Exercise Problems: Fuzzy Sets

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1. Consider two fuzzy sets, one representing a scooter and other van.

$$\text{Scooter} = \left\{ \frac{0.6}{\text{van}} + \frac{0.3}{\text{motorcycle}} + \frac{0.8}{\text{boat}} + \frac{0.9}{\text{scooter}} + \frac{0.1}{\text{house}} \right\},$$

$$\text{Van} = \left\{ \frac{1.0}{\text{van}} + \frac{0.2}{\text{motorcycle}} + \frac{0.5}{\text{boat}} + \frac{0.3}{\text{scooter}} + \frac{0.2}{\text{house}} \right\}$$

Find the following:

(a) $\text{Scooter} \cup \text{Van} = \left\{ \frac{\mathbf{1.0}}{\mathbf{van}} + \frac{\mathbf{0.3}}{\mathbf{motorcycle}} + \frac{\mathbf{0.8}}{\mathbf{boat}} + \frac{\mathbf{0.9}}{\mathbf{scooter}} + \frac{\mathbf{0.2}}{\mathbf{house}} \right\}$

(b) $\text{Scooter} \setminus \text{Van} = \text{Scooter} \cap \overline{\text{Van}} = \left\{ \frac{\mathbf{0.0}}{\mathbf{van}} + \frac{\mathbf{0.3}}{\mathbf{motorcycle}} + \frac{\mathbf{0.5}}{\mathbf{boat}} + \frac{\mathbf{0.7}}{\mathbf{scooter}} + \frac{\mathbf{0.1}}{\mathbf{house}} \right\}$

(c) $\text{Scooter} \cap \overline{\text{Van}} = \left\{ \frac{\mathbf{0.0}}{\mathbf{van}} + \frac{\mathbf{0.3}}{\mathbf{motorcycle}} + \frac{\mathbf{0.5}}{\mathbf{boat}} + \frac{\mathbf{0.7}}{\mathbf{scooter}} + \frac{\mathbf{0.1}}{\mathbf{house}} \right\}$

(d) $\overline{\text{Scooter} \cup \text{Scooter}} = \overline{\text{Scooter}} = \left\{ \frac{\mathbf{0.6}}{\mathbf{van}} + \frac{\mathbf{0.3}}{\mathbf{motorcycle}} + \frac{\mathbf{0.8}}{\mathbf{boat}} + \frac{\mathbf{0.9}}{\mathbf{scooter}} + \frac{\mathbf{0.1}}{\mathbf{house}} \right\}$

(e) $\overline{\text{Scooter} \cap \text{Scooter}} = \overline{\text{Scooter}} = \left\{ \frac{\mathbf{0.6}}{\mathbf{van}} + \frac{\mathbf{0.3}}{\mathbf{motorcycle}} + \frac{\mathbf{0.8}}{\mathbf{boat}} + \frac{\mathbf{0.9}}{\mathbf{scooter}} + \frac{\mathbf{0.1}}{\mathbf{house}} \right\}$

(f) $\text{Scooter} \cup \overline{\text{Van}} = \left\{ \frac{\mathbf{0.6}}{\mathbf{van}} + \frac{\mathbf{0.8}}{\mathbf{motorcycle}} + \frac{\mathbf{0.8}}{\mathbf{boat}} + \frac{\mathbf{0.9}}{\mathbf{scooter}} + \frac{\mathbf{0.8}}{\mathbf{house}} \right\}$

(g) $\text{Van} \cup \overline{\text{Van}} = \left\{ \frac{\mathbf{1.0}}{\mathbf{van}} + \frac{\mathbf{0.8}}{\mathbf{motorcycle}} + \frac{\mathbf{0.5}}{\mathbf{boat}} + \frac{\mathbf{0.7}}{\mathbf{scooter}} + \frac{\mathbf{0.8}}{\mathbf{house}} \right\}$

(h) $\text{Van} \cap \overline{\text{Van}} = \left\{ \frac{\mathbf{0.0}}{\mathbf{van}} + \frac{\mathbf{0.2}}{\mathbf{motorcycle}} + \frac{\mathbf{0.5}}{\mathbf{boat}} + \frac{\mathbf{0.3}}{\mathbf{scooter}} + \frac{\mathbf{0.2}}{\mathbf{house}} \right\}$

2. Consider flight simulator data, the determination of certain changes in creating conditions of the aircraft is made on the basis of hard breakpoint in the mach region. Let us define a fuzzy set to represent the condition of near a match number of 0.644. A second fuzzy sets in the region of mach number 0.74.

