Modelagem Matemática da Microrrede CC

Modelo Não-linear da Microrrede CC

Modelagem do Subsistema: Geração 1

O sistema da geração 1, apresentado na Figura 1, é composto por uma fonte de alimentação cuja tensão varia ao longo do tempo, a qual está conectada a um conversor do tipo *buck*. Este conversor, por sua vez, está ligado a um filtro RC. Finalmente, o conjunto é conectado ao barramento de corrente contínua (CC).

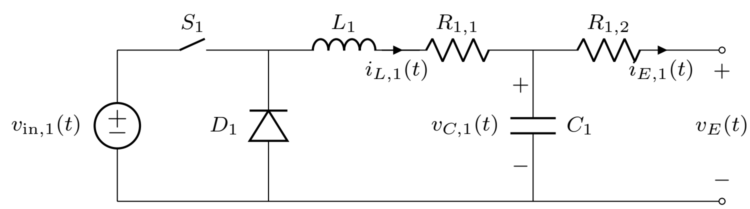


Figura 1: Circuito elétrico do sistema da geração 1.

Aplicando a LKT na segunda malha a partir da esquerda, obtemos:

$$
\begin{aligned}
d_1(t)v_{\text{in},1} - L_1\dot{i}_{L,1} - R_{1,1}i_{L,1}(t) - v_{C,1}(t) = 0 \\
\dot{i}_{L,1} = -\frac{R_{1,1}}{L_1}i_{L,1}(t) - \frac{1}{L_1}v_{C,1}(t) + \frac{v_{\text{in},1}}{L_1}d_1(t)
\end{aligned} \tag{1}
$$

Aplicando a LKC, obtemos:

$$
i_{L,1}(t) = C_1\dot{v}_{C,1} + i_{E,1}(t) \tag{2}
$$

Têm-se que, $i_{E,1}(t) = \dfrac{v_{C,1}(t) - v_E(t)}{R_{1,2}}$. Logo,

$$
\begin{aligned}
i_{L,1}(t) = C_1\dot{v}_{C,1} + \frac{1}{R_{1,2}}v_{C,1}(t) - \frac{1}{R_{1,2}}v_E(t) \\
i_{L,1}(t) = C_1\dot{v}_{C,1} + \frac{1}{R_{1,2}}v_{C,1}(t) - \frac{1}{R_{1,2}}v_E(t) \\
\dot{v}_{C,1} = \frac{1}{C_1}i_{L,1}(t) - \frac{1}{R_{1,2}C_1}v_{C,1}(t) + \frac{1}{R_{1,2}C_1}v_E(t)
\end{aligned} \tag{3}
$$

Portanto, o modelo do subsistema da geração é:

$$
\begin{cases}
\dot{i}_{L,1} = -\dfrac{R_{1,1}}{L_1}i_{L,1}(t) - \dfrac{1}{L_1}v_{C,1}(t) + \dfrac{v_{\text{in},1}}{L_1}d_1(t) \\
\dot{v}_{C,1} = \dfrac{1}{C_1}i_{L,1}(t) - \dfrac{1}{R_{1,2}C_1}v_{C,1}(t) + \dfrac{1}{R_{1,2}C_1}v_E(t)
\end{cases} \tag{4}
$$

Modelagem do Subsistema: Conversor Boost

O sistema da geração 2, apresentado na Figura 2, é composto por uma fonte de alimentação cuja tensão varia ao longo do tempo, a qual está conectada a um conversor do tipo *boost*. Este conversor, por sua vez, está ligado a um filtro RC. Finalmente, o conjunto é conectado ao barramento de corrente contínua (CC).

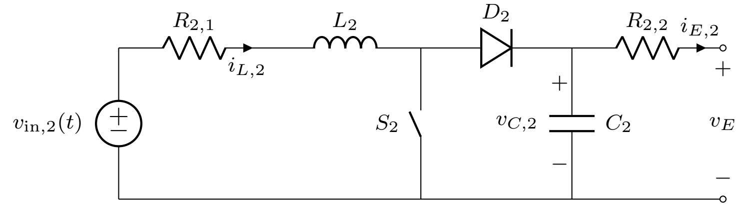


Figura 2: Circuito elétrico do sistema da geração 2.

Para S_2 fechada: $d_2(t) \cdot T_s$

Aplicando a LKT na malha mais a esquerda, obtemos:

$$v_{\text{in},2}(t) - R_{2,1} i_{L,2}(t) - L_2 \dot{i}_{L,2} = 0$$
$$\dot{i}_{L,2} = -\frac{R_{2,1}}{L_2} i_{L,2}(t) + \frac{1}{L_2} v_{in,2} \tag{5}$$

Aplicando a LKC na malha mais a direita, tém-se:

$$i_{E,2}(t) + C_2 \dot{v}_{C,2} = 0 \tag{6}$$

Como $i_{E,2}(t) = \dfrac{v_{C,2}(t) - v_E(t)}{R_{2,2}}$, têm-se:

$$\frac{v_{C,2}(t)}{R_{B,2}} - \frac{v_E(t)}{R_{B,2}} + C_2 \dot{v}_{C,2} = 0$$
$$\dot{v}_{C,2} = -\frac{1}{R_{B,2} C_2} v_{C,2}(t) + \frac{1}{R_{B,2} C_2} v_E(t) \tag{7}$$

Assim, neste modo, têm-se:

$$\begin{cases} \dot{i}_{L,2} = -\dfrac{R_{2,1}}{L_2} i_{L,2}(t) + \dfrac{1}{L_2} v_{in,2} \\ \dot{v}_{C,2} = -\dfrac{1}{R_{B,2} C_2} v_{C,2}(t) + \dfrac{1}{R_{B,2} C_2} v_E(t) \end{cases} \tag{8}$$

Para S_2 aberta: $[1 - d_2(t)] \cdot T_s$

Aplicando a LKT, obtemos:

$$v_{\text{in},2}(t) - L_2 \dot{i}_{L,2} - R_{2,1} i_{L,2}(t) - v_{C,2}(t) = 0$$
$$\dot{i}_{L,2} = -\frac{R_{2,1}}{L_2} i_{L,2}(t) - \frac{1}{L_2} v_{C,2}(t) + \frac{1}{L_2} v_{\text{in},2} \tag{9}$$

Aplicando a LKC, obtemos:

$$i_{L,2}(t) = C_2 \dot{v}_{C,2} + i_{E,2}(t)$$
$$i_{L,2}(t) = C_2 \dot{v}_{C,2} + \frac{v_{C,2}(t)}{R_{2,2}} - \frac{v_E(t)}{R_{2,2}}$$
$$\dot{v}_{C,2} = \frac{1}{C_2} i_{L,2}(t) - \frac{1}{R_{2,2} C_2} v_{C,2}(t) + \frac{1}{R_{2,2} C_2} v_E(t) \tag{10}$$

Assim, neste modo, têm-se:

$$\begin{cases} \dot{i}_{L,2} = -\dfrac{R_{2,1}}{L_2} i_{L,2}(t) - \dfrac{1}{L_2} v_{C,2}(t) + \dfrac{1}{L_2} v_{\text{in},2} \\ \dot{v}_{C,2} = \dfrac{1}{C_2} i_{L,2}(t) - \dfrac{1}{R_{2,2} C_2} v_{C,2}(t) + \dfrac{1}{R_{2,2} C_2} v_E(t) \end{cases} \tag{11}$$

