Medical Engineering - Imaging Systems

Image Processing

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Image Processing

The Basics of Image Processing

Further Readings

Images as 2-D Signals

Image f: a 2-D function $f(x, y)$ defined over the discrete *image domain* $\Omega \subset \mathbb{Z}^2$

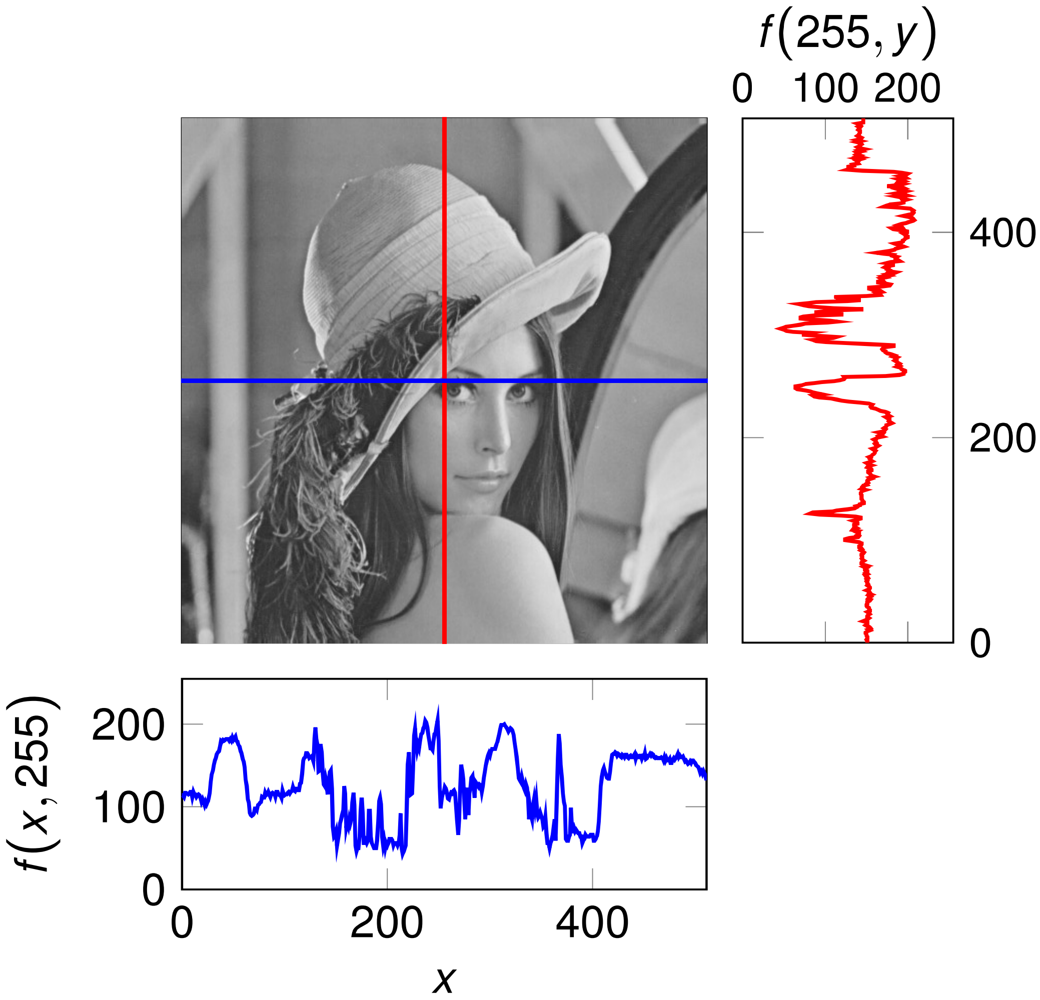
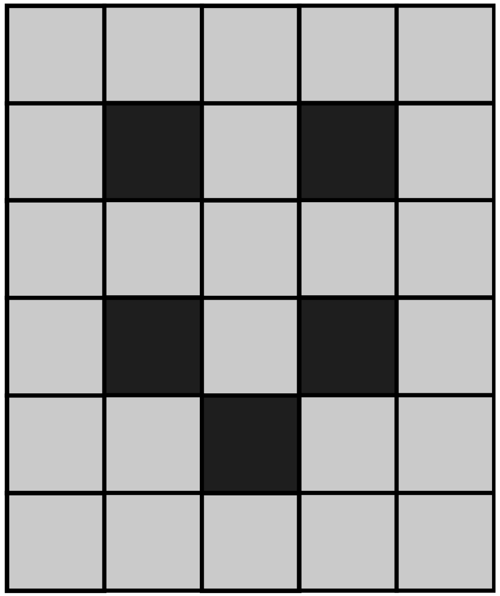


Image Storage

Images are stored as matrices on a computer.

