# Planning with Temporal Logic

April 25, 2016

# Motivation

- Consider a self-driving car…
- Regardless of our destination, we also want to make sure we always follow the rules of the road.

# Motivation

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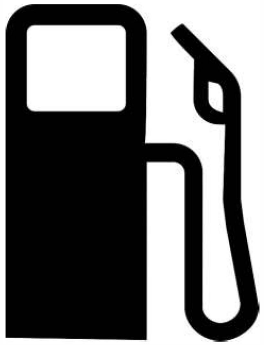
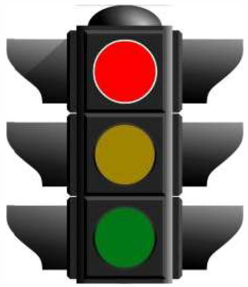


# Key Takeaways

- Modeling temporally-extended goals with linear temporal logic (LTL)
- Modeling preferences between alternative plans

# Outline

- **Introduction to Linear Temporal Logic**
  - Why use Linear Temporal Logic?
  - Linear Temporal Logic Operators
  - Example LTL Problems
- **Applications to Planning**
- **Planning with Preferences**
  - Expressing Preferences
  - Planning in LPP

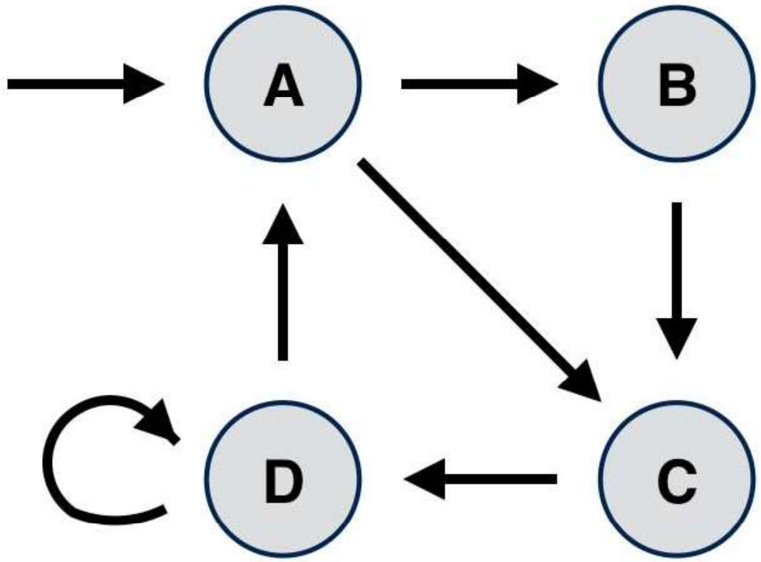
# Linear Temporal Logic

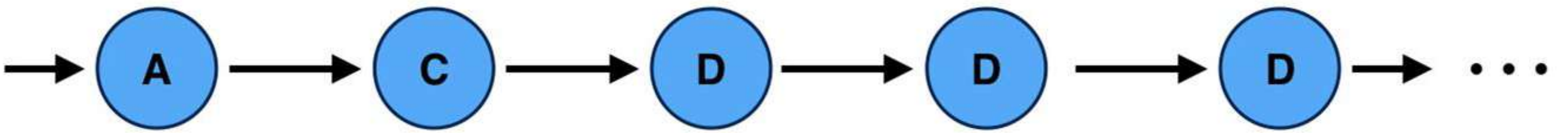
# Temporal Logic

- Formalism for specifying properties of systems that vary with time

# Temporal Logic

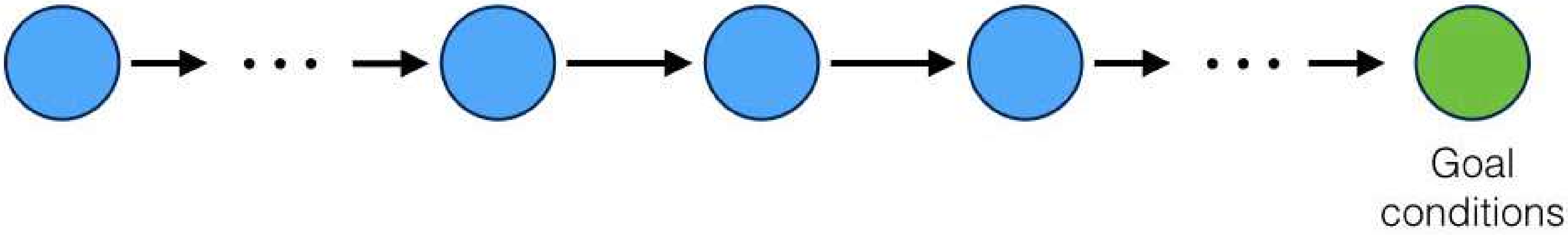
- Systems proceed through a sequence of discrete states



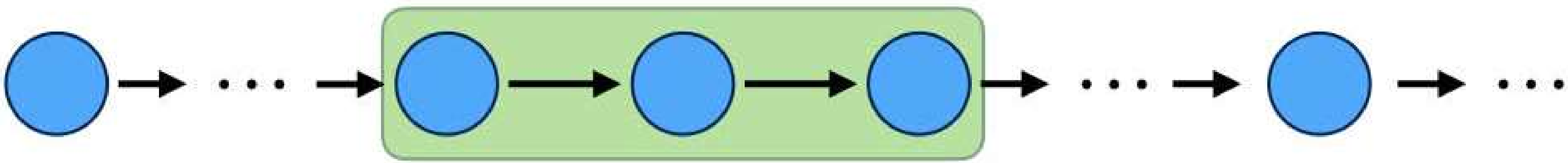



# Why Temporal Logic?

- Previously our planning algorithms have used propositional logic to specify goals dealing with a **single state** at a single point in time