Modelo Médio Completo

A equação completa da dinâmica da corrente $i_{L,2}(t)$ é:

$$\dot{i}_{L,2} = \left[-\frac{R_{2,1}}{L_2} i_{L,2}(t) + \frac{1}{L_2} v_{in,2} \right] d_2(t) + \left[-\frac{R_{2,1}}{L_2} i_{L,2}(t) - \frac{1}{L_2} v_{C,2}(t) + \frac{1}{L_2} v_{\text{in},2}(t) \right] [1 - d_2(t)]$$
$$\dot{i}_{L,2} = -\frac{R_{2,1}}{L_2} i_{L,2}(t) - \frac{1}{L_2} v_{C,2}(t) \left[1 - d_2(t)\right] + \frac{1}{L_2} v_{\text{in},2}(t) \tag{12}$$

E da tensão,

$$\dot{v}_{C,2} = \left[-\frac{1}{R_{B,2}C_2}v_{C,2}(t) + \frac{1}{R_{B,2}C_2}v_E(t)\right]d_2(t) + \left[\frac{1}{C_2}i_{L,2}(t) - \frac{1}{R_{2,2}C_2}v_{C,2}(t) + \frac{1}{R_{2,2}C_2}v_E(t)\right][1-d_2(t)]$$

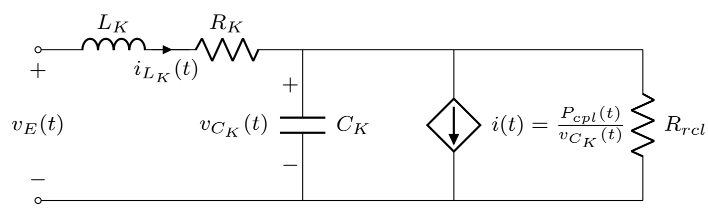
$$\dot{v}_{C,2} = \frac{1}{C_2}i_{L,2}(t)\,[1-d_2(t)] - \frac{1}{R_{2,2}C_2}v_{C,2}(t) + \frac{1}{R_{2,2}C_2}v_E(t) \tag{13}$$

Portanto, o modelo dinâmico do sistema da geração 2 é:

$$\begin{cases} \dot{i}_{L,2} = -\dfrac{R_{2,1}}{L_2}i_{L,2}(t) - \dfrac{1}{L_2}v_{C,2}(t)\,[1-d_2(t)] + \dfrac{1}{L_2}v_{\text{in},2} \\ \dot{v}_{C,2} = \dfrac{1}{C_2}i_{L,2}(t)\,[1-d_2(t)] - \dfrac{1}{R_{2,2}C_2}v_{C,2}(t) + \dfrac{1}{R_{2,2}C_2}v_E(t) \end{cases} \tag{14}$$

Modelagem do Subsistema: Cargas

O circuito que representa as duas cargas conectadas a redes, a CPL e a CRL, é:



Aplicando a LKT na malha mais a esquerda, têm-se:

$$v_E(t) - L_K\dot{i}_{L_K} - R_K i_{L_K}(t) - v_{C_K}(t) = 0$$

$$\dot{i}_{L_K} = -\frac{R_K}{L_K}i_{L_K}(t) - \frac{1}{L_K}v_{C_K}(t) + \frac{1}{L_K}v_E(t) \tag{15}$$

Aplicando a LKC, obtemos:

$$i_{L_K} = C_K\dot{v}_{C_K} + \frac{P_{cpl}(t)}{v_{C_K}(t)} + \frac{v_{C_K}(t)}{R_{crl}}$$

$$\dot{v}_{C_K} = \frac{1}{C_K}i_{L_K} - \frac{1}{R_{crl}C_K}v_{C_K}(t) - \frac{1}{C_K}\frac{P_{cpl}(t)}{v_{C_K}(t)} \tag{16}$$

Portanto, o modelo dinâmico do subsistema das cargas é:

$$\begin{cases} \dot{i}_{L_K} = -\dfrac{R_K}{L_K}i_{L_K}(t) - \dfrac{1}{L_K}v_{C_K}(t) + \dfrac{1}{L_K}v_E(t) \\ \dot{v}_{C_K} = \dfrac{1}{C_K}i_{L_K} - \dfrac{1}{R_{crl}C_K}v_{C_K}(t) - \dfrac{1}{C_K}\dfrac{P_{cpl}(t)}{v_{C_K}(t)} \end{cases} \tag{17}$$

Centralização dos Modelos

Do esquemático, têm-se que:

$$i_{E,1}(t) + i_{E,2}(t) = i_B(t) + i_{L_K}(t)$$

$$\frac{1}{R_{1,2}}v_{C,1}(t) - \frac{1}{R_{1,2}}v_E(t) + \frac{1}{R_{2,2}}v_{C,2}(t) - \frac{1}{R_{2,2}}v_E(t) = i_B(t) + i_{L_K}(t)$$

$$R_{2,2}v_{C,1}(t) - R_{2,2}v_E(t) + R_{1,2}v_{C,2}(t) - R_{1,2}v_E(t) = R_{1,2}R_{2,2}\,[i_B(t) + i_{L_K}(t)]$$

$$R_{2,2}v_{C,1}(t) + R_{1,2}v_{C,2}(t) - (R_{1,2} + R_{2,2})\,v_E(t) = R_{1,2}R_{2,2}\,[i_B(t) + i_{L_K}(t)] \tag{18}$$

$$v_E(t) = \frac{R_{2,2}}{R_{1,2}+R_{2,2}}v_{C,1}(t) + \frac{R_{1,2}}{R_{1,2}+R_{2,2}}v_{C,2}(t) - \frac{R_{1,2}R_{2,2}}{R_{1,2}+R_{2,2}}\left[i_B(t) + i_{L_K}(t)\right]$$

$$v_E(t) = \frac{R_E}{R_{1,2}}v_{C,1}(t) + \frac{R_E}{R_{2,2}}v_{C,2}(t) - R_E i_B(t) - R_E i_{L_K}(t) \tag{19}$$

onde, $R_E = \dfrac{R_{1,2}R_{2,2}}{R_{1,2}+R_{2,2}}$.

Reescrevendo a equação de i_{L_K}, obtêm-se:

$$\dot{i}_{L_K} = -\frac{R_K}{L_K}i_{L_K}(t) - \frac{1}{L_K}v_{C_K}(t) + \frac{1}{L_K}\left[\frac{R_E}{R_{1,2}}v_{C,1}(t) + \frac{R_E}{R_{2,2}}v_{C,2}(t) - R_E i_B(t) - R_E i_{L_K}(t)\right]$$

$$\dot{i}_{L_K} = -\frac{R_K+R_E}{L_K}i_{L_K}(t) - \frac{1}{L_K}v_{C_K}(t) + \frac{R_E}{R_{1,2}L_K}v_{C,1}(t) + \frac{R_E}{R_{2,2}L_K}v_{C,2}(t) - \frac{R_E}{L_K}i_B(t) \tag{20}$$

Reescrevendo a equação de $v_{C,1}$, obtêm-se:

$$\dot{v}_{C,1} = \frac{1}{C_1}i_{L,1}(t) - \frac{1}{R_{1,2}C_1}v_{C,1}(t) + \frac{1}{R_{1,2}C_1}\left[\frac{R_E}{R_{1,2}}v_{C,1}(t) + \frac{R_E}{R_{2,2}}v_{C,2}(t) - R_E i_B(t) - R_E i_{L_K}(t)\right]$$

$$\dot{v}_{C,1} = \frac{1}{C_1}i_{L,1}(t) - \frac{1}{R_{1,2}C_1}\left(1 - \frac{R_E}{R_{1,2}}\right)v_{C,1}(t) + \frac{1}{C_1(R_{1,2}+R_{2,2})}v_{C,2}(t) - \frac{R_E}{R_{1,2}C_1}i_B(t) - \frac{R_E}{R_{1,2}C_1}i_{L_K}(t) \tag{21}$$