200	200	200	200	200
200	35	200	35	200
200	200	200	200	200
200	35	200	35	200
200	200	35	200	200
200	200	200	200	200



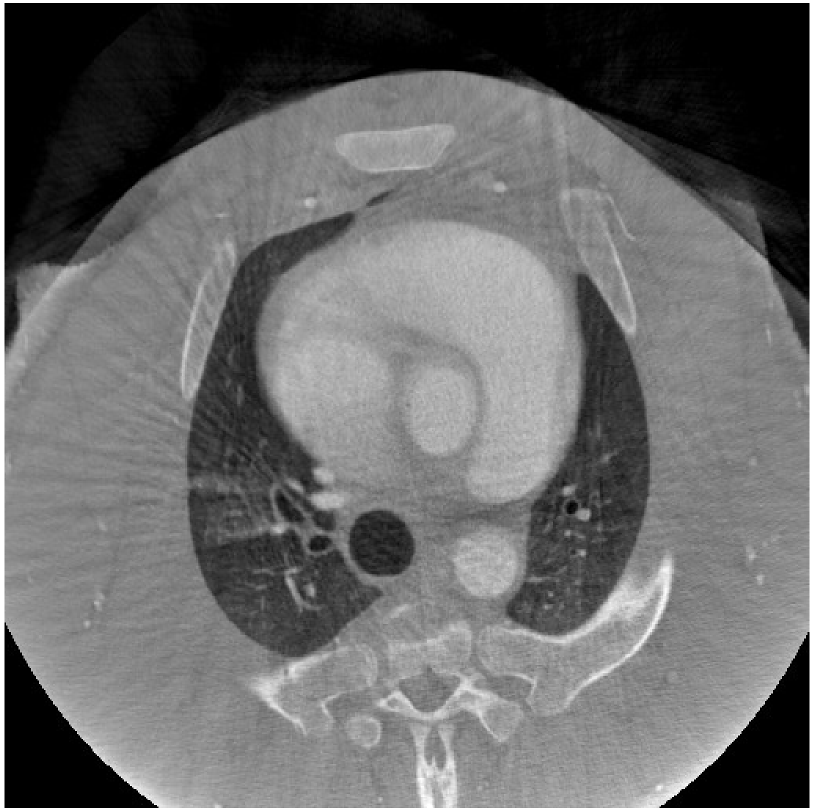
Gamma Correction – Example