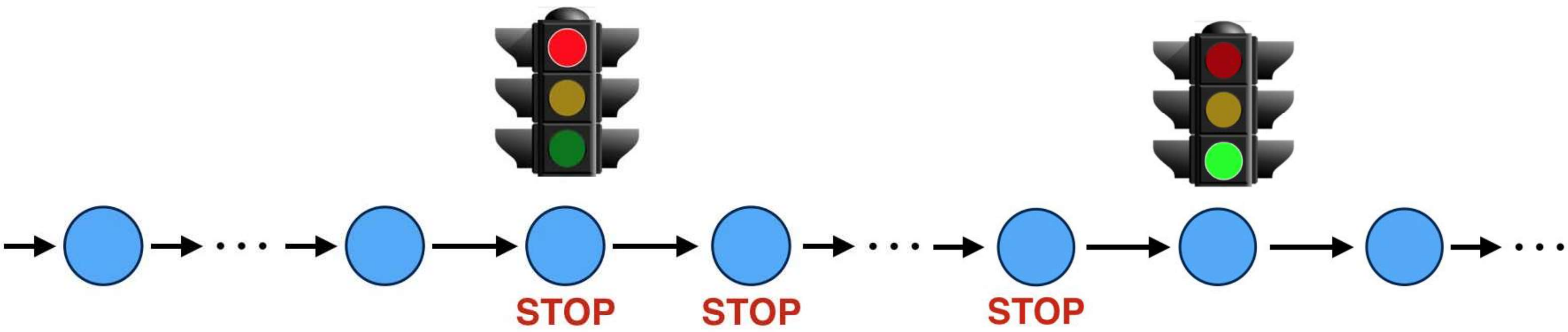

- Temporal logic allows these goals to be specified over a sequence of states



Goal conditions over a set of states

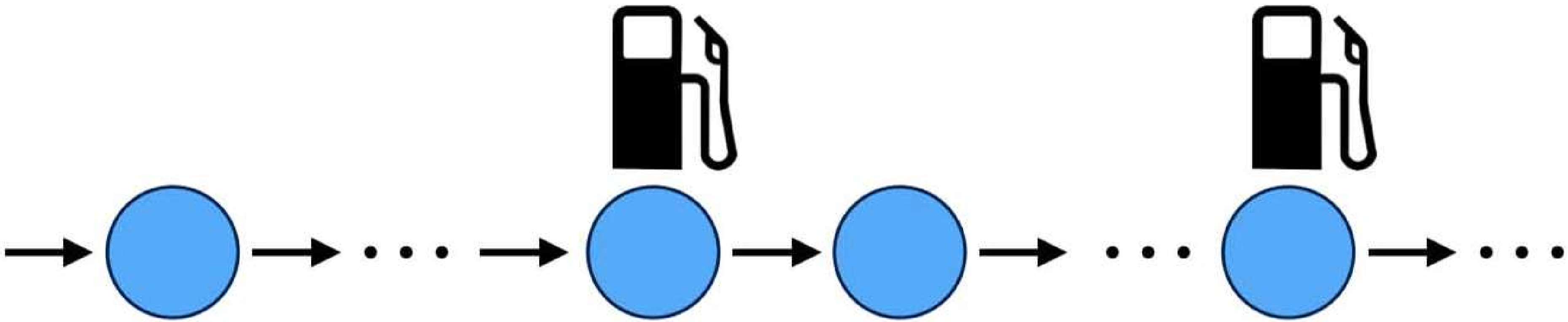
# Why Temporal Logic?

- What if the problem requires a condition to:
  - –Be met until another condition is met...
    - •For example: **red** implies (**stop** until **green**)



# Why Temporal Logic?

- What if the problem requires a condition to:
  - –Always eventually be met
    - •For example, always have some point in the future when you visit a gas station



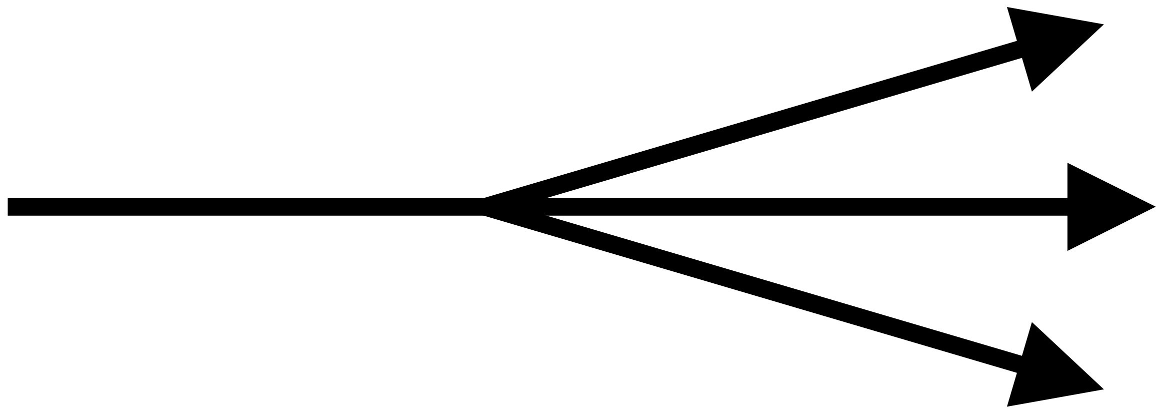
## Branching vs linear time

- Linear time
  - Models physical time
  - At each time instant, only one of the future behaviors is considered
  - We can reason about **always**

# Temporal Logic

## Branching vs linear time

- Branching time
  - At each time instant, all possible future behaviors are considered
  - Time may split into alternate courses
  - We can reason about **possibilities**