Reescrevendo a equação de $v_{C,2}$, obtêm-se:

$$\dot{v}_{C,2} = \frac{1}{C_2}i_{L,2}(t)\left[1 - d_2(t)\right] - \frac{1}{R_{2,2}C_2}v_{C,2}(t) + \frac{1}{R_{2,2}C_2}\left[\frac{R_E}{R_{1,2}}v_{C,1}(t) + \frac{R_E}{R_{2,2}}v_{C,2}(t) - R_E i_B(t) - R_E i_{L_K}(t)\right]$$

$$\begin{aligned}\dot{v}_{C,2} = \frac{1}{C_2}\left[1 - d_2(t)\right]i_{L,2}(t) - \frac{1}{R_{2,2}C_2}\left(1 - \frac{R_E}{R_{2,2}}\right)v_{C,2}(t) \\ + \frac{1}{C_2(R_{1,2}+R_{2,2})}v_{C,1}(t) - \frac{R_E}{R_{2,2}C_2}i_B(t) - \frac{R_E}{R_{2,2}C_2}i_{L_K}(t)\end{aligned} \tag{22}$$

Assim, o modelo centralizado é:

$$\begin{cases}
\dot{i}_{L,1} = -\frac{R_{1,1}}{L_1}i_{L,1}(t) - \frac{1}{L_1}v_{C,1}(t) + \frac{v_{\text{in},1}(t)}{L_1}d_1(t) \\
\dot{v}_{C,1} = \frac{1}{C_1}i_{L,1}(t) - \frac{1}{R_{1,2}C_1}\left(1 - \frac{R_E}{R_{1,2}}\right)v_{C,1}(t) + \frac{1}{C_1(R_{1,2}+R_{2,2})}v_{C,2}(t) - \frac{R_E}{R_{1,2}C_1}i_B(t) - \frac{R_E}{R_{1,2}C_1}i_{L_K}(t) \\
\dot{i}_{L,2} = -\frac{R_{2,1}}{L_2}i_{L,2}(t) - \frac{1}{L_2}v_{C,2}(t)\left[1 - d_2(t)\right] + \frac{1}{L_2}v_{\text{in},2}(t) \\
\dot{v}_{C,2} = \frac{1}{C_2}\left[1 - d_2(t)\right]i_{L,2}(t) - \frac{1}{R_{2,2}C_2}\left(1 - \frac{R_E}{R_{2,2}}\right)v_{C,2}(t) + \frac{1}{C_2(R_{1,2}+R_{2,2})}v_{C,1}(t) - \frac{R_E}{R_{2,2}C_2}i_B(t) - \frac{R_E}{R_{2,2}C_2}i_{L_K}(t) \\
\dot{i}_{L_K} = -\frac{R_K+R_E}{L_K}i_{L_K}(t) - \frac{1}{L_K}v_{C_K}(t) + \frac{R_E}{R_{1,2}L_K}v_{C,1}(t) + \frac{R_E}{R_{2,2}L_K}v_{C,2}(t) - \frac{R_E}{L_K}i_B(t) \\
\dot{v}_{C_K} = \frac{1}{C_K}i_{L_K} - \frac{1}{R_{crl}C_K}v_{C_K}(t) - \frac{1}{C_K}\frac{P_{cpl}(t)}{v_{C_K}(t)}
\end{cases} \tag{23}$$

Translação do Modelo

Parcelando os estados e as entradas do sistema em termos fixos e em termos variantes no tempo, obtemos:

$$\begin{array}{ccc}
i_{L,1}(t) = i^o_{L,1} + \delta i_{L,1}(t) & i_{L,2}(t) = i^o_{L,2} + \delta i_{L,2}(t) & i_{L_K}(t) = i^o_{L_K} + \delta i_{L_K}(t) \\
v_{C,1}(t) = v^o_{C,1} + \delta v_{C,1}(t) & v_{C,2}(t) = v^o_{C,2} + \delta v_{C,2}(t) & v_{C_K}(t) = v^o_{C_K} + \delta v_{C_K}(t) \\
d_1(t) = d^o_1 + \delta d_1(t) & d_2(t) = d^o_2 + \delta d_2(t) & P_{cpl}(t) = P^o_{cpl} + \delta P_{cpl}(t)
\end{array}$$

Corrente $i_{L,1}$ ***transladada***

Para $\dot{i}_{L,1}$, têm-se:

$$-\frac{R_{1,1}}{L_1}i^o_{L,1}-\frac{1}{L_1}v^o_{C,1}+\frac{v_{\text{in},1}}{L_1}d^o_1=0\Rightarrow -R_{1,1}i^o_{L,1}-v^o_{C,1}+v_{\text{in},1}d^o_1=0$$

$$d^o_1=\frac{R_{1,1}}{v_{\text{in},1}}i^o_{L,1}+\frac{1}{v_{\text{in},1}}v^o_{C,1} \tag{24}$$

Substituindo, obtemos:

$$\dot{i}_{L,1}=-\frac{R_{1,1}}{L_1}i_{L,1}(t)-\frac{1}{L_1}v_{C,1}(t)+\frac{v_{\text{in},1}}{L_1}d_1(t)$$

$$\delta\dot{i}_{L,1}=-\frac{R_{1,1}}{L_1}\left[i^o_{L,1}+\delta i_{L,1}(t)\right]-\frac{1}{L_1}\left[v^o_{C,1}+\delta v_{C,1}(t)\right]+\frac{v_{\text{in},1}}{L_1}\left[\frac{R_{1,1}}{v_{\text{in},1}}i^o_{L,1}+\frac{1}{v_{\text{in},1}}v^o_{C,1}+\delta d_1(t)\right] \tag{25}$$

$$\delta\dot{i}_{L,1}=-\frac{R_{1,1}}{L_1}\delta i_{L,1}(t)-\frac{1}{L_1}\delta v_{C,1}(t)+\frac{v_{\text{in},1}}{L_1}\delta d_1(t) \tag{26}$$

Corrente $i_{L,2}$ ***transladada***

Para $\dot{i}_{L,2}$, têm-se:

$$-\frac{R_{2,1}}{L_2}i^o_{L,2}-\frac{1}{L_2}\left[1-d^o_2\right]v^o_{C,2}+\frac{1}{L_2}v_{\text{in},2}=0\Rightarrow d^o_2=1+\frac{R_{2,1}i^o_{L,2}}{v^o_{C,2}}-\frac{v_{\text{in},2}}{v^o_{C,2}} \tag{27}$$

Substituindo, obtemos:

$$\delta\dot{i}_{L,2}=-\frac{R_{2,1}}{L_2}\left[i^o_{L,2}+\delta i_{L,2}(t)\right]-\frac{1}{L_2}\left[1-d^o_2-\delta d_2(t)\right]\left[v^o_{C,2}+\delta v_{C,2}(t)\right]+\frac{1}{L_2}v_{\text{in},2}$$

$$\delta\dot{i}_{L,2}=-\frac{R_{2,1}}{L_2}\delta i_{L,2}(t)+\left[\frac{R_{2,1}i^o_{L,2}}{L_2v^o_{C,2}}-\frac{v_{\text{in},2}}{L_2v^o_{C,2}}\right]\delta v_{C,2}(t)+\frac{1}{L_2}\left[v^o_{C,2}+\delta v_{C,2}(t)\right]\delta d_2(t) \tag{28}$$

Corrente i_{L_K} ***transladada***

Para $\dot{i}_{L_K}$, têm-se:

$$-\frac{R_K+R_E}{L_K}i^o_{L_K}-\frac{1}{L_K}v^o_{C_K}+\frac{R_E}{R_{1,2}L_K}v^o_{C,1}+\frac{R_E}{R_{2,2}L_K}v^o_{C,2}-\frac{R_E}{L_K}i^o_B=0$$

$$v^o_{C,2}=\frac{R_{2,2}(R_K+R_E)}{R_E}i^o_{L_K}+\frac{R_{2,2}}{R_E}v^o_{C_K}-\frac{R_{2,2}}{R_{1,2}}v^o_{C,1}+R_{2,2}i^o_B \tag{29}$$

