Exercise 9

First a comment about the exercise text, we have a mesh with 4 elements and 5 *nodes*, right? We look at the first system where the nodes are given as

$$\mathcal{N} = \{0, 1, 1.2, 1.6, 2\},$$

with elements

$$\begin{aligned}
\Omega_0 &= [0, 1] \\
\Omega_1 &= [1, 1.2] \\
\Omega_2 &= [1.2, 1.6] \\
\Omega_3 &= [1.6, 2].
\end{aligned}$$

This gives a 5×5 sparsity matrix

$$M = \begin{pmatrix} 1 & 1 & 0 & 0 & 0 \\ 1 & 1 & 1 & 0 & 0 \\ 0 & 1 & 1 & 1 & 0 \\ 0 & 0 & 1 & 1 & 1 \\ 0 & 0 & 0 & 1 & 1 \end{pmatrix}$$

which is tridiagonal because each node is in the same element as both of its neighbors. For the next system where the nodes are

$$\mathcal{N}' = \{2, 1.6, 1.2, 1, 0\},$$

and elements

$$\begin{aligned}
\Omega'_0 &= [1.6, 2] \\
\Omega'_1 &= [1.2, 1.6] \\
\Omega'_2 &= [1, 1.2] \\
\Omega'_3 &= [0, 1],
\end{aligned}$$

we get the matrix

$$M' = \begin{pmatrix} 1 & 1 & 0 & 0 & 0 \\ 1 & 1 & 1 & 0 & 0 \\ 0 & 1 & 1 & 1 & 0 \\ 0 & 0 & 1 & 1 & 1 \\ 0 & 0 & 0 & 1 & 1 \end{pmatrix}.$$

This gives the same matrix because the neighbor list is symmetric with a 1 dimensional reverse ordering.