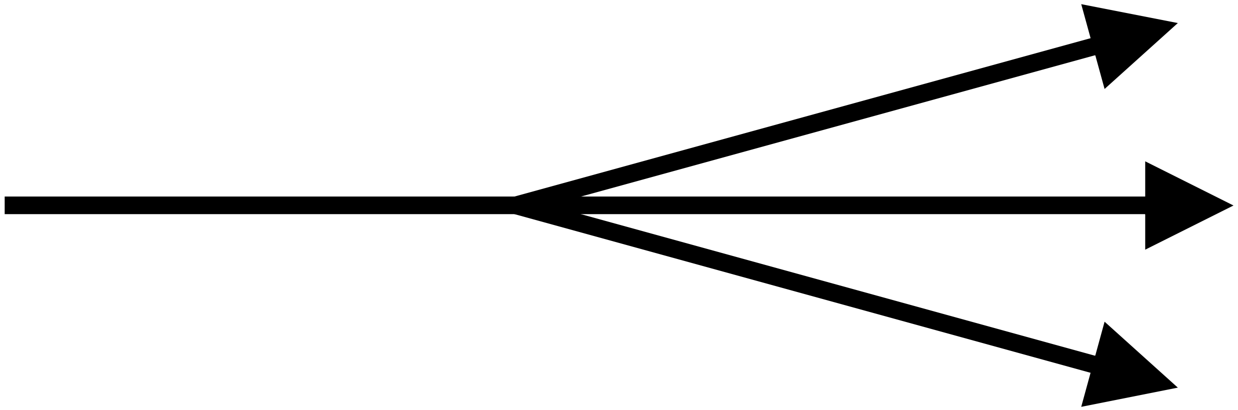


# Temporal Logic

## Branching vs linear time

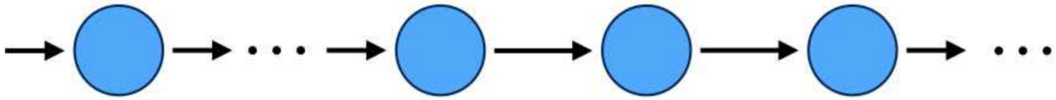
- Linear time

- Branching time



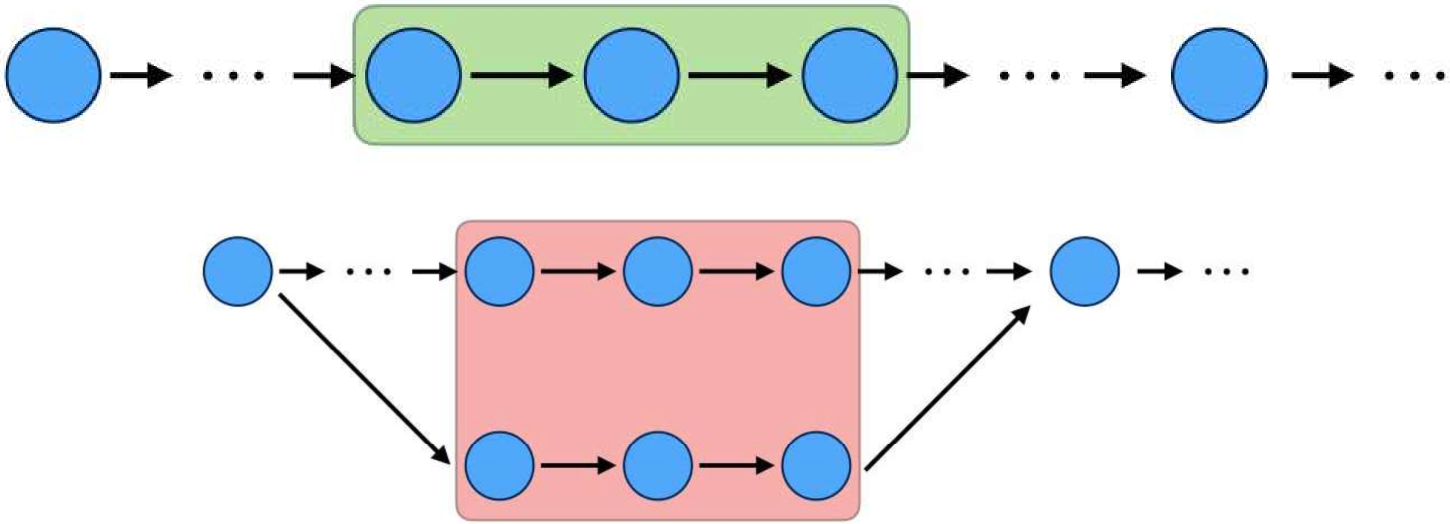
# Linear Temporal Logic

- Linear Temporal Logic (LTL) involves:
  - Linear time model
  - Infinite sequences of states



  - Forward-looking conditions

- Cannot express properties over a set of different paths



# Applications of Temporal Logic

- Temporal logic is used in:
  - Verification and Model Checking
    - Safety and Maintenance
  - **Planning**

# LTL Syntax

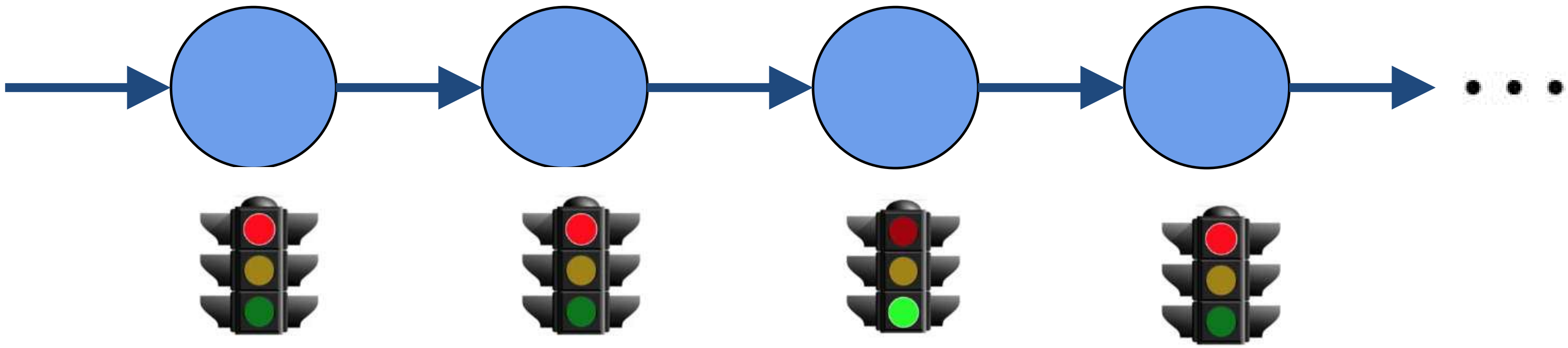
**LTL formula f := true | $p_i$ | $f_i \wedge f_j$ | $\neg f_i$ | X $f_i$ | $f_i$ U $f_j$**