$$A = \text{near mach } 0.64 = \left\{ \frac{0.1}{0.630} + \frac{0.6}{0.635} + \frac{1.0}{0.640} + \frac{0.8}{0.645} + \frac{0.2}{0.650} \right\},$$

$$B = \text{near mach } 0.64 = \left\{ \frac{0.0}{0.630} + \frac{0.5}{0.635} + \frac{0.8}{0.640} + \frac{1.0}{0.645} + \frac{0.4}{0.650} \right\}.$$

Find the following:

(a) $A \cup B = \left\{ \frac{\mathbf{0.1}}{\mathbf{0.630}} + \frac{\mathbf{0.6}}{\mathbf{0.635}} + \frac{\mathbf{1.0}}{\mathbf{0.640}} + \frac{\mathbf{1.0}}{\mathbf{0.645}} + \frac{\mathbf{0.4}}{\mathbf{0.650}} \right\}$

(b) $A \cap B = \left\{ \frac{\mathbf{0.0}}{\mathbf{0.630}} + \frac{\mathbf{0.5}}{\mathbf{0.635}} + \frac{\mathbf{0.8}}{\mathbf{0.640}} + \frac{\mathbf{0.8}}{\mathbf{0.645}} + \frac{\mathbf{0.2}}{\mathbf{0.650}} \right\}$

(c) $\overline{A} = \left\{ \frac{\mathbf{0.9}}{\mathbf{0.630}} + \frac{\mathbf{0.4}}{\mathbf{0.635}} + \frac{\mathbf{0.0}}{\mathbf{0.640}} + \frac{\mathbf{0.2}}{\mathbf{0.645}} + \frac{\mathbf{0.8}}{\mathbf{0.650}} \right\}$

(d) $\overline{B} = \left\{ \frac{\mathbf{1.0}}{\mathbf{0.630}} + \frac{\mathbf{0.5}}{\mathbf{0.635}} + \frac{\mathbf{0.2}}{\mathbf{0.640}} + \frac{\mathbf{0.0}}{\mathbf{0.645}} + \frac{\mathbf{0.6}}{\mathbf{0.650}} \right\}$

(e) $A \setminus B = A \cap \overline{B} = \left\{ \frac{\mathbf{0.1}}{\mathbf{0.630}} + \frac{\mathbf{0.5}}{\mathbf{0.635}} + \frac{\mathbf{0.2}}{\mathbf{0.640}} + \frac{\mathbf{0.0}}{\mathbf{0.645}} + \frac{\mathbf{0.2}}{\mathbf{0.650}} \right\}$

(f) $\overline{A \cup B} = \overline{A} \cap \overline{B} = \left\{ \frac{\mathbf{0.9}}{\mathbf{0.630}} + \frac{\mathbf{0.4}}{\mathbf{0.635}} + \frac{\mathbf{0.0}}{\mathbf{0.640}} + \frac{\mathbf{0.0}}{\mathbf{0.645}} + \frac{\mathbf{0.6}}{\mathbf{0.650}} \right\}$

(g) $\overline{A \cap B} = \overline{A} \cup \overline{B} = \left\{ \frac{\mathbf{1.0}}{\mathbf{0.630}} + \frac{\mathbf{0.5}}{\mathbf{0.635}} + \frac{\mathbf{0.2}}{\mathbf{0.640}} + \frac{\mathbf{0.2}}{\mathbf{0.645}} + \frac{\mathbf{0.8}}{\mathbf{0.650}} \right\}$

3. The continuos form of MOSFET and a transistor are shown in figure below. The discretized membership functions are given by the following equations.

$$\mu_m = \left\{ \frac{0}{0} + \frac{0.4}{2} + \frac{0.6}{4} + \frac{0.7}{6} + \frac{0.8}{8} + \frac{0.9}{10} \right\},$$

$$\mu_T = \left\{ \frac{0}{0} + \frac{0.1}{2} + \frac{0.2}{4} + \frac{0.3}{6} + \frac{0.4}{8} + \frac{0.5}{10} \right\}$$

For these two fuzzy calculate the following:

(a) $\mu_m \cup \mu_T = \left\{ \frac{\mathbf{0}}{\mathbf{0}} + \frac{\mathbf{0.4}}{\mathbf{2}} + \frac{\mathbf{0.6}}{\mathbf{4}} + \frac{\mathbf{0.7}}{\mathbf{6}} + \frac{\mathbf{0.8}}{\mathbf{8}} + \frac{\mathbf{0.9}}{\mathbf{10}} \right\}$