Gamma corrected $(\gamma = 2.2)$

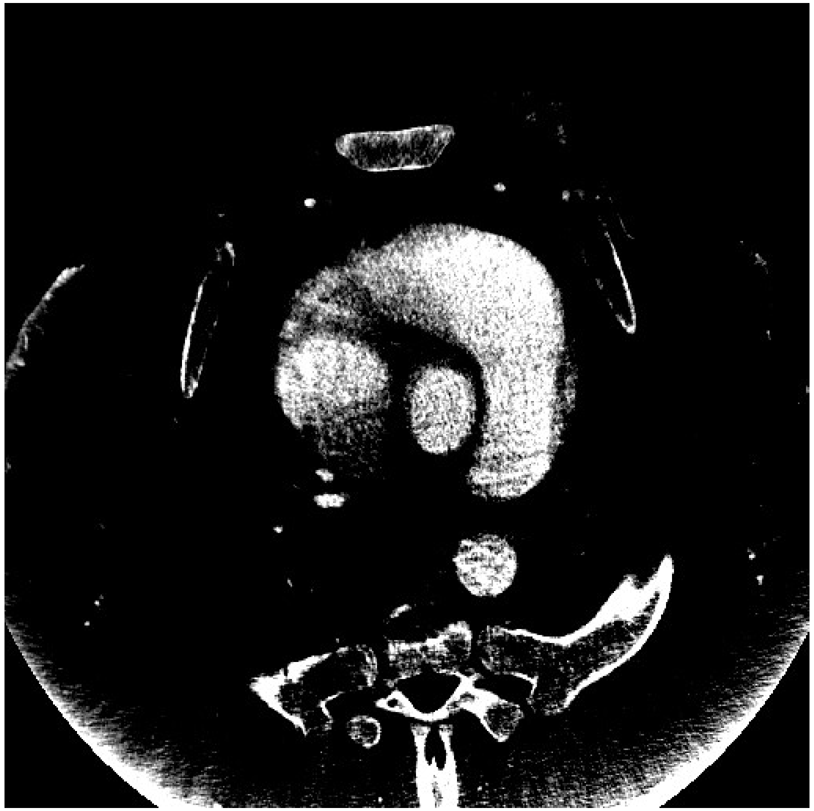
Original

Gamma corrected $(\gamma = \frac{1}{2.2})$

Window and Level – Example



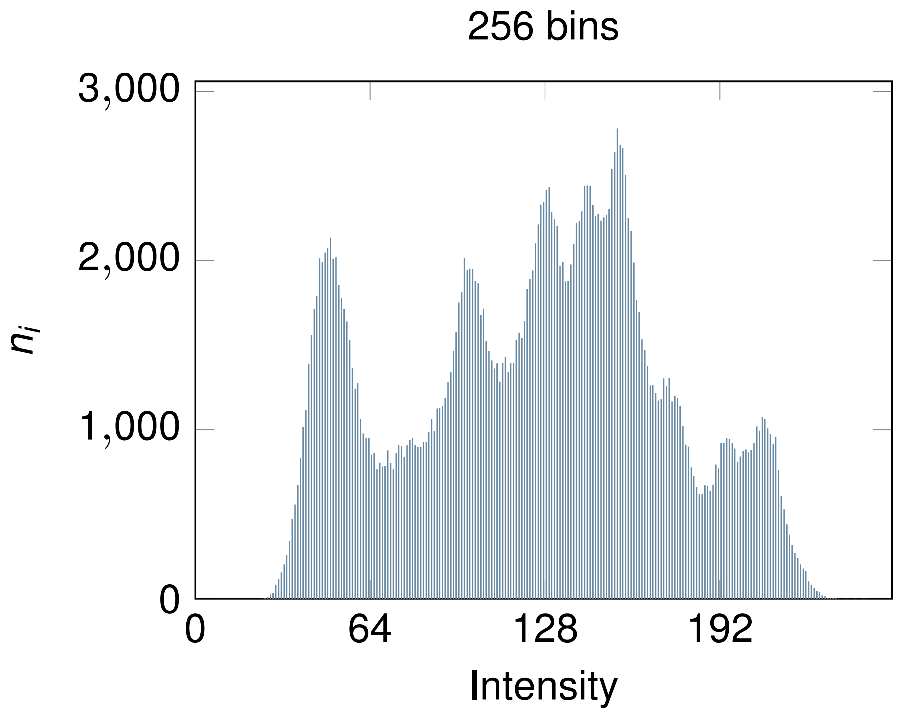
-1000 HU to 1000 HU