*An LTL formula is built from:*

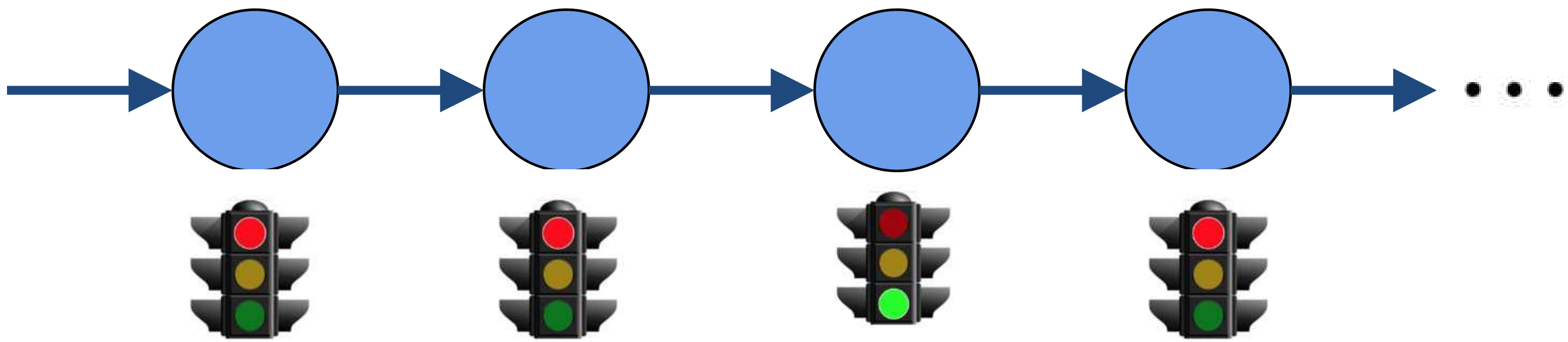
1. Propositional variables: p, ρ, ϕ, ω etc.
   Can be True or False
2. Logical Operators: ¬, ∨, ∧, →, ↔, True, False
   - ¬ = not
   - ∨ = or
   - ∧ = and
   - → = implies
   - ↔ = if and only if
   - True, False