Substituindo, obtemos:

$$\begin{aligned}\delta\dot{i}_{L_K}=&-\frac{R_K+R_E}{L_K}\left[i^o_{L_K}+\delta i_{L_K}(t)\right]-\frac{1}{L_K}\left[v^o_{C_K}+\delta v_{C_K}(t)\right]\\&+\frac{R_E}{R_{1,2}L_K}\left[v^o_{C,1}+\delta v_{C,1}(t)\right]+\frac{R_E}{R_{2,2}L_K}\left[v^o_{C,2}+\delta v_{C,2}(t)\right]-\frac{R_E}{L_K}\left[i^o_B+\delta i_B(t)\right]=0\end{aligned}$$

$$\delta\dot{i}_{L_K}=-\frac{R_K+R_E}{L_K}\delta i_{L_K}(t)-\frac{1}{L_K}\delta v_{C_K}(t)+\frac{R_E}{R_{1,2}L_K}\delta v_{C,1}(t)+\frac{R_E}{R_{2,2}L_K}\delta v_{C,2}(t)-\frac{R_E}{L_K}\delta i_B(t) \tag{30}$$

Tensão $v_{C,1}$ ***transladada***

Para $\dot{v}_{C,1}$, têm-se:

$$\frac{1}{C_1}i^o_{L,1}-\frac{1}{R_{1,2}C_1}\left(1-\frac{R_E}{R_{1,2}}\right)v^o_{C,1}-\frac{R_E}{R_{1,2}C_1}i^o_{L_K}+\frac{1}{C_1(R_{1,2}+R_{2,2})}v^o_{C,2}-\frac{R_E}{R_{1,2}C_1}i^o_B=0$$

$$i^o_{L,1}=\frac{1}{R_{1,2}}\left(1-\frac{R_E}{R_{1,2}}\right)v^o_{C,1}+\frac{R_E}{R_{1,2}}i^o_{L_K}-\frac{1}{(R_{1,2}+R_{2,2})}v^o_{C,2}+\frac{R_E}{R_{1,2}}i^o_B \tag{31}$$

Substituindo, obtemos:

$$\delta\dot{v}_{C,1} = \frac{1}{C_1}\left[i^o_{L,1} + \delta i_{L,1}(t)\right] - \frac{1}{R_{1,2}C_1}\left(1 - \frac{R_E}{R_{1,2}}\right)\left[v^o_{C,1} + \delta v_{C,1}(t)\right] - \frac{R_E}{R_{1,2}C_1}\left[i^o_{L_K} + \delta i_{L_K}(t)\right] + \frac{1}{C_1(R_{1,2}+R_{2,2})}\left[v^o_{C,2} + \delta v_{C,2}(t)\right] - \frac{R_E}{R_{1,2}C_1}\left[i^o_B + \delta i_B(t)\right]$$

$$\delta\dot{v}_{C,1} = \frac{1}{C_1}\delta i_{L,1}(t) - \frac{1}{R_{1,2}C_1}\left(1 - \frac{R_E}{R_{1,2}}\right)\delta v_{C,1}(t) - \frac{R_E}{R_{1,2}C_1}\delta i_{L_K}(t) + \frac{1}{C_1(R_{1,2}+R_{2,2})}\delta v_{C,2}(t) - \frac{R_E}{R_{1,2}C_1}\delta i_B(t) \tag{32}$$

Tensão $v_{C,2}$ ***transladada***

Para $\dot{v}_{C,2}$, têm-se:

$$\frac{1}{C_2}\left(1 - d^o_2\right) i^o_{L,2} - \frac{1}{R_{2,2}C_2}\left(1 - \frac{R_E}{R_{2,2}}\right) v^o_{C,2} - \frac{R_E}{R_{2,2}C_2} i^o_{L_K} + \frac{1}{C_2(R_{1,2}+R_{2,2})} v^o_{C,1} - \frac{R_E}{R_{2,2}C_2} i^o_B = 0$$

$$i^o_{L,2} = \frac{1}{R_{2,2}\left(1-d^o_2\right)}\left(1 - \frac{R_E}{R_{2,2}}\right) v^o_{C,2} + \frac{R_E}{R_{2,2}\left(1-d^o_2\right)} i^o_{L_K} - \frac{1}{\left(1-d^o_2\right)\left(R_{1,2}+R_{2,2}\right)} v^o_{C,1} + \frac{R_E}{R_{2,2}\left(1-d^o_2\right)} i^o_B \tag{33}$$

Substituindo, obtemos:

$$\delta\dot{v}_{C,2} = \frac{1}{C_2}\left[1 - d^o_2 - \delta d_2(t)\right]\left[i^o_{L,2} + \delta i_{L,2}(t)\right] - \frac{1}{R_{2,2}C_2}\left(1 - \frac{R_E}{R_{2,2}}\right)\left[v^o_{C,2} + \delta v_{C,2}(t)\right] - \frac{R_E}{R_{2,2}C_2}\left[i^o_{L_K} + \delta i_{L_K}(t)\right] + \frac{1}{C_2(R_{1,2}+R_{2,2})}\left[v^o_{C,1} + \delta v_{C,1}(t)\right] - \frac{R_E}{R_{2,2}C_2}\left[i^o_B + \delta i_B(t)\right]$$

$$\delta\dot{v}_{C,2} = \frac{1}{C_2}(1-d^o_2)i^o_{L,2} + \frac{1}{C_2}(1-d^o_2)\delta i_{L,2}(t) - \frac{1}{C_2}\delta d_2(t) i^o_{L,2} - \frac{1}{C_2}\delta d_2(t)\delta i_{L,2}(t) - \frac{1}{R_{2,2}C_2}\left(1 - \frac{R_E}{R_{2,2}}\right)\left[v^o_{C,2} + \delta v_{C,2}(t)\right] - \frac{R_E}{R_{2,2}C_2}\left[i^o_{L_K} + \delta i_{L_K}(t)\right] + \frac{1}{C_2(R_{1,2}+R_{2,2})}\left[v^o_{C,1} + \delta v_{C,1}(t)\right] - \frac{R_E}{R_{2,2}C_2}\left[i^o_B + \delta i_B(t)\right]$$

$$\delta\dot{v}_{C,2} = \frac{1}{C_2}(1-d^o_2)\delta i_{L,2}(t) - \frac{1}{C_2}\left[i^o_{L,2} + \delta i_{L,2}(t)\right]\delta d_2(t) - \frac{1}{R_{2,2}C_2}\left(1 - \frac{R_E}{R_{2,2}}\right)\delta v_{C,2}(t) - \frac{R_E}{R_{2,2}C_2}\delta i_{L_K}(t) + \frac{1}{C_2(R_{1,2}+R_{2,2})}\delta v_{C,1}(t) - \frac{R_E}{R_{2,2}C_2}\delta i_B(t)$$