(b) $\mu_m \cap \mu_T = \left\{ \frac{\mathbf{0}}{\mathbf{0}} + \frac{\mathbf{0.1}}{\mathbf{2}} + \frac{\mathbf{0.2}}{\mathbf{4}} + \frac{\mathbf{0.3}}{\mathbf{6}} + \frac{\mathbf{0.4}}{\mathbf{8}} + \frac{\mathbf{0.5}}{\mathbf{10}} \right\}$

(c) $\overline{\mu}_T = 1 - \mu_T = \left\{ \frac{\mathbf{0}}{\mathbf{0}} + \frac{\mathbf{0.9}}{\mathbf{2}} + \frac{\mathbf{0.8}}{\mathbf{4}} + \frac{\mathbf{0.7}}{\mathbf{6}} + \frac{\mathbf{0.6}}{\mathbf{8}} + \frac{\mathbf{0.5}}{\mathbf{10}} \right\}$

(d) $\overline{\mu_m} = 1 - \mu_m = \left\{ \frac{\mathbf{0}}{\mathbf{0}} + \frac{\mathbf{0.6}}{\mathbf{2}} + \frac{\mathbf{0.4}}{\mathbf{4}} + \frac{\mathbf{0.3}}{\mathbf{6}} + \frac{\mathbf{0.2}}{\mathbf{8}} + \frac{\mathbf{0.1}}{\mathbf{10}} \right\}$

(e) De Morgan's law

$\overline{\boldsymbol{\mu_m \cup \mu_T}} = \overline{\boldsymbol{\mu_m}} \cap \overline{\boldsymbol{\mu_T}} = \left\{ \frac{\mathbf{0}}{\mathbf{0}} + \frac{\mathbf{0.6}}{\mathbf{2}} + \frac{\mathbf{0.4}}{\mathbf{4}} + \frac{\mathbf{0.3}}{\mathbf{6}} + \frac{\mathbf{0.2}}{\mathbf{8}} + \frac{\mathbf{0.1}}{\mathbf{10}} \right\}$

$\overline{\boldsymbol{\mu_m \cap \mu_T}} = \overline{\boldsymbol{\mu_m}} \cup \overline{\boldsymbol{\mu_T}} = \left\{ \frac{\mathbf{0}}{\mathbf{0}} + \frac{\mathbf{0.9}}{\mathbf{2}} + \frac{\mathbf{0.8}}{\mathbf{4}} + \frac{\mathbf{0.7}}{\mathbf{6}} + \frac{\mathbf{0.6}}{\mathbf{8}} + \frac{\mathbf{0.5}}{\mathbf{10}} \right\}$

4. Samples of new microprocessors IC chip are to be sent to several customers for beta testing. The chips are sorted to meet certain maximum electrical characteristics say frequency, and temperature rating, so that the "best" chips are distributed to preferred customer 1. Suppose that each sample chip is screened and all chips are found to have a maximum operating frequency in the range 7-15 MHz at 20^{o}C. Also the maximum operating temperature range (20^{o}C $\pm\Delta T$) at 8 MHz is determined. Suppose there are eight sample chips with the following electrical characteristics:

Table 1: Chip number

	1	2	3	4	5	6	7	8
$F_{\max}(MHz)$	6	7	8	9	10	11	12	13
$\Delta T_{\max}$	0	0	20	40	30	50	40	60

The following fuzzy sets are defined.

$$A = \text{Set of "Fast" chips} = \text{chips with } f_{\max} \geq 12\,\text{MHz}$$
$$= \left\{ \frac{0.0}{1} + \frac{0.0}{2} + \frac{0.1}{3} + \frac{0.1}{4} + \frac{0.2}{5} + \frac{0.8}{6} + \frac{1.0}{7} + \frac{1.0}{8} \right\}$$

$$B = \text{Set of "Fast" chips} = \text{chips with } f_{\max} \geq 8\,\text{MHz}$$
$$= \left\{ \frac{0.1}{1} + \frac{0.5}{2} + \frac{1.0}{3} + \frac{1.0}{4} + \frac{1.0}{5} + \frac{1.0}{6} + \frac{1.0}{7} + \frac{1.0}{8} \right\}$$

$$C = \text{Set of "Fast" chips} = \text{chips with } T_{\max} \geq 10\,^{\circ}\text{C}$$
$$= \left\{ \frac{0.0}{1} + \frac{0.0}{2} + \frac{1.0}{3} + \frac{1.0}{4} + \frac{1.0}{5} + \frac{1.0}{6} + \frac{1.0}{7} + \frac{1.0}{8} \right\}$$

$$D = \text{Set of "Fast" chips} = \text{chips with } T_{\max} \geq 50\,^{\circ}\text{C}$$
$$= \left\{ \frac{0.0}{1} + \frac{0.6}{2} + \frac{0.1}{3} + \frac{0.2}{4} + \frac{0.5}{5} + \frac{0.8}{6} + \frac{1.0}{7} + \frac{1.0}{8} \right\}$$