# Logical Operator Examples

| Logical Operators | Example |
|---|---|
| true | **true** |

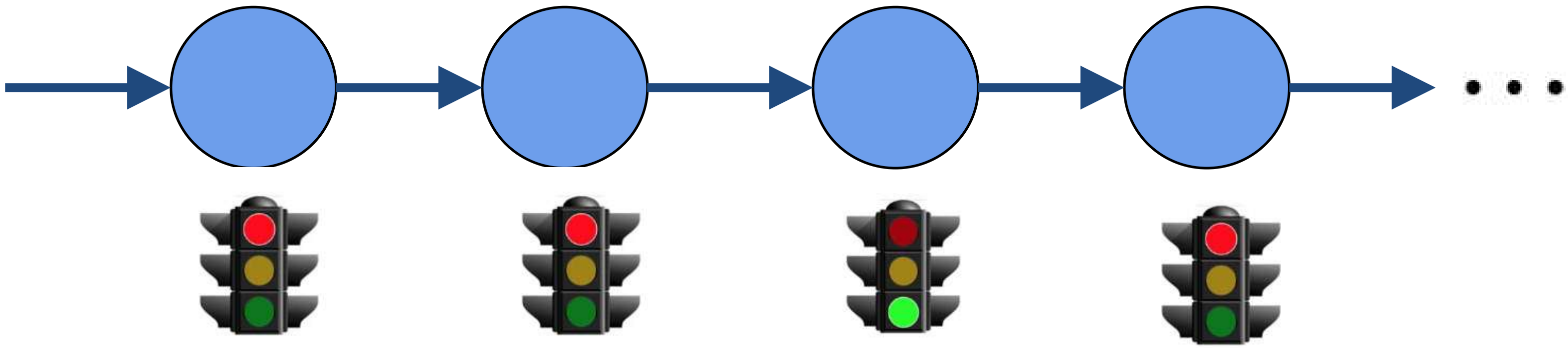


| Logical Operators | Example |
|---|---|
| p = true | **R =** red light |

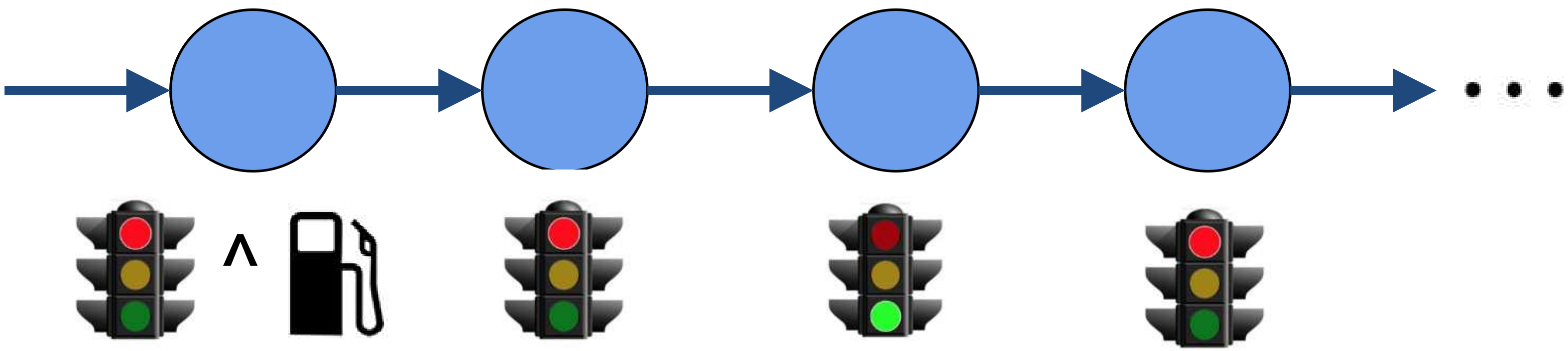


# Logical Operator Examples

| Logical Operators | Example |
| --- | --- |
| not, ¬ | ¬ **G** = green light |

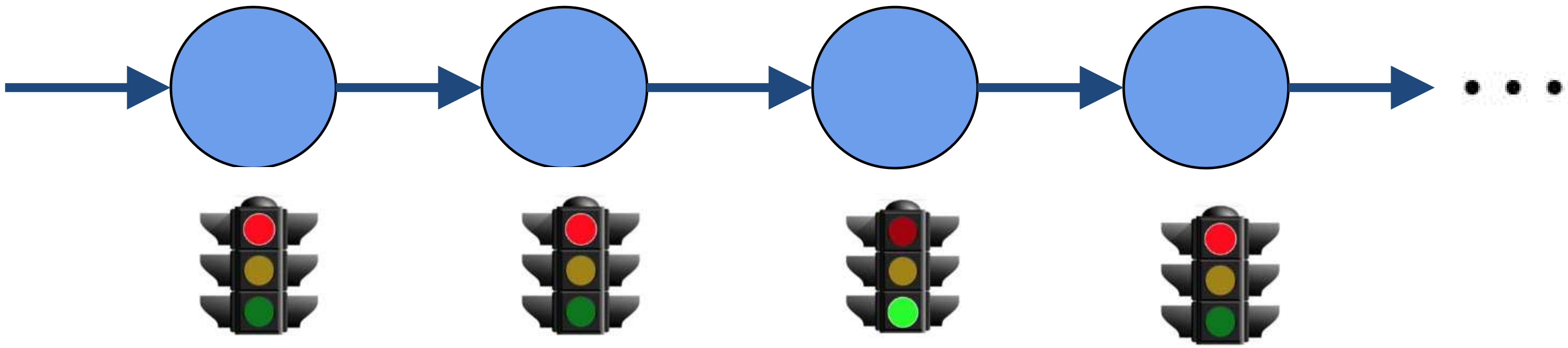


| Logical Operators | Example |
| --- | --- |
| and, ∧ | **R** ∧ **B** = gas station |



# Logical Operator Examples

| Logical Operators | Example |
|---|---|
| or, ∨ | **R** ∨ **G** |



Or (∨) can be rewritten with and (∧) and not (¬)

R ∨ G = ¬(¬R ∧ ¬G)

Similar process can be done for implies and iff, but we won't be explaining them due to time constraints

# LTL Syntax

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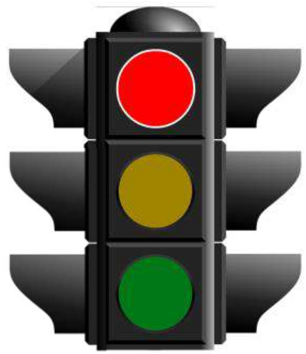
   - True, False
3. Temporal Operators

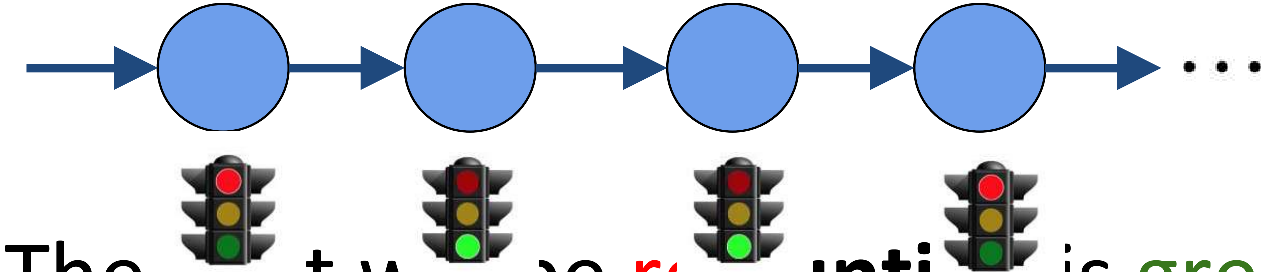
# Temporal Operators

What are some useful operators we may want to describe our car?