Tensão v_{C_K} ***transladada***

Para $\dot{v}_{C_K}$, têm-se:

$$\frac{1}{C_K} i^o_{L_K} - \frac{1}{R_{crl}C_K} v^o_{C_K} - \frac{1}{C_K}\frac{P^o_{cpl}}{v^o_{C_K}} = 0$$

$$i^o_{L_K} = \frac{1}{R_{crl}} v^o_{C_K} + \frac{P^o_{cpl}}{v^o_{C_K}}$$

Substituindo, obtemos:

$$\delta\dot{v}_{C_K} = \frac{1}{C_K}\left[i^o_{L_K} + \delta i_{L_K}(t)\right] - \frac{1}{R_{crl}C_K}\left[v^o_{C_K} + \delta v_{C_K}(t)\right]$$

$$\dot{v}_{C_K} = \frac{1}{C_K}\left[\frac{1}{R_{crl}} v^o_{C_K} + \frac{P^o_{cpl}}{v^o_{C_K}} + \delta i_{L_K}(t)\right] - \frac{1}{R_{crl}C_K}\left[v^o_{C_K} + \delta v_{C_K}(t)\right]$$

$$\dot{v}_{C_K} = \frac{1}{C_K}\delta i_{L_K}(t) - \frac{1}{R_{crl}C_K}\delta v_{C_K}(t) + \frac{P^o_{cpl}\delta v_{C_K}(t) - v^o_{C_K}\delta P_{cpl}(t)}{C v^o_{C_K}\left[v^o_{C_K} + \delta v_{C_K}(t)\right]}$$

Translação da saída

A saída transladada é:

$$\delta v_E(t) = \frac{R_E}{R_{1,2}}\delta v_{C,1}(t) + \frac{R_E}{R_{2,2}}\delta v_{C,2}(t) - R_E\delta i_B(t) - R_E\delta i_{L_K}(t) \tag{34}$$

Modelo dinâmico transladado

Têm-se,

$$i^o_{L,2}\left(1-d^o_2\right) = \frac{1}{R_{2,2}}\left(1-\frac{R_E}{R_{2,2}}\right)v^o_{C,2} + \frac{R_E}{R_{2,2}}i^o_{L_K} - \frac{1}{R_{1,2}+R_{2,2}}v^o_{C,1} + \frac{R_E}{R_{2,2}}i^o_B$$

$$i^o_{L,2}\left(-\frac{R_{2,1}i^o_{L,2}}{v^o_{C,2}} + \frac{v_{\text{in},2}}{v^o_{C,2}}\right) = \frac{1}{R_{2,2}}\left(1-\frac{R_E}{R_{2,2}}\right)v^o_{C,2} + \frac{R_E}{R_{2,2}}i^o_{L_K} - \frac{1}{R_{1,2}+R_{2,2}}v^o_{C,1} + \frac{R_E}{R_{2,2}}i^o_B$$

$$-\frac{R_{2,1}}{v^o_{C,2}}(i^o_{L,2})^2 + \frac{v_{\text{in},2}}{v^o_{C,2}}i^o_{L,2} - \left\{\frac{1}{R_{2,2}}\left(1-\frac{R_E}{R_{2,2}}\right)v^o_{C,2} + \frac{R_E}{R_{2,2}}i^o_{L_K} - \frac{1}{R_{1,2}+R_{2,2}}v^o_{C,1} + \frac{R_E}{R_{2,2}}i^o_B\right\} = 0$$

E os demais pontos de operação são:

$$d^o_1 = \frac{R_{1,1}}{v_{\text{in},1}(t)}i^o_{L,1} + \frac{1}{v_{\text{in},1}(t)}v^o_{C,1}, \qquad d^o_2 = 1 + \frac{R_{2,1}i^o_{L,2}}{v^o_{C,2}} - \frac{v_{\text{in},2}}{v^o_{C,2}}, \qquad i^o_{L_K} = \frac{1}{R_{crl}}v^o_{C_K} + \frac{P^o_{cpl}}{v^o_{C_K}}, \tag{35}$$

$$i^o_{L,1} = \frac{1}{R_{1,2}}\left(1-\frac{R_E}{R_{1,2}}\right)v^o_{C,1} + \frac{R_E}{R_{1,2}}i^o_{L_K} - \frac{1}{(R_{1,2}+R_{2,2})}v^o_{C,2} + \frac{R_E}{R_{1,2}}i^o_B \tag{36}$$

$$v^o_{C,2} = \frac{R_{2,2}(R_K+R_E)}{R_E}i^o_{L_K} + \frac{R_{2,2}}{R_E}v^o_{C_K} - \frac{R_{2,2}}{R_{1,2}}v^o_{C,1} + R_{2,2}i^o_B \tag{37}$$

O modelo dinâmico transladado é:

$$\begin{cases}
\delta\dot{i}_{L,1} = -\frac{R_{1,1}}{L_1}\delta i_{L,1}(t) - \frac{1}{L_1}\delta v_{C,1}(t) + \frac{v_{\text{in},1}+\delta v_{\text{in},1}(t)}{L_1}\delta d_1(t) + \frac{1}{L_1}\left(\frac{R_{1,1}}{v_{\text{in},1}}i^o_{L,1} + \frac{1}{v_{\text{in},1}}v^o_{C,1}\right)\delta v_{\text{in},1}(t) \\
\delta\dot{i}_{L,2} = -\frac{R_{2,1}}{L_2}\delta i_{L,2}(t) + \left[\frac{R_{2,1}i^o_{L,2}}{L_2 v^o_{C,2}} - \frac{v_{\text{in},2}}{L_2 v^o_{C,2}}\right]\delta v_{C,2}(t) + \frac{1}{L_2}\left[v^o_{C,2}+\delta v_{C,2}(t)\right]\delta d_2(t) + \frac{1}{L_2}\delta v_{\text{in},2}(t) \\
\delta\dot{i}_{L_K} = -\frac{R_K+R_E}{L_K}\delta i_{L_K}(t) - \frac{1}{L_K}\delta v_{C_K}(t) + \frac{R_E}{R_{1,2}L_K}\delta v_{C,1}(t) + \frac{R_E}{R_{2,2}L_K}\delta v_{C,2}(t) - \frac{R_E}{L_K}\delta i_B(t) \\
\delta\dot{v}_{C,1} = \frac{1}{C_1}\delta i_{L,1}(t) - \frac{1}{R_{1,2}C_1}\left(1-\frac{R_E}{R_{1,2}}\right)\delta v_{C,1}(t) - \frac{R_E}{R_{1,2}C_1}\delta i_{L_K}(t) + \frac{1}{C_1(R_{1,2}+R_{2,2})}\delta v_{C,2}(t) - \frac{R_E}{R_{1,2}C_1}\delta i_B(t) \\
\delta\dot{v}_{C,2} = \dfrac{1}{C_2}(1-d^o_2)\delta i_{L,2}(t) - \dfrac{1}{C_2}\left[i^o_{L,2}+\delta i_{L,2}(t)\right]\delta d_2(t) - \dfrac{1}{R_{2,2}C_2}\left(1-\dfrac{R_E}{R_{2,2}}\right)\delta v_{C,2}(t) - \dfrac{R_E}{R_{2,2}C_2}\delta i_{L_K}(t) \\
\qquad + \dfrac{1}{C_2(R_{1,2}+R_{2,2})}\delta v_{C,1}(t) - \dfrac{R_E}{R_{2,2}C_2}\delta i_B(t) \\
\dot{v}_{C_K} = \frac{1}{C_K}\delta i_{L_K}(t) - \frac{1}{R_{crl}C_K}\delta v_{C_K}(t) + \frac{P^o_{cpl}\delta v_{C_K}(t) - v^o_{C_K}\delta P_{cpl}(t)}{C v^o_{C_K}\left[v^o_{C_K}+\delta v_{C_K}(t)\right]}
\end{cases} \tag{38}$$

E a saída é:

$$\delta v_E(t) = -R_{1,2}R_{E_1}\delta i_{L_K}(t) + R_{E_1}\delta v_{C,1}(t) + R_{E_2}\delta v_{C,2}(t) \tag{39}$$

