EMSR-b and DAVN Heuristics in Airline Revenue Management

1 Introduction

Airline revenue management revolves around the challenge of determining which customer bookings to accept and which to reject, aiming to maximize overall revenue. Airlines offer multiple fare classes for the same origin-destination itinerary, each associated with different conditions and privileges. As tickets are sold over time, airlines must strategically adjust fare availability, closing lower fare classes and opening higher ones to optimize profitability.

Passengers in the economy cabin typically have access to several fare classes, even though all seats in that cabin are physically identical. Lower fare classes tend to sell out earlier due to higher demand, while higher fare classes may remain available longer, often offering additional benefits such as flexible cancellations or priority boarding. While customers who book early often choose lower fares, some may still opt for premium fares despite cheaper options being available. Airlines frequently adjust pricing based on factors such as remaining time before departure, demand patterns, and customer preferences to enhance revenue.

The key decision in revenue management is how many seats to allocate to each fare class to strike a balance between high occupancy and maximizing revenue. Allocating too many seats to lower fares may lead to full flights but lower profits, while reserving too many seats for higher fares risks unsold inventory, reducing potential revenue. Thus, airlines must carefully manage seat allocations to avoid these extremes.

Additionally, airline seats are a perishable resource—once a flight departs, any unsold seat represents lost revenue. To mitigate this, airlines employ overbooking strategies, selling more tickets than available seats to compensate for expected cancellations and no-shows. However, if more passengers show up than seats available, the airline incurs costs by compensating displaced travelers and arranging alternative flights. The complexity of revenue management arises from fluctuating demand, cancellation rates, and overbooking risks, all of which require airlines to make data-driven decisions on pricing and seat allocation.

The Expected Marginal Seat Revenue (EMSR-b) heuristic is widely used to determine seat allocations among different fare classes dynamically. It extends Littlewood's rule by incorporating multiple fare classes and probabilistic demand forecasts. The Displacement Adjusted Virtual Nesting (DAVN) heuristic builds upon EMSR-b to handle multi-leg airline networks, taking into account how bookings on one leg impact availability across the network, thereby optimizing revenue across multiple connected flights.

2 Definitions of Key Concepts

Before delving into the EMSR-b and DAVN heuristics, we define some key technical terms:

- **Fare Class** (i): A category of airline tickets distinguished by price and associated conditions (e.g., refundable vs. non-refundable).
- **Protection Level** (Q_i): The number of seats reserved for fare classes higher than i to ensure that they are available for potential high-revenue passengers.
- **Booking Limit** (B_i): The maximum number of seats that can be sold at a specific fare class i.
- **Demand Distribution**: The statistical representation of expected customer demand for each fare class, typically modeled using a normal distribution.

- **Revenue Management**: The practice of dynamically adjusting seat allocations to maximize revenue while considering demand uncertainty.
- **Displacement Cost**: The opportunity cost of accepting a booking, which might prevent a more profitable booking later.

3 Littlewood's Rule and the EMSR-b Heuristic

Littlewood's rule applies to a single-resource, two-fare class problem and states:

> A seat should be sold at the lower fare f_1 if the expected revenue from selling it now is at least as great as the expected revenue from reserving it for a higher-fare customer who might arrive later.

Mathematically, we accept a booking at the lower fare f_1 if:

$$f_1 \geq f_2 \cdot \Pr[D_2 > Q_2] \tag{1}$$

where:

- f_1 = lower fare price
- f_2 = higher fare price
- D_2 = random demand for the higher fare
- Q_2 = protection level for the higher fare
- $\Pr[D_2 > Q_2]$ = probability that demand for the higher fare exceeds the protection level

Extending this to multiple fare classes, the EMSR-b heuristic computes protection levels as:

$$Q_i = \mu_{\geq i} + z_i \sigma_{\geq i}, \tag{2}$$

where $z_i = \Phi^{-1}\left(\frac{f_i}{f_n}\right)$ is determined by the ratio of fares.

4 The DAVN Heuristic: Extending EMSR-b to Networks

While EMSR-b optimizes seat allocations for a single flight leg, the DAVN heuristic extends this concept to multi-leg itineraries.

4.1 Linear Program Formulation

The DAVN heuristic solves the following linear program:

$$\max \sum_{j=1}^{n} f_j x_j \tag{3}$$

subject to:

$$0 \leq x_j \leq E[D_j], \quad \forall j = 1, 2, ..., n \tag{4}$$

$$\sum_{j \in A_l} x_j \leq C_l, \quad \forall l = 1, 2, ..., L \tag{5}$$

where f_j is the fare price of product j, $E[D_j]$ is the expected demand for product j, C_l is the capacity of leg l, and A_l is the set of products using leg l.

4.2 Computing Displacement Adjusted Revenue (DARE)

The displacement adjusted revenue (DARE) for product j on leg l is computed as:

$$DARE_j^l = f_j - \sum_{i \neq l} \lambda_i, \tag{6}$$

where λ_i is the dual price of constraint (3) for leg i. These DARE values are then used to apply EMSR-b on each leg separately.

5 Conclusion

The EMSR-b heuristic is a foundational approach for single-leg revenue management, while the DAVN heuristic extends it to network-wide optimization by considering displacement costs and virtual nesting. Together, these methods enable airlines to maximize revenue across complex route structures.