Using fuzzy set, illustrate various set operations possible.

$$\boldsymbol{A \cup B} = \left\{ \frac{\mathbf{0.0}}{\mathbf{1}} + \frac{\mathbf{0.0}}{\mathbf{2}} + \frac{\mathbf{1.0}}{\mathbf{3}} + \frac{\mathbf{1.0}}{\mathbf{4}} + \frac{\mathbf{1.0}}{\mathbf{5}} + \frac{\mathbf{1.0}}{\mathbf{6}} + \frac{\mathbf{1.0}}{\mathbf{7}} + \frac{\mathbf{1.0}}{\mathbf{8}} \right\}$$
$$\boldsymbol{A \cup C} = \left\{ \frac{\mathbf{0.0}}{\mathbf{1}} + \frac{\mathbf{0.0}}{\mathbf{2}} + \frac{\mathbf{1.0}}{\mathbf{3}} + \frac{\mathbf{1.0}}{\mathbf{4}} + \frac{\mathbf{1.0}}{\mathbf{5}} + \frac{\mathbf{1.0}}{\mathbf{6}} + \frac{\mathbf{1.0}}{\mathbf{7}} + \frac{\mathbf{1.0}}{\mathbf{8}} \right\}$$
$$\boldsymbol{A \cup D} = \left\{ \frac{\mathbf{0.0}}{\mathbf{1}} + \frac{\mathbf{0.6}}{\mathbf{2}} + \frac{\mathbf{0.1}}{\mathbf{3}} + \frac{\mathbf{0.2}}{\mathbf{4}} + \frac{\mathbf{0.5}}{\mathbf{5}} + \frac{\mathbf{0.8}}{\mathbf{6}} + \frac{\mathbf{1.0}}{\mathbf{7}} + \frac{\mathbf{1.0}}{\mathbf{8}} \right\}$$
$$\boldsymbol{B \cup C} = \left\{ \frac{\mathbf{0.1}}{\mathbf{1}} + \frac{\mathbf{0.5}}{\mathbf{2}} + \frac{\mathbf{1.0}}{\mathbf{3}} + \frac{\mathbf{1.0}}{\mathbf{4}} + \frac{\mathbf{1.0}}{\mathbf{5}} + \frac{\mathbf{1.0}}{\mathbf{6}} + \frac{\mathbf{1.0}}{\mathbf{7}} + \frac{\mathbf{1.0}}{\mathbf{8}} \right\}$$
$$\boldsymbol{B \cup D} = \left\{ \frac{\mathbf{0.1}}{\mathbf{1}} + \frac{\mathbf{0.6}}{\mathbf{2}} + \frac{\mathbf{1.0}}{\mathbf{3}} + \frac{\mathbf{1.0}}{\mathbf{4}} + \frac{\mathbf{1.0}}{\mathbf{5}} + \frac{\mathbf{1.0}}{\mathbf{6}} + \frac{\mathbf{1.0}}{\mathbf{7}} + \frac{\mathbf{1.0}}{\mathbf{8}} \right\}$$
$$\boldsymbol{C \cup D} = \left\{ \frac{\mathbf{0.0}}{\mathbf{1}} + \frac{\mathbf{0.6}}{\mathbf{2}} + \frac{\mathbf{1.0}}{\mathbf{3}} + \frac{\mathbf{1.0}}{\mathbf{4}} + \frac{\mathbf{1.0}}{\mathbf{5}} + \frac{\mathbf{1.0}}{\mathbf{6}} + \frac{\mathbf{1.0}}{\mathbf{7}} + \frac{\mathbf{1.0}}{\mathbf{8}} \right\}$$
$$\boldsymbol{A \cap B} = \left\{ \frac{\mathbf{0.0}}{\mathbf{1}} + \frac{\mathbf{0.0}}{\mathbf{2}} + \frac{\mathbf{0.1}}{\mathbf{3}} + \frac{\mathbf{0.1}}{\mathbf{4}} + \frac{\mathbf{0.2}}{\mathbf{5}} + \frac{\mathbf{0.8}}{\mathbf{6}} + \frac{\mathbf{1.0}}{\mathbf{7}} + \frac{\mathbf{1.0}}{\mathbf{8}} \right\}$$
$$\boldsymbol{A \cap C} = \left\{ \frac{\mathbf{0.0}}{\mathbf{1}} + \frac{\mathbf{0.0}}{\mathbf{2}} + \frac{\mathbf{0.1}}{\mathbf{3}} + \frac{\mathbf{0.1}}{\mathbf{4}} + \frac{\mathbf{0.2}}{\mathbf{5}} + \frac{\mathbf{0.8}}{\mathbf{6}} + \frac{\mathbf{1.0}}{\mathbf{7}} + \frac{\mathbf{1.0}}{\mathbf{8}} \right\}$$