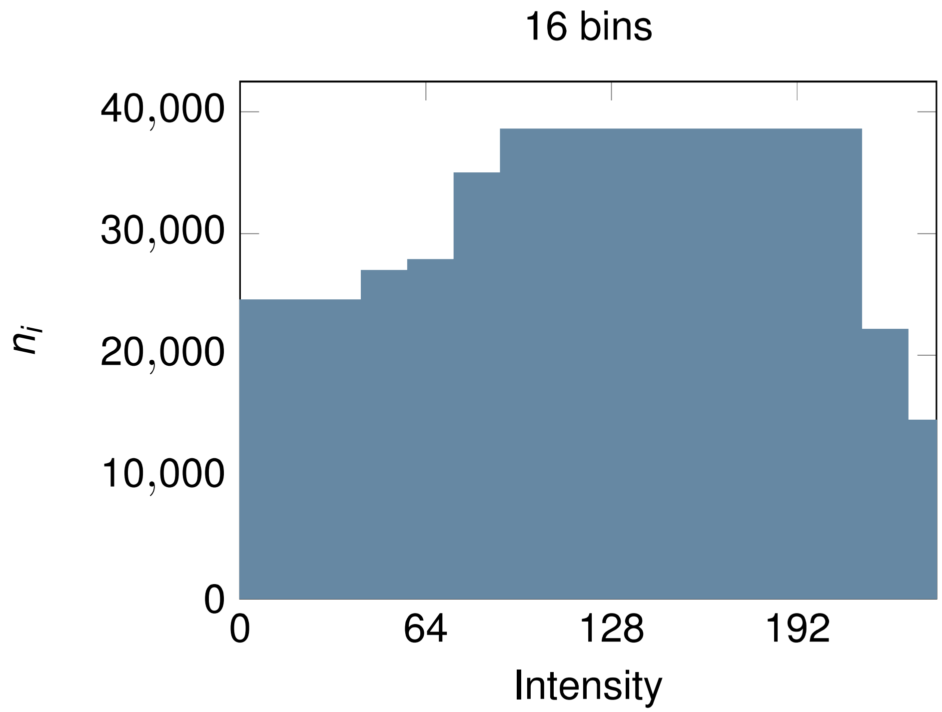


200 HU to 500 HU

Histograms of Images

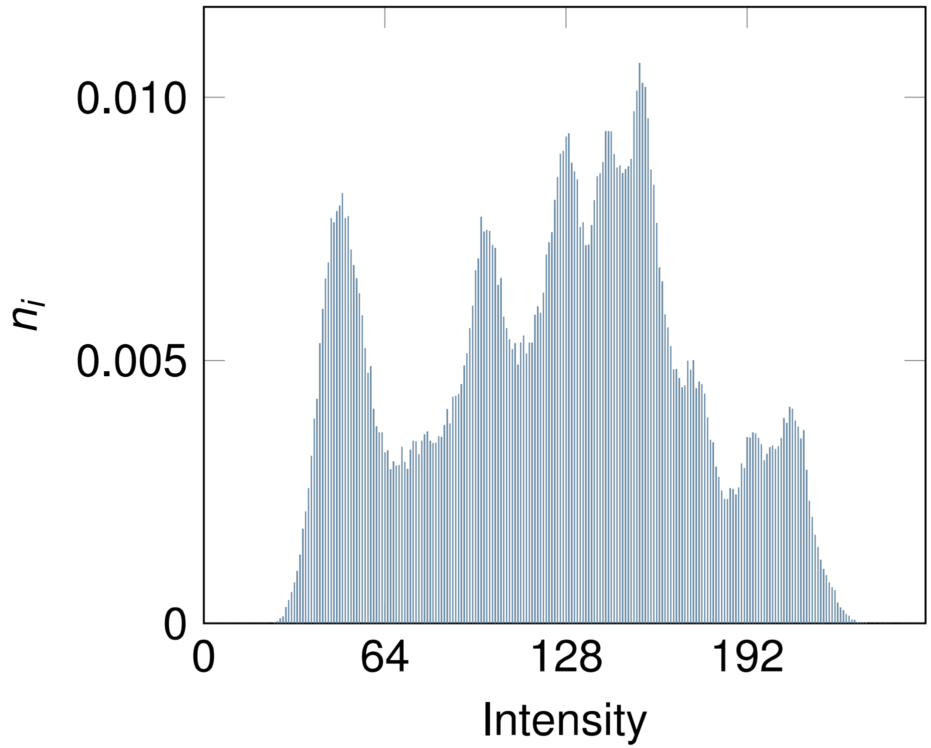
Histograms show the distribution of intensity values, grouped into *bins*.





