diode. The output voltage filter consists of an inductor with winding resistance R_L and inductance L, and a capacitor with capacitance C. Finally, two types of loads are supplied: a Constant Resistance Load with resistance $R(t)$ and a Constant Power Load (CPL) with power $P_\ell(t)$, represented by a controlled current source.

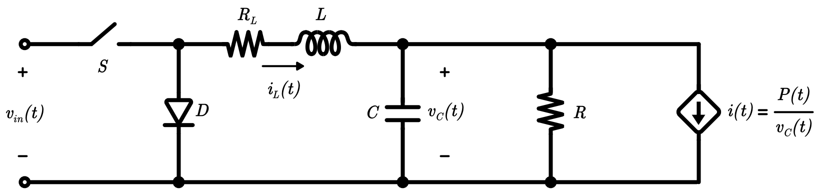

Fig. 1: Buck converter circuit.

2.1 Non-linear Buck Converter Model

When the switch S is in the off state fot the duration t_{off}, the dynamic equations describing the circuit's behavior at this moment, derived from Kirchhoff's laws, are as follows:

$$\begin{cases} \dot{i}_L = -\dfrac{R_L}{L} i(t) - \dfrac{1}{L} v_C(t) \\ \dot{v}_C = \dfrac{1}{C} i_L(t) - \dfrac{1}{CR} v_C(t) - \dfrac{1}{C v_C(t)} P_\ell(t) \end{cases} \tag{6}$$

And, when the switch S is in the on state for the duration t_{on},

$$\begin{cases} \dot{i}_L = -\dfrac{R_L}{L} i(t) - \dfrac{1}{L} v_C(t) + \dfrac{1}{L} v_{\text{in}}(t) \\ \dot{v}_C = \dfrac{1}{C} i_L(t) - \dfrac{1}{CR} v_C(t) - \dfrac{1}{C v_C(t)} P_\ell(t) \end{cases} \tag{7}$$

Building on the equations (6) and (7), the average dynamic model that represents the behavior of the buck converter throughout its operation is:

$$\begin{cases} \dot{i}_L = \left[-\frac{R_L}{L}i(t) - \frac{1}{L}v_C(t)\right]\frac{t_{\text{off}}}{t_{\text{off}}+t_{\text{on}}} + \left[-\frac{R_L}{L}i(t) - \frac{1}{L}v_C(t) + \frac{1}{L}v_{\text{in}}(t)\right]\frac{t_{\text{on}}}{t_{\text{off}}+t_{\text{on}}} \\ \dot{v}_C = \left[-\frac{1}{C}i_L(t) - \frac{1}{CR}v_C(t) - \frac{1}{Cv_C(t)}P_\ell(t)\right]\frac{t_{\text{off}}}{t_{\text{off}}+t_{\text{on}}} + \left[-\frac{1}{C}i_L(t) - \frac{1}{CR}v_C(t) - \frac{1}{Cv_C(t)}P_\ell(t)\right]\frac{t_{\text{on}}}{t_{\text{off}}+t_{\text{on}}} \end{cases} \tag{8}$$

Defining the following expression:

$$d = \frac{t_{\text{on}}}{t_{\text{off}}+t_{\text{on}}}, \tag{9}$$

yields,

$$\frac{t_{\text{off}}+t_{\text{on}}}{t_{\text{off}}+t_{\text{on}}} = 1 \Rightarrow \frac{t_{\text{off}}}{t_{\text{off}}+t_{\text{on}}} + \frac{t_{\text{on}}}{t_{\text{off}}+t_{\text{on}}} = 1 \Rightarrow \frac{t_{\text{off}}}{t_{\text{off}}+t_{\text{on}}} = 1 - \frac{t_{\text{on}}}{t_{\text{off}}+t_{\text{on}}} \Rightarrow \frac{t_{\text{off}}}{t_{\text{off}}+t_{\text{on}}} = 1-d \tag{10}$$

Therefore, the equation (8) can be rewritten as follows:

$$\begin{cases} \dot{i}_L = \left[-\frac{R_L}{L}i(t) - \frac{1}{L}v_C(t)\right](1-d) + \left[-\frac{R_L}{L}i(t) - \frac{1}{L}v_C(t) + \frac{1}{L}v_{\text{in}}(t)\right]d \\ \dot{v}_C = \left[-\frac{1}{C}i_L(t) - \frac{1}{CR}v_C(t) - \frac{1}{Cv_C(t)}P_\ell(t)\right](1-d) + \left[-\frac{1}{C}i_L(t) - \frac{1}{CR}v_C(t) - \frac{1}{Cv_C(t)}P_\ell(t)\right]d \end{cases}$$