$$\boldsymbol{A \cap D} = \left\{ \frac{\mathbf{0.0}}{\mathbf{1}} + \frac{\mathbf{0.0}}{\mathbf{2}} + \frac{\mathbf{0.1}}{\mathbf{3}} + \frac{\mathbf{0.1}}{\mathbf{4}} + \frac{\mathbf{0.2}}{\mathbf{5}} + \frac{\mathbf{0.8}}{\mathbf{6}} + \frac{\mathbf{1.0}}{\mathbf{7}} + \frac{\mathbf{1.0}}{\mathbf{8}} \right\}$$
$$\boldsymbol{B \cap C} = \left\{ \frac{\mathbf{0.0}}{\mathbf{1}} + \frac{\mathbf{0.0}}{\mathbf{2}} + \frac{\mathbf{1.0}}{\mathbf{3}} + \frac{\mathbf{1.0}}{\mathbf{4}} + \frac{\mathbf{1.0}}{\mathbf{5}} + \frac{\mathbf{1.0}}{\mathbf{6}} + \frac{\mathbf{1.0}}{\mathbf{7}} + \frac{\mathbf{1.0}}{\mathbf{8}} \right\}$$
$$\boldsymbol{B \cap D} = \left\{ \frac{\mathbf{0.0}}{\mathbf{1}} + \frac{\mathbf{0.5}}{\mathbf{2}} + \frac{\mathbf{0.1}}{\mathbf{3}} + \frac{\mathbf{0.2}}{\mathbf{4}} + \frac{\mathbf{0.5}}{\mathbf{5}} + \frac{\mathbf{0.8}}{\mathbf{6}} + \frac{\mathbf{1.0}}{\mathbf{7}} + \frac{\mathbf{1.0}}{\mathbf{8}} \right\}$$
$$\boldsymbol{C \cap D} = \left\{ \frac{\mathbf{0.0}}{\mathbf{1}} + \frac{\mathbf{0.6}}{\mathbf{2}} + \frac{\mathbf{0.1}}{\mathbf{3}} + \frac{\mathbf{0.2}}{\mathbf{4}} + \frac{\mathbf{0.5}}{\mathbf{5}} + \frac{\mathbf{0.8}}{\mathbf{6}} + \frac{\mathbf{1.0}}{\mathbf{7}} + \frac{\mathbf{1.0}}{\mathbf{8}} \right\}$$
$$\boldsymbol{\overline{A}} = \left\{ \frac{\mathbf{1.0}}{\mathbf{1}} + \frac{\mathbf{1.0}}{\mathbf{2}} + \frac{\mathbf{0.9}}{\mathbf{3}} + \frac{\mathbf{0.9}}{\mathbf{4}} + \frac{\mathbf{0.8}}{\mathbf{5}} + \frac{\mathbf{0.2}}{\mathbf{6}} + \frac{\mathbf{0.0}}{\mathbf{7}} + \frac{\mathbf{0.0}}{\mathbf{8}} \right\}$$
$$\boldsymbol{\overline{B}} = \left\{ \frac{\mathbf{0.9}}{\mathbf{1}} + \frac{\mathbf{0.5}}{\mathbf{2}} + \frac{\mathbf{0.0}}{\mathbf{3}} + \frac{\mathbf{0.0}}{\mathbf{4}} + \frac{\mathbf{0.0}}{\mathbf{5}} + \frac{\mathbf{0.0}}{\mathbf{6}} + \frac{\mathbf{0.0}}{\mathbf{7}} + \frac{\mathbf{0.0}}{\mathbf{8}} \right\}$$
$$\boldsymbol{\overline{C}} = \left\{ \frac{\mathbf{1.0}}{\mathbf{1}} + \frac{\mathbf{1.0}}{\mathbf{2}} + \frac{\mathbf{0.0}}{\mathbf{3}} + \frac{\mathbf{0.0}}{\mathbf{4}} + \frac{\mathbf{0.0}}{\mathbf{5}} + \frac{\mathbf{0.0}}{\mathbf{6}} + \frac{\mathbf{0.0}}{\mathbf{7}} + \frac{\mathbf{0.0}}{\mathbf{8}} \right\}$$
$$\boldsymbol{\overline{D}} = \left\{ \frac{\mathbf{1.0}}{\mathbf{1}} + \frac{\mathbf{0.4}}{\mathbf{2}} + \frac{\mathbf{0.9}}{\mathbf{3}} + \frac{\mathbf{0.8}}{\mathbf{4}} + \frac{\mathbf{0.5}}{\mathbf{5}} + \frac{\mathbf{0.2}}{\mathbf{6}} + \frac{\mathbf{0.0}}{\mathbf{7}} + \frac{\mathbf{0.0}}{\mathbf{8}} \right\}$$