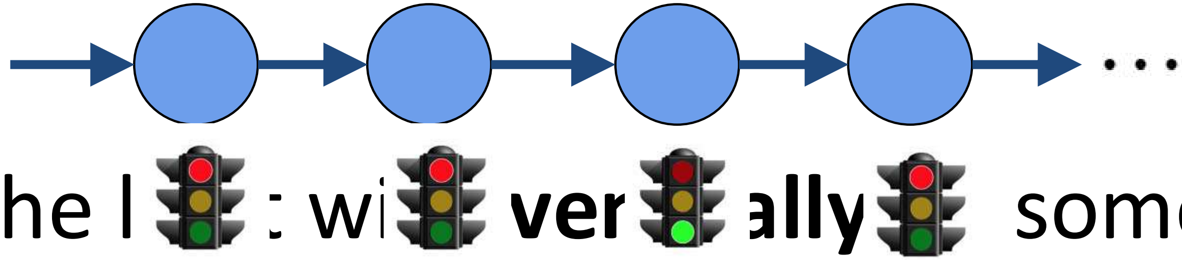


# Temporal Operators

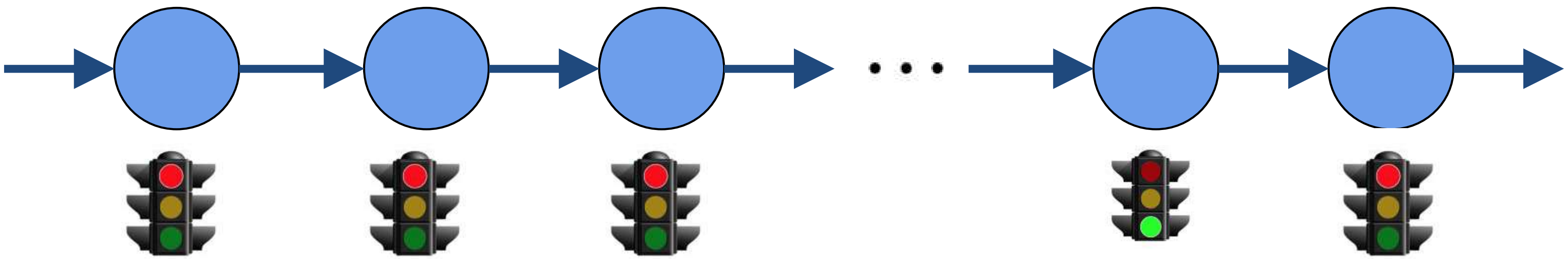
- The **next** light to be green



- The light will be red **until** it is green

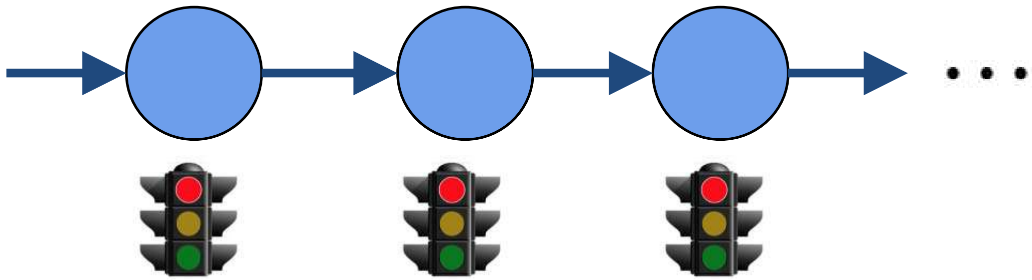


- The light will **eventually** at some point in the **future**, turn green

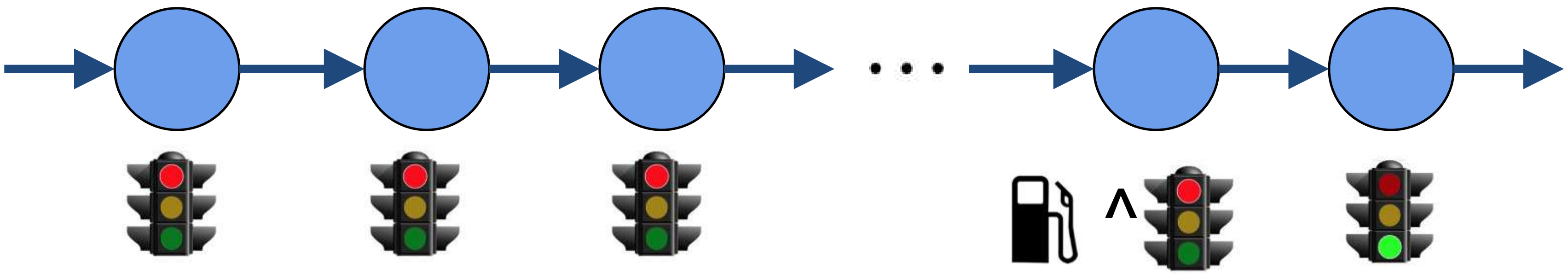


# Temporal Operators

- The light will **always** be red



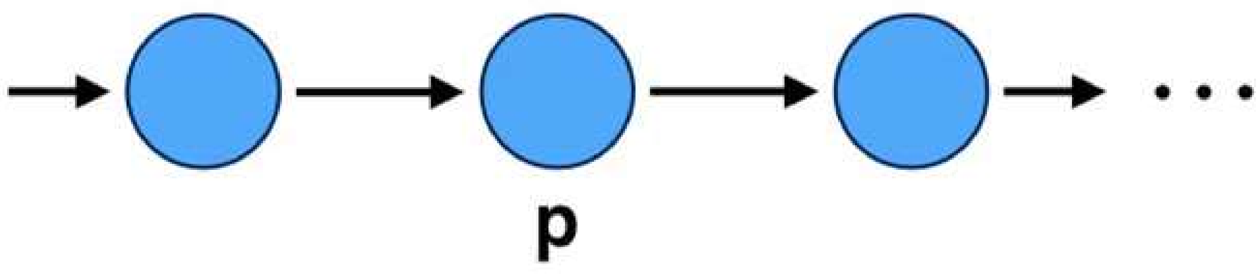
- The light will be red until the car gets gas and the state after it's **released**, the light can be whatever



# Next

| Operator | Textual Operator |
|---|---|
| ne**X**t | **X**ρ |

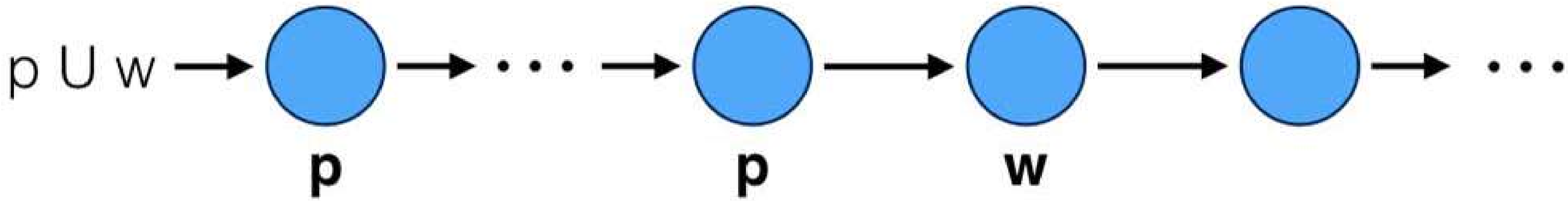
*Definition*: Variable ρ must be true in the next state



# Until

| Operator | Textual Operator |
| --- | --- |
| **U**ntil | ρ**U**ω |

*Definition*: Variable ρ must remain true up until the state where variable ω becomes true, at which point ρ becomes unconstrained