Modelo Fuzzy da Microrrede

O modelo dinâmico transladado pode ser descrito como:

$$\dot{x}(t) = A(x) \cdot x(t) + B(x) \cdot u(t) + F(x) \cdot \omega(t)$$

com,

$$A = \begin{bmatrix} -\dfrac{R_{1,1}}{L_1} & 0 & 0 & -\dfrac{1}{L_1} & 0 & 0 \\ 0 & -\dfrac{R_{2,1}}{L_2} & 0 & 0 & \left[\dfrac{R_{2,1} i^o_{L,2}}{L_2 v^o_{C,2}} - \dfrac{v^o_{in,2}}{L_2 v^o_{C,2}}\right] & 0 \\ 0 & 0 & -\dfrac{R_K + R_{EQ}}{L_K} & \dfrac{R_{EQ}}{R_{1,2} L_K} & \dfrac{R_{EQ}}{R_{2,2} L_K} & -\dfrac{1}{L_K} \\ \dfrac{1}{C_1} & 0 & -\dfrac{R_{EQ}}{R_{1,2} C_1} & -\dfrac{1}{R_{1,2} C_1}\left[1 - \dfrac{R_{EQ}}{R_{1,2}}\right] & \dfrac{1}{C_1 \left(R_{1,2} + R_{2,2}\right)} & 0 \\ 0 & \dfrac{1}{C_2}\left(1 - d^o_2\right) & -\dfrac{R_{EQ}}{R_{1,2} C_2} & \dfrac{1}{C_2 \left(R_{1,2} + R_{2,2}\right)} & -\dfrac{1}{R_{2,2} C_1}\left[1 - \dfrac{R_{EQ}}{R_{2,2}}\right] & 0 \\ 0 & 0 & \dfrac{1}{C_K} & 0 & 0 & a_{66} \end{bmatrix}, \quad x(t) = \begin{bmatrix} \delta i_{L,1}(t) \\ \delta i_{L,2}(t) \\ \delta i_{L_K}(t) \\ \delta v_{C,1}(t) \\ \delta v_{C,2}(t) \\ \delta v_{C_K}(t) \end{bmatrix},$$

$$B = \begin{bmatrix} \dfrac{v_{in,1}}{L_1} & 0 & 0 \\ 0 & \dfrac{1}{L_2}\left[v^o_{C,2} + \delta v_{C,2}(t)\right] & 0 \\ 0 & 0 & -\dfrac{R_{EQ}}{L_K} \\ 0 & 0 & -\dfrac{R_{EQ}}{R_{1,2} C_1} \\ 0 & -\dfrac{1}{C_2}\left[i^o_{L,2} + \delta i_{L,2}(t)\right] & -\dfrac{R_{EQ}}{R_{2,2} C_2} \\ 0 & 0 & 0 \end{bmatrix}, \quad u(t) = \begin{bmatrix} \delta d_1(t) \\ \delta d_2(t) \\ \delta i_B(t) \end{bmatrix},$$

$$F = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ -\dfrac{v^o_{C_K}}{C v^o_{C_K}\left[v^o_{C_K} + \delta v_{C_K}(t)\right]} \end{bmatrix}, \quad \omega(t) = \begin{bmatrix} \delta v_{in,1}(t) \\ \delta v_{in,2}(t) \\ \delta P_{cpl}(t) \end{bmatrix}. \tag{40}$$

Em que:

$$a_{66} = \frac{P^o_{cpl}}{C v^o_{C_K}\left[v^o_{C_K} + \delta v_{C_K}(t)\right]} - \frac{1}{R_{crl} C_K},$$

$$f_{11} = \frac{1}{L_1}\left(\frac{R_{1,1}}{v^o_{in,1}} i^o_{L,1} + \frac{1}{v^o_{in,1}} v^o_{C,1}\right).$$

As variáveis premissas:

$$z_1(t) = \frac{1}{Cv^o_{C_K}\left[v^o_{C_K} + \delta v_{C_K}(t)\right]}$$

$$z_2(t) = \delta v_{in,1}(t)$$

$$z_3(t) = \delta v_{C,2}(t)$$

$$z_4(t) = \delta i_{L,2}(t) \tag{41}$$

Definição das funções de pertinência

Termo não linear $z_1(t)$:

$$\max_{\delta v_{C_K}(t)} z_1(t) = \frac{1}{Cv^o_{C_K}\left[v^o_{C_K} + \delta v_{C_K}(t)\right]} = q_1 \quad ; \quad \min \delta v_{C_K}(t)$$

$$\min_{\delta v_{C_K}(t)} z_1(t) = \frac{1}{Cv^o_{C_K}\left[v^o_{C_K} + \delta v_{C_K}(t)\right]} = q_2 \quad ; \quad \max \delta v_{C_K}(t) \tag{42}$$

$z_1(t)$ pode ser definido:

$$z_1(t) = \sum_{i=1}^{2} E_i(z_1(t))q_i$$

$$E_1(z_1(t))q_1 + E_2(z_1(t))q_2 = z_1(t) \tag{43}$$

O somatório das funções de pertinência devem ser iguais a 1, então:

$$E_1(z_1(t)) + E_2(z_1(t)) = 1 \tag{44}$$

Usando as equações 43 e 44:

$$E_1(z_1(t))q_1 = z_1(t) - E_2(z_1(t))q_2$$

$$E_1(z_1(t)) = \frac{z_1(t) - E_2(z_1(t))q_2}{q_1}$$

$$E_2(z_1(t)) + \frac{z_1(t) - E_2(z_1(t))q_2}{q_1} = 1$$

$$E_2(z_1(t))\,(q_1 - q_2) = q_1 - z_1(t)$$

$$E_2(z_1(t)) = \frac{q_1 - z_1(t)}{q_1 - q_2} \tag{45}$$

$$E_1(z_1(t)) = 1 - E_2(z_1(t))$$

$$E_1(z_1(t)) = \frac{z_1(t) - q_2}{q_1 - q_2} \tag{46}$$

As funções de pertinência dos demais termos podem ser obtidas de forma análoga.

Termo não linear $z_2(t)$:

$$\max_{v_{in,1}(t)} z_2(t) = \max \delta v_{in,1}(t) = b_1$$

$$\min_{v_{in,1}(t)} z_2(t) = \min \delta v_{in,1}(t) = b_2 \tag{47}$$

$$z_2(t) = \sum_{j=1}^{2} M_j(z_2(t)) b_i$$

$$M_1(z_2(t)) = \frac{z_2(t) - b_2}{b_1 - b_2} \tag{48}$$

$$M_2(z_2(t)) = \frac{b_1 - z_2(t)}{b_1 - b_2} \tag{49}$$

Termo não linear $z_3(t)$:

$$\max_{\delta v_{C,2}(t)} z_3(t) = \max \delta v_{C,2}(t) = c_1$$

$$\min_{\delta v_{C,2}(t)} z_3(t) = \min \delta v_{C,2}(t) = c_2 \tag{50}$$

$$z_3(t) = \sum_{k=1}^{2} N_k(z_3(t)) c_i$$

$$N_1(z_3(t)) = \frac{z_3(t) - c_2}{c_1 - c_2} \tag{51}$$

$$N_2(z_3(t)) = \frac{c_1 - z_3(t)}{c_1 - c_2} \tag{52}$$

Termo não linear $z_4(t)$:

$$\max_{\delta i_{L,2}(t)} z_4(t) = \max \delta i_{L,2}(t) = d_1$$

$$\min_{\delta i_{L,2}(t)} z_4(t) = \min \delta i_{L,2}(t) = d_2 \tag{53}$$

$$z_4(t) = \sum_{l=1}^{2} S_l(z_4(t))d_i$$

$$S_1(z_4(t)) = \frac{z_4(t) - d_2}{d_1 - d_2} \tag{54}$$

$$S_2(z_4(t)) = \frac{d_1 - z_4`(t)}{d_1 - d_2} \tag{55}$$

O modelo fuzzy da microrrede é representado como:

$$\dot{x}(t) = \sum_{i=1}^{2}\sum_{j=1}^{2}\sum_{k=1}^{2}\sum_{l=1}^{2} E_i(z_1(t))M_j(z_2(t))N_k(z_3(t))S_l(z_4(t))$$

$$\times \{A_{ijkl}x(t) + B_{ijkl}u(t) + F_{ijkl}\omega(t)\} \tag{56}$$

Os somatórios podem ser reduzidos em um único somatório:

$$\dot{x}(t) = \sum_{\mathbf{i}=1}^{16} h_{\mathbf{i}}(z(t)) \{A_{\mathbf{i}}x(t) + B_{\mathbf{i}}u(t) + F_{\mathbf{i}}\omega(t)\} \tag{57}$$