5. Consider two fuzzy sets A and B as shown in figure below. Write the fuzzy set using membership definition and find the following properties:

$$\boldsymbol{A} = \left\{ \frac{\mathbf{0.0}}{\mathbf{2}} + \frac{\mathbf{0.5}}{\mathbf{4}} + \frac{\mathbf{1.0}}{\mathbf{6}} + \frac{\mathbf{0.5}}{\mathbf{8}} + \frac{\mathbf{0.0}}{\mathbf{10}} + \frac{\mathbf{0.0}}{\mathbf{12}} + \frac{\mathbf{0.0}}{\mathbf{14}} + \frac{\mathbf{0.0}}{\mathbf{16}} \right\}$$
$$\boldsymbol{B} = \left\{ \frac{\mathbf{0.0}}{\mathbf{2}} + \frac{\mathbf{0.0}}{\mathbf{4}} + \frac{\mathbf{0.0}}{\mathbf{6}} + \frac{\mathbf{0.5}}{\mathbf{8}} + \frac{\mathbf{0.75}}{\mathbf{10}} + \frac{\mathbf{0.75}}{\mathbf{12}} + \frac{\mathbf{0.5}}{\mathbf{14}} + \frac{\mathbf{0.0}}{\mathbf{16}} \right\}$$

(a) $A \cup B = \left\{ \frac{\mathbf{0.0}}{\mathbf{2}} + \frac{\mathbf{0.5}}{\mathbf{4}} + \frac{\mathbf{1.0}}{\mathbf{6}} + \frac{\mathbf{0.5}}{\mathbf{8}} + \frac{\mathbf{0.75}}{\mathbf{10}} + \frac{\mathbf{0.75}}{\mathbf{12}} + \frac{\mathbf{0.5}}{\mathbf{14}} + \frac{\mathbf{0.0}}{\mathbf{16}} \right\}$

(b) $A \cap B = \left\{ \frac{\mathbf{0.0}}{\mathbf{2}} + \frac{\mathbf{0.0}}{\mathbf{4}} + \frac{\mathbf{0.0}}{\mathbf{6}} + \frac{\mathbf{0.5}}{\mathbf{8}} + \frac{\mathbf{0.0}}{\mathbf{10}} + \frac{\mathbf{0.0}}{\mathbf{12}} + \frac{\mathbf{0.0}}{\mathbf{14}} + \frac{\mathbf{0.0}}{\mathbf{16}} \right\}$

(c) $\overline{A} = \left\{ \frac{\mathbf{1.0}}{\mathbf{2}} + \frac{\mathbf{0.5}}{\mathbf{4}} + \frac{\mathbf{0.0}}{\mathbf{6}} + \frac{\mathbf{0.5}}{\mathbf{8}} + \frac{\mathbf{1.0}}{\mathbf{10}} + \frac{\mathbf{1.0}}{\mathbf{12}} + \frac{\mathbf{1.0}}{\mathbf{14}} + \frac{\mathbf{1.0}}{\mathbf{16}} \right\}$

(d) $\overline{B} = \left\{ \frac{\mathbf{1.0}}{\mathbf{2}} + \frac{\mathbf{1.0}}{\mathbf{4}} + \frac{\mathbf{1.0}}{\mathbf{6}} + \frac{\mathbf{0.5}}{\mathbf{8}} + \frac{\mathbf{0.25}}{\mathbf{10}} + \frac{\mathbf{0.25}}{\mathbf{12}} + \frac{\mathbf{0.5}}{\mathbf{14}} + \frac{\mathbf{1.0}}{\mathbf{16}} \right\}$