Then,

$$\begin{cases} \dot{i}_L = -\frac{R_L}{L}i(t) - \frac{1}{L}v_C(t) + \frac{1}{L}v_{\text{in}}(t)d \\ \dot{v}_C = -\frac{1}{C}i_L(t) - \frac{1}{CR}v_C(t) - \frac{1}{Cv_C(t)}P_\ell(t) \end{cases} \tag{11}$$

The variable d is commonly known as the switching duty cycle, which plays a crucial role in controlling switch states. Its value can be determined based on the control input $u_d(t)$, defined by:

$$d(u_d(t)) = \max\left\{\min\{u_d(t), 1\}, 0\right\}. \tag{12}$$

Choosing the operation point $P^{\text{o}} = (i_L^{\text{o}}, v_C^{\text{o}}, u_d^{\text{o}}, v_{\text{in}}^{\text{o}}, P_\ell^{\text{o}})$, where $u_d^{\text{o}} \in [0,1]$, the following coordinate change can be performed:

$$\delta i_L(t) = i_L(t) - i_L^{\text{o}}, \qquad \delta v_C(t) = v_C(t) - v_C^{\text{o}}, \qquad \delta u_d(t) = u_d(t) - u_d^{\text{o}} \tag{13}$$

$$\delta v_{\text{in}}(t) = v_{\text{in}}(t) - v_{\text{in}}^{\text{o}} \qquad \delta P_\ell(t) = P_\ell(t) - P_\ell^{\text{o}} \tag{14}$$

Moreover, let the control input saturation be modeled by means of the function $\text{sat}: \mathbb{R} \to [-\upsilon, \upsilon]$ such that:

$$\delta d = \text{sat}(\delta u_d(t)) = \max\left\{\min\left\{\delta u_d(t), \upsilon\right\}, -\upsilon\right\} \tag{15}$$

$$\upsilon = \min\left\{1 - u_d^{\text{o}}, u_d^{\text{o}}\right\} \tag{16}$$

Thus, the following non-linear model is obtained:

$$\begin{cases} \delta \dot{i}_L = -\dfrac{R_L}{L}\delta i(t) - \dfrac{1}{L}\delta v_C(t) + \dfrac{v_{\text{in}}^{\text{o}} + \delta v_{\text{in}}(t)}{L}\text{sat}(\delta u_d(t)) + \dfrac{u_d^{\text{o}}}{L}\delta v_{\text{in}}(t) \\ \delta \dot{v}_C = \dfrac{1}{C}\delta i_L(t) + \left[-\dfrac{1}{CR} + \dfrac{P_\ell^o}{Cv_C^{\text{o}}\left[v_C^{\text{o}} + \delta v_C(t)\right]} \right]\delta v_C(t) - \dfrac{1}{C\left[v_C^{\text{o}} + \delta v_C(t)\right]}\delta P_\ell(t) \end{cases} \tag{17}$$

where,

$$i_L^{\text{o}} = \frac{1}{R}v_C^{\text{o}} + \frac{1}{v_C^{\text{o}}}P_\ell^{\text{o}}, \qquad u_d^{\text{o}} = \frac{R_L}{v_{\text{in}}^{\text{o}}}i_L^{\text{o}} + \frac{v_C^{\text{o}}}{v_{\text{in}}^{\text{o}}}.$$

Finally, the model (17) can be rewritten as:

$$\dot{x} = A(x)x(t) + B(w)\text{sat}(u(t)) + E(x,u)w(t) \tag{18}$$

where $x(t) = \begin{bmatrix} i_L(t) & v_C(t) \end{bmatrix}^{\text{T}}$, $u(t) = \delta u_d(t)$, $w(t) = \begin{bmatrix} \delta v_{\text{in}}(t) & \delta P_\ell(t) \end{bmatrix}^{\text{T}}$, and

$$A(x) = \begin{bmatrix} -\dfrac{R_L}{L} & -\dfrac{1}{L} \\ \dfrac{1}{C} & -\dfrac{1}{CR} + \dfrac{P_\ell^o}{Cv_C^{\text{o}}\left[v_C^{\text{o}} + \delta v_C(t)\right]} \end{bmatrix} \qquad B(w) = \begin{bmatrix} \dfrac{v_{\text{in}}^{\text{o}} + \delta v_{\text{in}}(t)}{L} \\ 0 \end{bmatrix}$$

$$E(x,u) = \begin{bmatrix} \dfrac{u_d^{\text{o}}}{L} & 0 \\ 0 & -\dfrac{1}{C\left[v_C^{\text{o}} + \delta v_C(t)\right]} \end{bmatrix} \tag{18}$$