Em que,

$$\begin{gathered}
\mathbf{i} = 2^0 l + 2^1(k-1) + 2^2(j-1) + 2^3(i-1), \\
h_{\mathbf{i}}(z(t)) = E_i\left(z_1(t)\right) M_j\left(z_2(t)\right) N_k\left(z_3(t)\right) S_l\left(z_4(t)\right), \\
\boldsymbol{A}_{\mathbf{i}} = \boldsymbol{A}_{ijkl}, \quad \boldsymbol{B}_{\mathbf{i}} = \boldsymbol{B}_{ijkl}, \quad \boldsymbol{F}_{\mathbf{i}} = \boldsymbol{B}_{ijkl}.
\end{gathered}$$

Sistema em malha fechada

Lei de controle não linear aplicada:

$$u(t) = K(\hat{x}(t))\hat{x}(t) + L(\hat{x}(t))\hat{\omega}(t) = \sum_{\mathbf{j}} \mathrm{h}_{\mathbf{j}}(\hat{x}(t))\left(K_{\mathbf{j}}\hat{x}(t) + L_{\mathbf{j}}\hat{\omega}(t)\right) \tag{58}$$

Em que $\hat{x}(t)$ são os dados disponíveis sobre os estados para o controlador.

Considerando o erros de transmissão:

$$e_x(t) = \hat{x}(t) - x(t), \quad e_\omega(t) = \hat{\omega}(t) - \omega(t) \tag{59}$$

O sistema em malha fechada é

$$\dot{x}(t) = \sum_{\mathbf{i}}\sum_{\mathbf{j}} h_{\mathbf{i}}(x(t))h_{\mathbf{j}}(\hat{x})\left\{(A_{\mathbf{i}} + B_{\mathbf{i}}K_{\mathbf{j}})x + (F_{\mathbf{i}} + B_{\mathbf{i}}L_{\mathbf{j}})\omega + B_{\mathbf{i}}K_{\mathbf{j}}e_x + B_{\mathbf{j}}L_{\mathbf{j}}e_\omega\right\}. \tag{60}$$

1 LMI de Estabilidade

Sistema em malha fechada:

$$\dot{x} = [A(x) + B(x)K(x(t))]\,x(t) + B(x)K(x(t))e(t) + B(x)K(e(t))\,[e(t) + x(t)] + E(x)w(t) \tag{61}$$

ETM dinâmico:

$$t_0 = 0, t_{k+1} = \inf\{t > t_k : \eta(t) + \theta\Gamma(x(t), e_x(t)) < 0\},\ \forall k \in \mathbb{N}. \tag{62}$$

Função de ativação:

$$\Gamma(x, e) = x^T(t)\Psi x(t) - e^T \Xi e - \zeta(x, e) \tag{63}$$

com,

$$\zeta(x, e) = 2x^T P\,[B(x)\,(K(x+e) - x)\,(x+e)] \tag{64}$$

LMI de restrição:

$$\sum_{i,j \in B^p} \Upsilon_{ij} < 0 \tag{65}$$

onde,

$$\Upsilon_{ij} = \begin{bmatrix} \mathrm{He}(A_i X + B_i \tilde{K}_j) & B_i \tilde{K}_j & X E_i^\top & X & X \\ \star & -\tilde{\Xi} & 0 & 0 & 0 \\ \star & \star & -\mu I & 0 & 0 \\ \star & \star & \star & -\tilde{\Psi} & 0 \\ \star & \star & \star & \star & -I \end{bmatrix} \tag{66}$$

Prova:

$$\begin{bmatrix} \mathrm{He}(AX + B(x)\tilde{K}(x)) & B(x)\tilde{K}(x) & X E(x)^\top & X & X \\ \star & -\tilde{\Xi} & 0 & 0 & 0 \\ \star & \star & -\mu X & 0 & 0 \\ \star & \star & \star & -\tilde{\Psi} & 0 \\ \star & \star & \star & \star & -I \end{bmatrix} < 0 \tag{67}$$

Multiplicando por $\mathrm{diag}(X^{-1}, X^{-1}, X^{-1}, I, I)$

$$K_j = \tilde{K}_j X^{-1}, \quad \Xi = X^{-1}\tilde{\Xi}X^{-1}, \quad \Psi = \tilde{\Psi}^{-1}, \quad P = X^{-1} \tag{68}$$

$$y = C(x)x(t) + D(x)u(t) \tag{69}$$

$$y = C(x)x(t) + D(x)K(\hat{x})\hat{x} \tag{70}$$

$$y = C(x)x(t) + D(x)K(x)e + D(x)\,(K(x+e) - x)\,(x+e) \tag{71}$$

$$\zeta_y = D(x)\,(K(x+e) - x)\,(x+e) \tag{72}$$

$$\zeta_2 = He(\zeta_y^\top Cx + \zeta_y^\top(D(x)K(x)e)) + \zeta_y^\top \zeta_y \tag{73}$$

$$y = C(x)x(t) + D(x)K(x)e + \zeta_y \tag{74}$$

$$y^\top y = x^\top C^\top Cx + x^\top C^\top DKe + (DKe)^\top Cx + (D(x)K(x)e)^\top D(x)K(x)e \tag{75}$$

$$\begin{bmatrix} \mathrm{He}(PA(x) + PB(x)K(x)) & PB(x)K(x) & PE(x) & I & I \\ \star & -\Xi & 0 & 0 & 0 \\ \star & \star & -\mu I & 0 & 0 \\ \star & \star & \star & -\tilde{\Psi} & 0 \\ \star & \star & \star & \star & -\tilde{C}(x) \end{bmatrix} < 0 \tag{76}$$

onde, $\tilde{C}(x) = (C^{\top}(x)C(x))^{-1}$.
Por Schur:

$$\begin{bmatrix} \mathrm{He}(PA(x) + PB(x)K(x)) + \Psi + C^{\top}(x)C(x) & PB(x)K(x) + C^{\top}(x)D(x)K(x) & PE(x) \\ \star & -\Xi + K^{\top}(x)D^{\top}(x)D(x)K(x) & 0 \\ \star & \star & -\mu I \end{bmatrix} < 0 \tag{77}$$