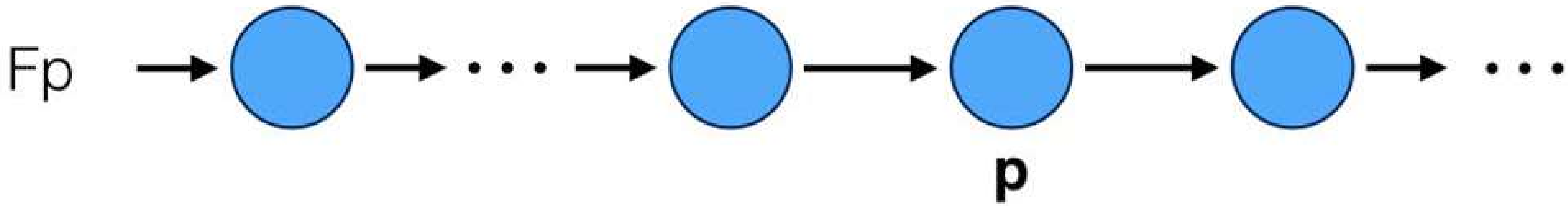



Note that ω is required to become true in some future state

# Future

| Operator | Textual Operator |
|---|---|
| **F**uture/Eventually | **F**ρ |

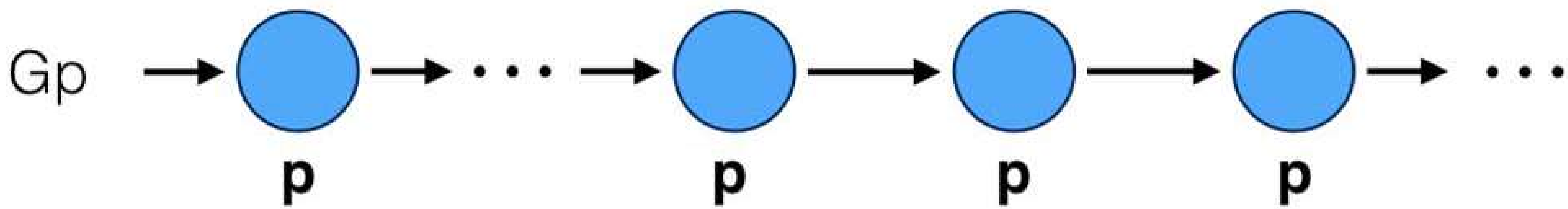
*Definition*: Variable ρ must become true in some future state



# Global

| Operator | Textual Operator |
| --- | --- |
| **G**lobally | **G**ρ |

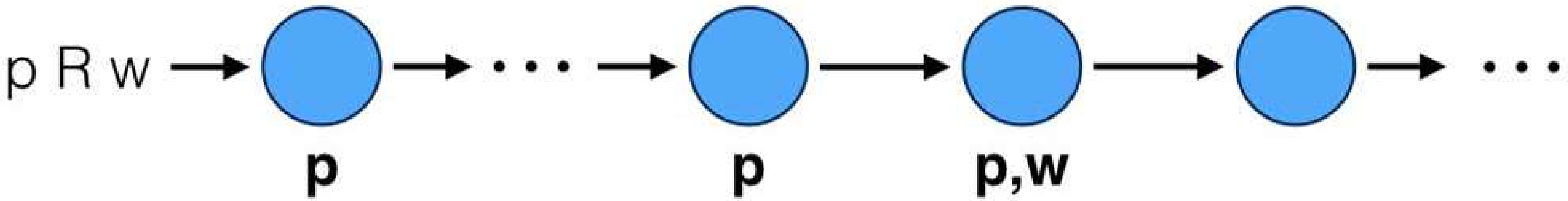
*Definition*: Variable ρ must be true in all future states



# Release

| Operator | Textual Operator |
|---|---|
| **R**elease | $\rho \mathbf{R} \omega$ |

*Definition*: Variable ρ must be true up until and including the state where ω becomes true, after which ω is unconstrained. If ρ is not true in any future state, then ω is true in all future states



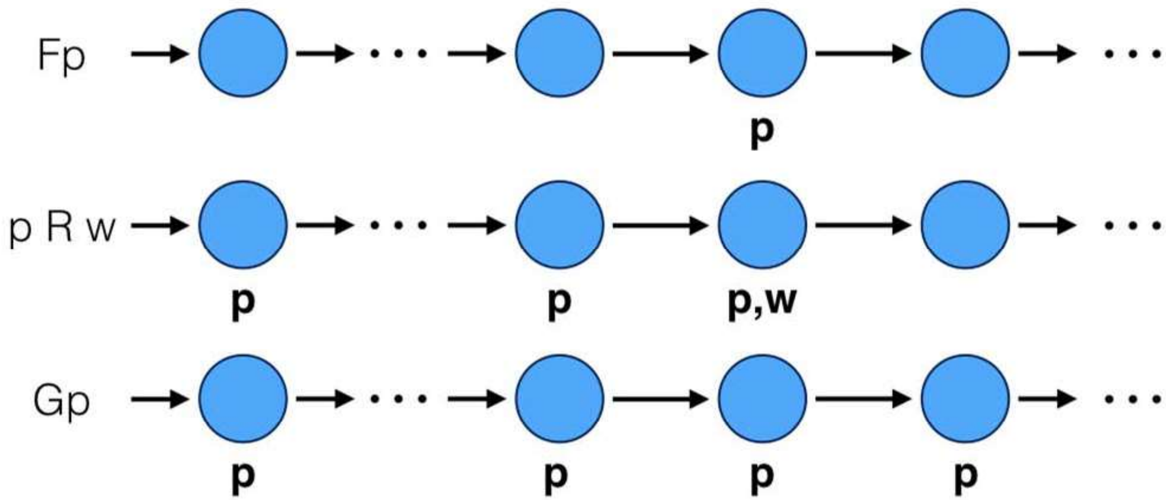

Different from **U** in that both ρ and ω are true in one state

# Which describe the other?

**F**uture/Eventually

**R**elease

**G**lobally




?

?

? $\equiv$ True U ρ

$\equiv$ ¬F¬ρ

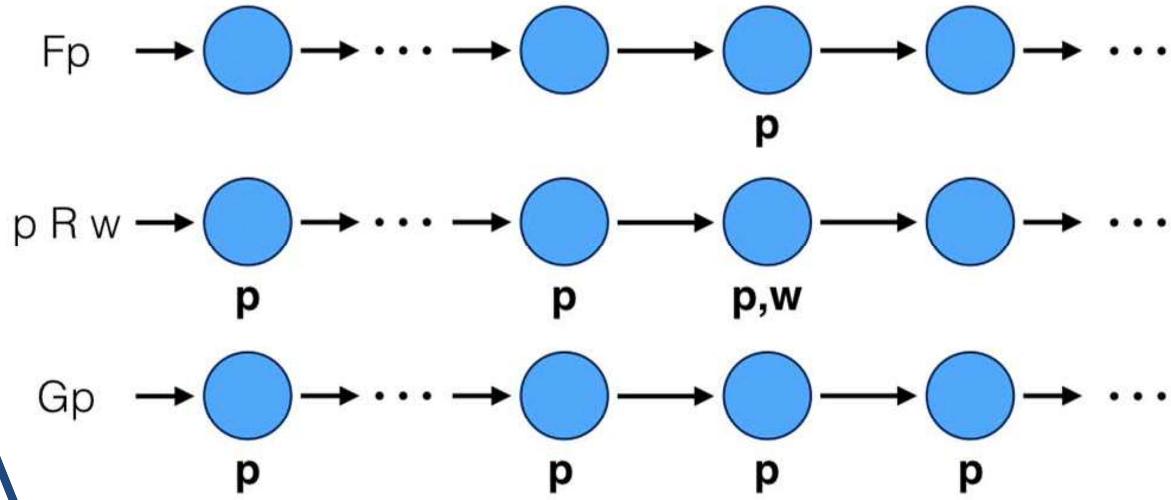
$\equiv$ ¬(¬ρ U ¬ω)