2.2 Buck Converter Fuzzy Model

In order to address the nonlinearity introduced by saturation, an approach based on substituting sat($\circ$) with a dead-zone type nonlinearity is used. The dead-zone nonlinearity is defined as:

$$\psi(u(t)) \triangleq u(t) - \text{sat}(u(t)) \tag{19}$$

From the matrices presented in equation (18), it follows $\dfrac{1}{v_C^{\text{o}} + \delta v_C^{\text{o}}}$ and $v_{\text{in}}^{\text{o}} + \delta v_{\text{in}}(t)$ are non-linear terms. For the non-linear terms, are defined

$$z_0(t) \equiv \frac{1}{v_C^{\text{o}} + \delta v_C^{\text{o}}} \qquad \text{and} \qquad z_1(t) \equiv v_{\text{in}}^{\text{o}} + \delta v_{\text{in}}(t). \tag{20}$$

Thus, the equation (18) can be rewritten as:

$$\dot{x} = A(z(t))x(t) + B(z(t))u(t) - B(w)\psi(u(t)) + E(z(t))w(t), \tag{21}$$

where $z(t) = \begin{bmatrix} z_0(t) & z_1(t) \end{bmatrix}$, $A(z(t)) = \begin{bmatrix} -\dfrac{R_L}{L} & -\dfrac{1}{L} \\ \dfrac{1}{C} & -\dfrac{1}{CR} + \dfrac{P_\ell^o}{Cv_C^o} z_0(t) \end{bmatrix}$, $B(z(t)) = \begin{bmatrix} \dfrac{1}{L} z_1(t) \\ 0 \end{bmatrix}$ and $E(z(t)) = \begin{bmatrix} \dfrac{u_d^o}{L} & 0 \\ 0 & -\dfrac{1}{C} z_0(t) \end{bmatrix}$.

Next, the minimum and maximum values of $z_0(t)$ e $z_1(t)$ under $v_C(t) \in \left[v_C^{\min}, v_C^{\max}\right]$ and $v_{\text{in}}(t) \in \left[v_{\text{in}}^{\min}, v_{\text{in}}^{\max}\right]$, are obtained as follows:

$$\begin{cases} z_i^0 & = \min\limits_{v_C(t), v_{\text{in}}(t)} z_i(t) \\ z_i^1 & = \max\limits_{v_C(t), v_{\text{in}}(t)} z_i(t) \end{cases}, \quad i = 1, 2 \tag{22}$$

From the maximum and minimum values of $z_0(t)$ and $z_1(t)$, the membership functions can be calculated as:

$$z_i(t) = \sum_{j=0}^{1} M_i^j(z_i(t)) z_i^j, \qquad \text{where} \qquad M_i^1 = \frac{z_i(t) - z_i^0}{z_i^1 - z_i^0} \quad \text{and} \quad M_i^0 = 1 - M_i^1, \qquad \text{for } i = \{1, 2\}. \tag{23}$$

Therefore, the Takagi-Sugeno fuzzy model for the buck converter is:

$$\dot{x} = \sum_{i=0}^{1} \sum_{j=0}^{1} \prod_{k=1}^{2} M_k^i(z_k(t)) \left[A\left(z^{\{i,j\}}\right) x(t) + B\left(z^{\{i,j\}}\right) u(t) - B\left(z^{\{i,j\}}\right) \psi(u(t)) + E\left(z^{\{i,j\}}\right) w(t) \right] \tag{24}$$

where $z^{\{i,j\}}(t)$ is a shorthand for $\begin{bmatrix} z_1^i(t) & z_2^j(t) \end{bmatrix}$.

The summations in (24) can be aggregated as one summations:

$$\dot{x} = \sum_{p=\{0,0\}}^{\{1,1\}} h_p(z(t)) \left\{ A_p x(t) + B_p u(t) - B_p \psi(u(t)) + E_p w(t) \right\} \tag{25}$$

where $p = \{b_1, b_2\} \in \mathbb{B}^2$, $\mathbb{B} = \{0, 1\}$,

$$h_p(z(t)) = \prod_{k=1}^{2} M_k^{b_k}(z_k(t)) \tag{26}$$

$$A_p = A(z^p) \qquad\qquad B_p = B(z^p) \qquad\qquad E_p = E(z^p) \tag{27}$$