Histogram Equalization

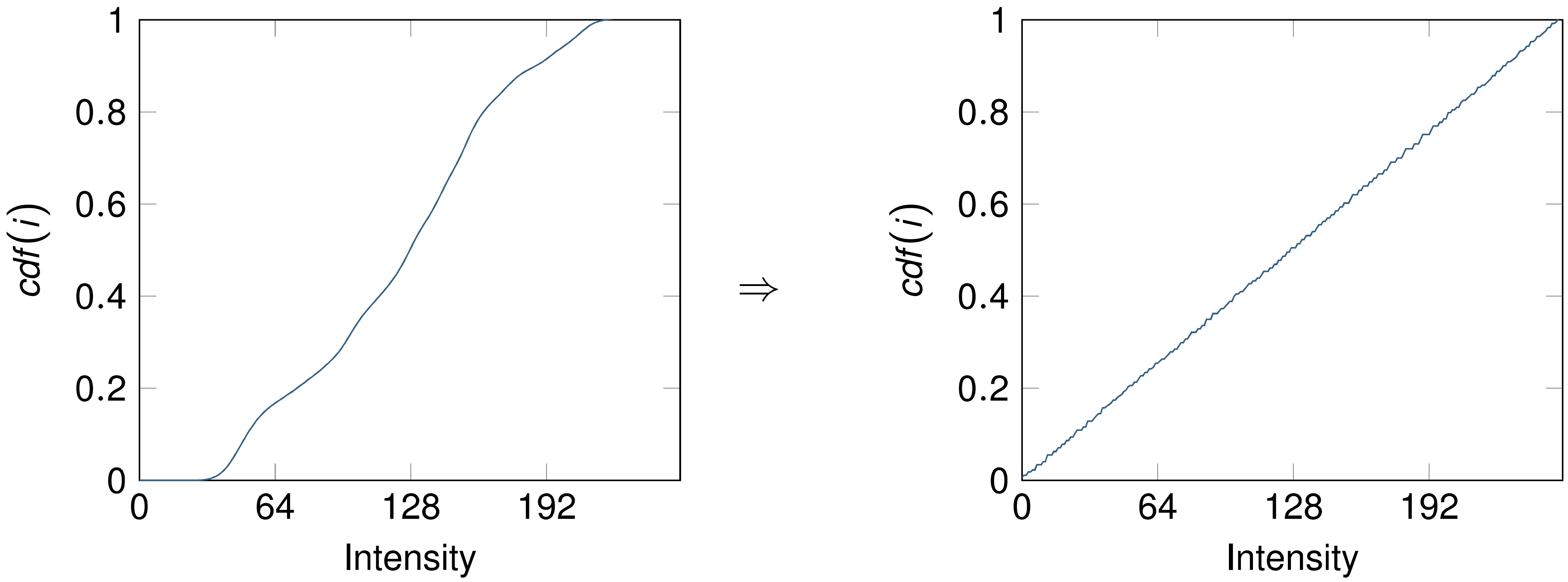
Intensities occupy only a small range in the histogram