(e) $A \setminus B = \boldsymbol{A} \cap \overline{\boldsymbol{B}} = \left\{ \frac{\mathbf{0.0}}{\mathbf{2}} + \frac{\mathbf{0.5}}{\mathbf{4}} + \frac{\mathbf{1.0}}{\mathbf{6}} + \frac{\mathbf{0.5}}{\mathbf{8}} + \frac{\mathbf{0.0}}{\mathbf{10}} + \frac{\mathbf{0.0}}{\mathbf{12}} + \frac{\mathbf{0.0}}{\mathbf{14}} + \frac{\mathbf{0.0}}{\mathbf{16}} \right\}$

(f) $\overline{A \cup B} = \overline{\boldsymbol{A}} \cap \overline{\boldsymbol{B}} = \left\{ \frac{\mathbf{1.0}}{\mathbf{2}} + \frac{\mathbf{0.5}}{\mathbf{4}} + \frac{\mathbf{0.0}}{\mathbf{6}} + \frac{\mathbf{0.5}}{\mathbf{8}} + \frac{\mathbf{0.25}}{\mathbf{10}} + \frac{\mathbf{0.25}}{\mathbf{12}} + \frac{\mathbf{0.5}}{\mathbf{14}} + \frac{\mathbf{1.0}}{\mathbf{16}} \right\}$

6. Consider two fuzzy sets A and B as shown

$$A = \left\{ \frac{0.0}{1} + \frac{0.5}{2} + \frac{0.3}{3} + \frac{0.7}{4} + \frac{0.9}{5} \right\},$$

$$B = \left\{ \frac{0.2}{1} + \frac{0.4}{2} + \frac{0.6}{3} + \frac{0.9}{4} + \frac{0.4}{5} \right\}.$$

Find

(a) $A \cup B = \left\{ \frac{\mathbf{0.2}}{\mathbf{1}} + \frac{\mathbf{0.5}}{\mathbf{2}} + \frac{\mathbf{0.6}}{\mathbf{3}} + \frac{\mathbf{0.9}}{\mathbf{4}} + \frac{\mathbf{0.9}}{\mathbf{5}} \right\}$

(b) $A \cap B = \left\{ \frac{\mathbf{0.0}}{\mathbf{1}} + \frac{\mathbf{0.4}}{\mathbf{2}} + \frac{\mathbf{0.3}}{\mathbf{3}} + \frac{\mathbf{0.7}}{\mathbf{4}} + \frac{\mathbf{0.4}}{\mathbf{5}} \right\}$

(c) $\overline{A} = \left\{ \frac{\mathbf{1.0}}{\mathbf{1}} + \frac{\mathbf{0.5}}{\mathbf{2}} + \frac{\mathbf{0.7}}{\mathbf{3}} + \frac{\mathbf{0.3}}{\mathbf{4}} + \frac{\mathbf{0.1}}{\mathbf{5}} \right\}$

(d) $\overline{B} = \left\{ \frac{\mathbf{0.8}}{\mathbf{1}} + \frac{\mathbf{0.6}}{\mathbf{2}} + \frac{\mathbf{0.4}}{\mathbf{3}} + \frac{\mathbf{0.1}}{\mathbf{4}} + \frac{\mathbf{0.6}}{\mathbf{5}} \right\}$

(e) $A \setminus B = \boldsymbol{A} \cap \overline{\boldsymbol{B}} = \left\{ \frac{\mathbf{0.0}}{\mathbf{1}} + \frac{\mathbf{0.5}}{\mathbf{2}} + \frac{\mathbf{0.3}}{\mathbf{3}} + \frac{\mathbf{0.1}}{\mathbf{4}} + \frac{\mathbf{0.6}}{\mathbf{5}} \right\}$

(f) $\overline{A \cup B} = \overline{\boldsymbol{A}} \cap \overline{\boldsymbol{B}} = \left\{ \frac{\mathbf{0.8}}{\mathbf{1}} + \frac{\mathbf{0.5}}{\mathbf{2}} + \frac{\mathbf{0.4}}{\mathbf{3}} + \frac{\mathbf{0.1}}{\mathbf{4}} + \frac{\mathbf{0.1}}{\mathbf{5}} \right\}$

7. Prove why law of excluded middle and law of contradiction does not hold good for fuzzy.

Ans: In crisp logic, two contradictory statements cannot be true at the same time. Therefore, $A \cap \overline{A} = \emptyset$. However, in fuzzy logic, the law of non-contradiction does not hold.

