Exercise 3

We have given the vector $f = (1,1,1)$ and a plane spanned by $\phi_1 = (1,0)$ and $\phi_2 = (0,1)$. The best approximation u is then given as

$$\begin{aligned} u &= \frac{(f,\phi_1)}{(\phi_1,\phi_1)}\phi_1 + \frac{(f,\phi_2)}{(\phi_2,\phi_2)}\phi_2 \\ &= \phi_1 + \phi_2 = (1,1) \end{aligned}$$

With the plane spanned by $\psi_1 = (2,1)$ and $\psi_2 = (1,2)$ we first need to create an orthogonal basis

$$\begin{aligned} k\tilde{\psi_2} &= \psi_2 - \frac{(\psi_2, \psi_1}{(\psi_1,\psi_1)}\psi_1 \\ &= \psi_2 - \frac{4}{5}\psi_1 = \frac{1}{5}(-3,6) \\ \tilde{\psi_2} &= (-3,6) \end{aligned}$$

where we dropped the factor $1/5$ for simplicity. The same plane is now spanned by the orthogonal basis $\{\psi_1, \tilde{\psi_2}\}$, and we can find the projection

$$\begin{aligned} v &= \frac{(f,\psi_1)}{(\psi_1,\psi_1)}\psi_1 + \frac{(f,\tilde{\psi_2})}{(\tilde{\psi_2},\tilde{\psi_2})}\tilde{\psi_2} \\ &= \frac{3}{5}\psi_1 + \frac{3}{45}\tilde{\psi_2} = (1,1) \end{aligned}$$

i.e. the same as we got in the first part. This is expected because it is the same plane or vector space.