Pré-multiplicando a matriz anterior por

$$\begin{bmatrix} x^T & e^T & w^T \end{bmatrix} \tag{78}$$

têm-se:
wT -¿ 1,nw P -¿ nx,nx E -¿ nx,nw
Cx x -¿ 6,1 C -¿ ny,6

$$\begin{bmatrix} A_1 & A_2 & A_3 \end{bmatrix} < 0 \tag{79}$$

onde,

$$A_1 = x^T \left[\mathrm{He}(PA(x) + PB(x)K(x)) + \Psi\right] + x^T C(x) + e^T PB(x)K(x) + w^T PE(x) \tag{80}$$

$$A_2 = x^T PB(x)K(x) - e^T \Xi \tag{81}$$

$$A_3 = x^T PE(x) - \mu w^T \tag{82}$$

Pós-multiplicando por:

$$\begin{bmatrix} x & e & w \end{bmatrix}^T \tag{83}$$

têm-se:

$$\begin{aligned} &x^T \left[\mathrm{He}(PA(x) + PB(x)K(x)) + \Psi\right] x + e^T PB(x)K(x)x + w^T PE(x)x \\ &\qquad + x^T PB(x)K(x)e - e^T \Xi e + x^T PE(x)w + Cx - \mu w^T w < 0 \end{aligned} \tag{84}$$

$$\begin{aligned} &x^T \left[\mathrm{He}(PA(x) + PB(x)K(x))\right] x + e^T PB(x)K(x)x + w^T PE(x)x \\ &\qquad + x^T PB(x)K(x)e - e^T \Xi e + x^T \Xi x + x^T PE(x)w + y^T y - \mu w^T w < 0 \end{aligned} \tag{85}$$

$$\begin{aligned} &2x^T P \left[A(x) + B(x)K(x)\right] x + 2x^T PB(x)K(x)e + 2x^T PE(x)w \\ &\qquad - e^T \Xi e + x^T(t)\Psi x(t) + y^T y - \mu w^T w < 0 \end{aligned} \tag{86}$$

$$2x^T P \left\{\left[A(x) + B(x)K(x)\right] x + B(x)K(x)e + E(x)w\right\} - e^T \Xi e + x^T(t)\Psi x(t) + y^T y - \mu w^T w < 0 \tag{87}$$

$$2x^T(t) P \left\{\left[A(x) + B(x)K(x(t))\right] x(t) + B(x)K(x(t))e(t) + E(x)w(t)\right\} + y^T(t)\Psi y(t) - e^T \Xi e < 0 \tag{88}$$

$$\dot{x}^T(t)Px(t) + x^T(t)P\dot{x}(T) + \dot{\eta}(t) + y^T(t)y(t) - \mu w^T(t)w(t) < 0 \tag{89}$$

2 Lidar com o Comportamento Zeno

3 Tentativa 1

Assumindo que $w(t)$ satisfaz:

$$w^{\mathsf{T}}(t)w(t) \leq \varrho \tag{90}$$

ϱ é uma constante dada.
Para $t \in [t_k, t_{k+1}]$, têm-se:

$$\|\dot{e}(t)\| = \|\dot{x}(t)\| = \| \left[A(x) + B(x)K(x(t))\right] x(t) + B(x)K(x(t))e(t) + B(x)K(e(t)) \left[e(t) + x(t)\right] + E(x)w(t)\| \tag{91}$$

$$\|\dot{e}(t)\| = \| \left[A(x) + B(x)K(x(t))\right] \left[\hat{x}(t) - e(t)\right] + B(x)K(x(t))e(t) + B(x)K(e(t)) \left[e(t) + x(t)\right] + E(x)w(t)\| \tag{92}$$

$$\|\dot{e}(t)\| \leq \|A(x)\|\|e(t)\| + \|A(x) + B(x)K(x(t))\|\|\hat{x}(t)\| + \|E(x)\|\|w(t)\| + \|B(x)K(e(t))\|\|e(t) + x(t)\| \tag{93}$$

$$\|\dot{e}(t)\| \leq c_1\|e(t)\| + c_2 \tag{94}$$

onde,

$$c_1 = \max\{|\lambda(A_i)|\} \tag{95}$$

$$c_2 = \max\{|\lambda(A_i + B_iK_j)|\}\|\hat{x}(t)\| + \max\{\lambda(B(x)K(e(t)))\}\|\hat{x}(t)\| + \max\{|\lambda(D_i)|\}\sqrt{\varrho} \tag{96}$$

Seja a variável auxiliar:

$$\|\dot{\upsilon}(t)\| = c_1\|\upsilon(t)\| + c_2 \tag{97}$$

Pode-se deduzir que $\upsilon(t) \geq e(t)$ pelo lema da comparação. Além disso, um pode 'alcançar' qur para $t \in [t_k, t_{k+1})$.
(i) Se $c_1 \neq 0$,

$$\|\upsilon(t)\| = \frac{c_2}{c_1}\left(e^{c_1(t-t_k)} - 1\right) \tag{98}$$

(ii) Se $c_1 = 0$,

$$\|\upsilon\| = c_2(t - t_k) \tag{99}$$

Dado $\varsigma = \min\{1, \lambda_{\min}(P)\}$, têm-se:

$$\varsigma\left(\eta(t) + \|x(t)\|^2\right) \leq x^{\mathsf{T}}(t)Px(t) + \eta(t) \leq W(x,\eta) \leq c \tag{100}$$

4 Tentativa 2

Quando evento é acionado, têm-se:

$$\eta(t) + \theta\Gamma(x(t), e(t)) < 0 \tag{101}$$

$$\Gamma(x(t), e(t)) < -\frac{\eta(t)}{\theta} \tag{102}$$

$$\Gamma(x(t), e(t)) < -\frac{\eta(t)}{\theta} \tag{103}$$

$$x^T(t)\Psi x(t) - e^T(t)\Xi e(t) - \zeta(x(t), e(t)) < 0 \tag{104}$$

$$e^T(t)\Xi e(t) > x^T(t)\Psi x(t) - \zeta(x(t), e(t)) \tag{105}$$

$$\frac{e^T(t)\Xi e(t)}{x^T(t)\Psi x(t)} > 1 - \frac{\zeta(x(t), e(t))}{x^T(t)\Psi x(t)} \tag{106}$$

$$\mathcal{G}(x(t), e(t)) > 1 - \mathcal{V}(x(t), e(t)) \tag{107}$$

com,

$$\zeta(x,e) = 2x^T P \left[B(x) \left(K(x+e) - x \right) (x+e) \right] \tag{108}$$

Como $\mathcal{G}(x(t), e(t)) > 0$, então $\mathcal{V}(x(t), e(t)) < 1$, enquanto nenhum evento é acionado.

$$\|\dot{x}(t)\| \leq L(\|x(t)\| + \|e(t)\| + \|w(t)\|) \tag{109}$$

$$\frac{\|\dot{x}(t)\|}{\|x(t)\|} \leq L \left(1 + \frac{\|e(t)\|}{\|x(t)\|} + \frac{\|w(t)\|}{\|x(t)\|} \right) \tag{110}$$

$$\frac{\mathsf{d}}{\mathsf{d}t} \left(\frac{\|e(t)\|}{\|x(t)\|} \right) \leq L \left(1 + \frac{\|e(t)\|}{\|x(t)\|} \right) \left(1 + \frac{\|e(t)\|}{\|x(t)\|} + \frac{\|w(t)\|}{\|x(t)\|} \right) \tag{111}$$

Como $\|x(t)\|^2 < \mu \|w(t)\|^2$, então:

$$\frac{\mathsf{d}}{\mathsf{d}t} \left(\frac{\|e(t)\|}{\|x(t)\|} \right) \leq L \left\{ \left(1 + \frac{\|e(t)\|}{\|x(t)\|} \right)^2 + \frac{\|w(t)\|}{\|x(t)\|} \right\} \tag{112}$$

$$\frac{\mathsf{d}}{\mathsf{d}t} \left(\frac{\|e(t)\|}{\|x(t)\|} \right) \leq L \left\{ \left(1 + \frac{\|e(t)\|}{\|x(t)\|} \right)^2 + \frac{\sqrt{\varrho}}{\|x(t)\|} \right\} \tag{113}$$

$$\dot{\varphi} \leq L \left\{ (1+\varphi)^2 + \frac{\sqrt{\varrho}}{\|x(t)\|} \right\} \tag{114}$$