Proof. Since

$$\overline{A} = \{(x, \mu_{\overline{A}}(x)) \mid \mu_{\overline{A}}(x) = 1 - \mu_A(x), \forall x \in U\}$$

and

$$A \cap B = \{(x, \mu_{A \cap B}(x)) \mid \mu_{A \cap B}(x) = \min\left(\mu_A(x), \mu_B(x)\right), \forall x \in U\}$$

then,

$$A \cap \overline{A} = \{(x, \mu_{A \cap \overline{A}}(x)) \mid \mu_{A \cap \overline{A}}(x) = \min\left(\mu_A(x), \mu_{\overline{A}}(x)\right), \forall x \in U\}$$
$$A \cap \overline{A} = \{(x, \mu_{A \cap \overline{A}}(x)) \mid \mu_{A \cap \overline{A}}(x) = \min\left(\mu_A(x), 1 - \mu_A(x)\right), \forall x \in U\}$$

Therefore, if $A \neq \emptyset$, then $A \cap \overline{A} \neq \emptyset$, such that $\mu_{A \cap \overline{A}}(x) \in (0, 0.5)$, and thus, there will always be at least one singleton $\frac{\mu_{A \cap \overline{A}}(x)}{x}$.

The law of excluded middle in crisp logic states that a proposition is true or its negation is true, that is, $A \cup \overline{A} = U$. However, in fuzzy logic, this is not true.

Proof. Using De Morgan's law, we have:

$$A \cup \overline{A} = \overline{\overline{A} \cap A} = \overline{A \cap \overline{A}}$$

thus, $\mu_{A \cup \overline{A}} = 1 - \mu_{A \cap \overline{A}}(x)$, and therefore, $\mu_{A \cup \overline{A}} \in (0.5, 1)$. Thus, $A \cup \overline{A} = U$ will not always be true, as long as at least one $\mu_A \in (0, 0.5)$ in a given universe of discourse, the proposition becomes false.

8. Consider the universe with two elements $X = \{a, b\}$ and consider Y with $Y = \{0, 1\}$. Find the power set.

Ans: $\mathcal{P}(X) = \{\emptyset, \{a\}, \{b\}, \{a, b\}\}$ $e\, \mathcal{P}(Y) = \{\emptyset, \{0\}, \{1\}, \{0, 1\}\}$

9. Consider a universe of four elements $x = \{1, 2, 3, 4, 5, 6\}$. Find the cardinal number power set and cardinality.

Ans: $n_x = 6$ e $\eta_{\mathcal{P}(x)} = 2^{n_x} = 2^6 = 64$

10. Consider the following fuzzy sets:

$$A = \left\{ \frac{1.0}{2} + \frac{0.1}{3} + \frac{0.8}{4} + \frac{0.6}{5} \right\},$$

$$B = \left\{ \frac{0.3}{2} + \frac{0.9}{3} + \frac{0.0}{4} + \frac{0.4}{5} \right\}.$$

Calculate, $A \cup B$, $A \cap B$, $\overline{A}$, $\overline{B}$ by Matlab program.

Ans:

$A \cup B = \{ \frac{\mathbf{1.0}}{\mathbf{2}} + \frac{\mathbf{0.9}}{\mathbf{3}} + \frac{\mathbf{0.8}}{\mathbf{4}} + \frac{\mathbf{0.6}}{\mathbf{5}} \}$

$A \cap B = \{ \frac{\mathbf{0.3}}{\mathbf{2}} + \frac{\mathbf{0.1}}{\mathbf{3}} + \frac{\mathbf{0.0}}{\mathbf{4}} + \frac{\mathbf{0.4}}{\mathbf{5}} \}$

$\overline{A} = \{ \frac{\mathbf{0.0}}{\mathbf{2}} + \frac{\mathbf{0.9}}{\mathbf{3}} + \frac{\mathbf{0.2}}{\mathbf{4}} + \frac{\mathbf{0.4}}{\mathbf{5}} \}$

$\overline{B} = \{ \frac{\mathbf{0.7}}{\mathbf{2}} + \frac{\mathbf{0.1}}{\mathbf{3}} + \frac{\mathbf{1.0}}{\mathbf{4}} + \frac{\mathbf{0.6}}{\mathbf{5}} \}$

11. For the above problem perform the De Morgan's law by writing M-file.

Ans:

$\overline{A \cup B} = \{ \frac{\mathbf{0.0}}{\mathbf{2}} + \frac{\mathbf{0.1}}{\mathbf{3}} + \frac{\mathbf{0.2}}{\mathbf{4}} + \frac{\mathbf{0.4}}{\mathbf{5}} \}$

$\overline{A \cap B} = \{ \frac{\mathbf{0.7}}{\mathbf{2}} + \frac{\mathbf{0.9}}{\mathbf{3}} + \frac{\mathbf{1.0}}{\mathbf{4}} + \frac{\mathbf{0.6}}{\mathbf{5}} \}$