Histogram Equalization

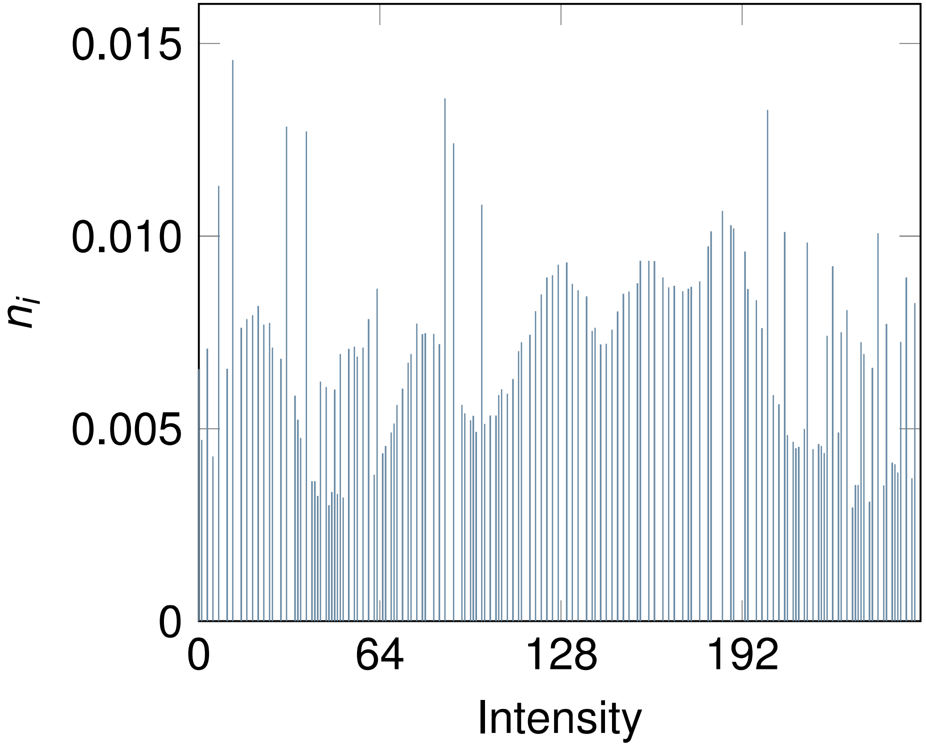
Goal of Histogram Equalization

Achieve a uniform distribution of intensities $\Rightarrow$ **linear CDF**



Histogram Equalization

The histogram may contain gaps after histogram equalization!



Histogram Equalization

Before

After

Derivatives of Images

Continuous derivative

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

Discrete derivatives

Several choices:

forward difference	$\Delta_x f(x) = f(x+1) - f(x)$
central difference	$\delta_x f(x) = f(x+1) - f(x-1)$
backwards difference	$\nabla_x f(x) = f(x) - f(x-1)$

Derivatives of Images – Example

Derivative in x-direction

Derivative in y-direction

Image Filtering

Filters as operators

A filter $\mathscr{H}$ can be applied on an image f:

$$\mathscr{H}(f(x,y)) = r(x,y)$$

It can have the same properties as in 1-D:

linearity

$$\mathscr{H}\{\alpha \cdot f(x,y)\} = \alpha \cdot \mathscr{H}\{f(x,y)\}$$

$$\mathscr{H}\{f_1(x,y) + f_2(x,y)\} = \mathscr{H}\{f_1(x,y)\} + \mathscr{H}\{f_2(x,y)\}$$

shift-invariance

$$\mathscr{H}\{(f(x-x_0, y-y_0)\} = r(x-x_0, y-y_0)$$

Linear, Shift-invariant Filters

Filter kernels

Linear, shift-invariant filters are characterized by **filter kernels** k and are applied to an image by a convolution.

Convolution with a filter

Image f, filter kernel k

$$\mathscr{H}\{f\}(x,y) = f * k = \sum_{i=-\frac{w_k}{2}}^{\frac{w_k}{2}} \sum_{j=-\frac{h_k}{2}}^{\frac{h_k}{2}} f(x-i, y-j) \cdot k(i,j)$$

Efficient Filtering

Convolution in the Fourier domain

For large images and / or filters:

1. Transform f and k to the Fourier domain

$$F = \mathscr{F}(f) \qquad\qquad K = \mathscr{F}(k)$$

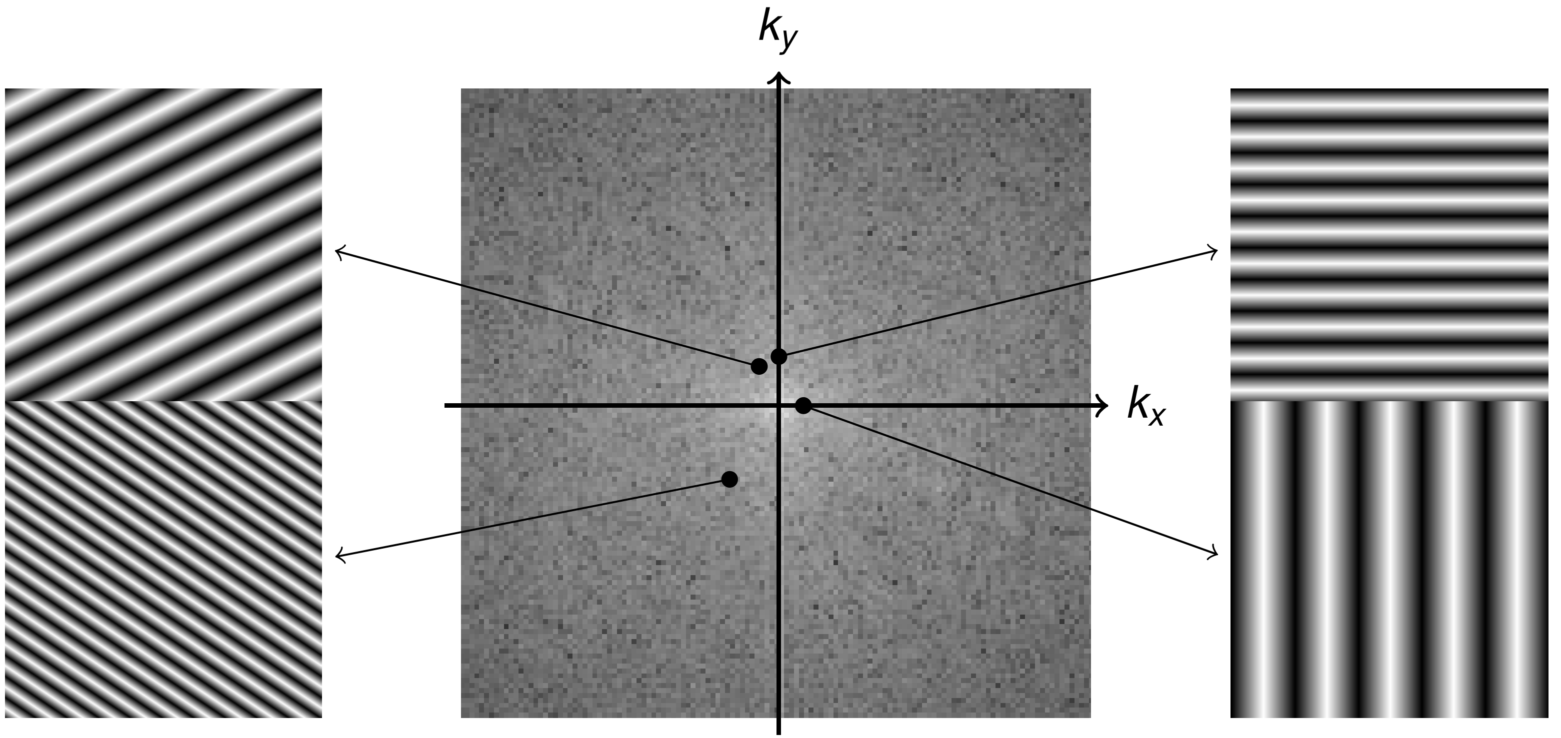
2. Like in 1-D, convolution then is a simple multiplication

$$\mathscr{F}\{f * k\} = F \cdot K$$

3. Get the filtered image by transforming the result back to the spatial domain

2-D Fourier Space

- 2-D Fourier Space with phase patterns associated with some positions:



Filter Kernel – Example: Central Difference

Derivatives are usually calculated by applying a filter.

The kernel can be constructed from the corresponding equation:

$$\delta_x f(x,y) = -1 \cdot f(x-1,y) + 0 \cdot f(x,y) + 1 \cdot f(x+1,y)$$

-1	0	1

Filter Kernels – Derivative Filters

First order derivatives

$$\delta_x \longrightarrow \begin{array}{|c|c|c|}\hline -1 & 0 & 1 \\ \hline\end{array} \qquad \delta_y \longrightarrow \begin{array}{|c|}\hline -1 \\ \hline 0 \\ \hline 1 \\ \hline\end{array}$$

Second order derivative

$$\delta_x^2 \longrightarrow \begin{array}{|c|c|c|}\hline 1 & -2 & 1 \\ \hline\end{array} \qquad \delta_y^2 \longrightarrow \begin{array}{|c|}\hline 1 \\ \hline -2 \\ \hline 1 \\ \hline\end{array}$$

Smoothing Filters

Used to reduce noise in images.

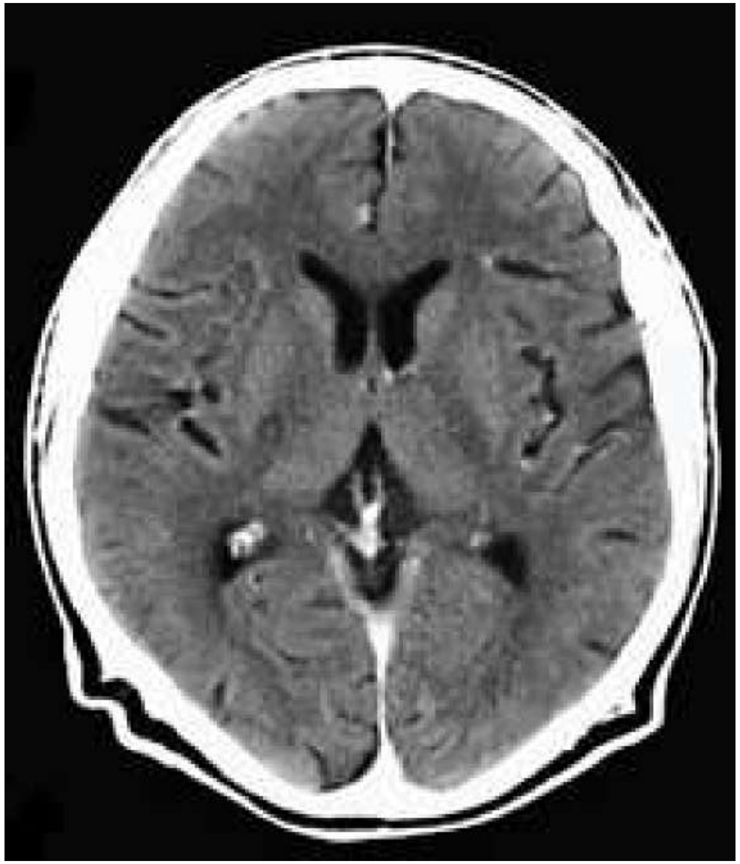
Mean filter

$$\begin{array}{|c|c|c|}\hline \frac{1}{9} & \frac{1}{9} & \frac{1}{9} \\ \hline \frac{1}{9} & \frac{1}{9} & \frac{1}{9} \\ \hline \frac{1}{9} & \frac{1}{9} & \frac{1}{9} \\ \hline \end{array}$$

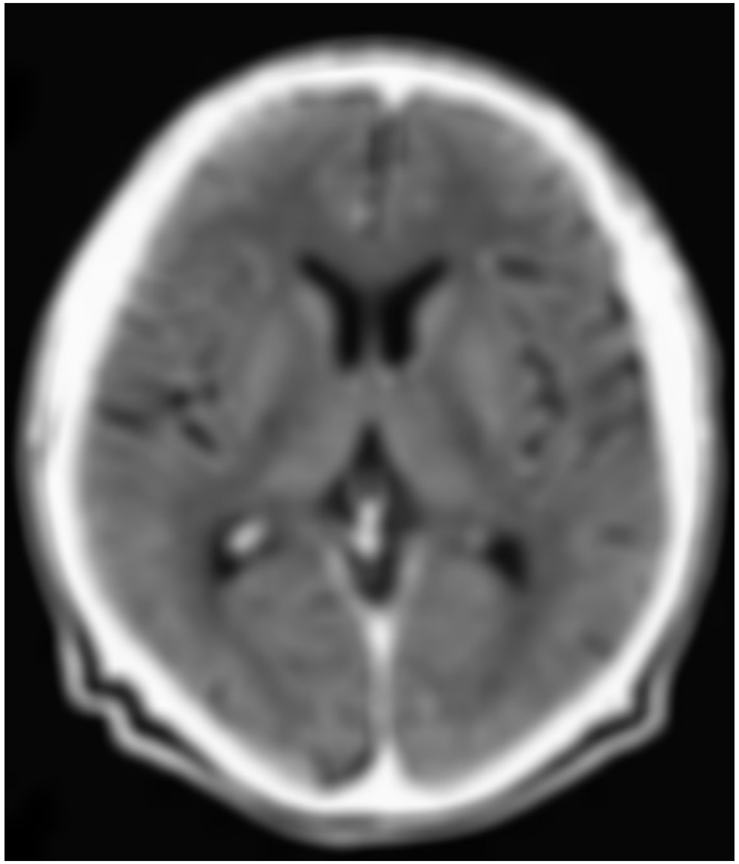
Gaussian filter

$$\frac{1}{52} \cdot \begin{array}{|c|c|c|c|c|}\hline 1 & 1 & 2 & 1 & 1 \\ \hline 1 & 2 & 4 & 2 & 1 \\ \hline 2 & 4 & 8 & 4 & 2 \\ \hline 1 & 2 & 4 & 2 & 1 \\ \hline 1 & 1 & 2 & 1 & 1 \\ \hline \end{array}$$

Gaussian Filtering



Original



Blurred