# Which describe the other?

**F**uture/Eventually

**R**elease

**G**lobally




$\equiv \text{True U } \rho$

$\equiv \neg F \neg \rho$

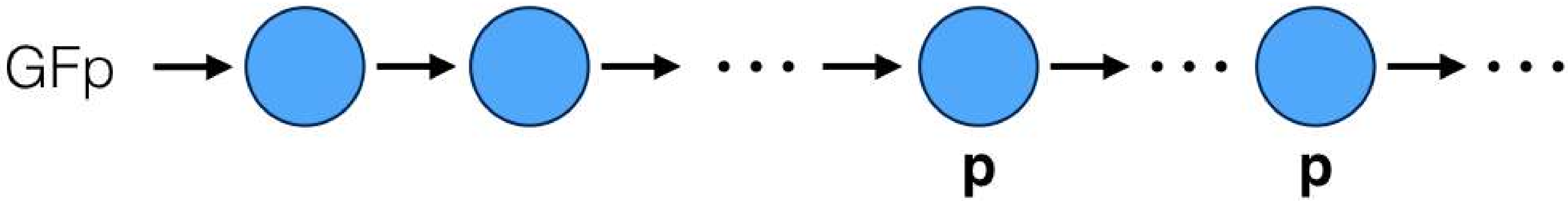
$\equiv \neg(\neg \rho \text{ U } \neg \omega)$

# Temporal Operators (Recap)

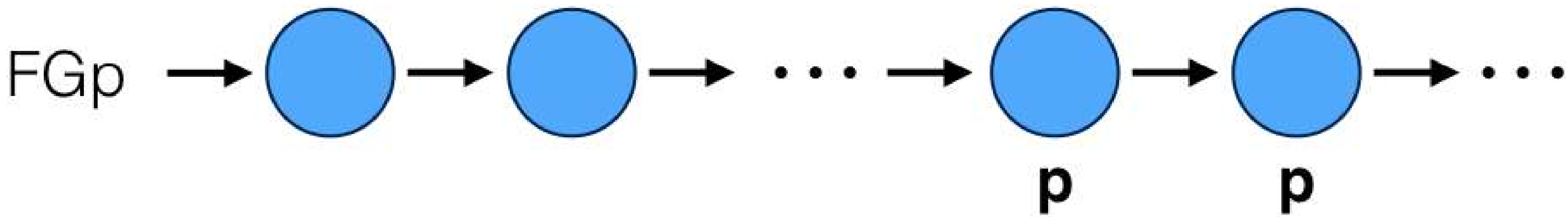
| Operator | Textual Operator | |
|---|---|---|
| ne**X**t | **X**ρ | |
| **U**ntil | ρ**U**ω | |
| **F**uture/Eventually | **F**ρ | ≡ True U ρ |
| **G**lobally | **G**ρ | ≡ ¬F¬ρ |
| **R**elease | ρ**R**ω | ≡ ¬(¬ρ U ¬ω) |

# Combination of Operators

## Infinitely Often

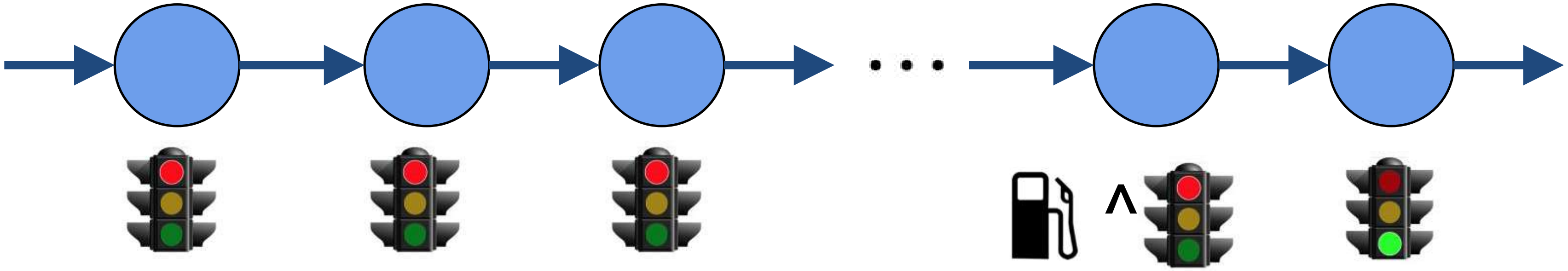



## Eventually Forever



# Example Problem

What are some true statements about this LTL formation?




- **X**R
- **F**G
- R**U**G
- (R**U**G)∧(**F**G)∧(**X**R)

# Expressing Temporal Logic in PDDL

## PDDL3 Goal Description

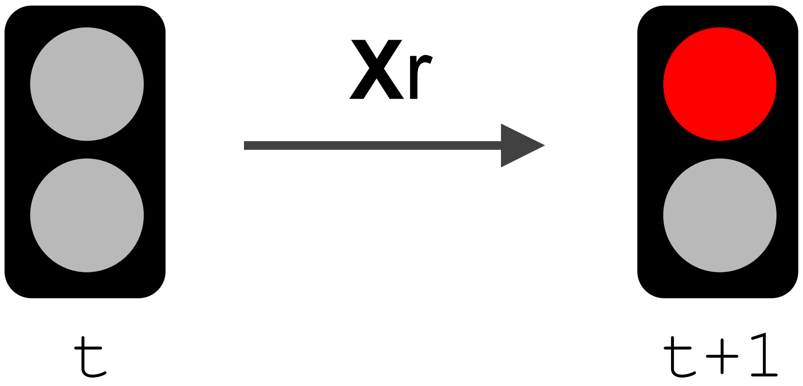
```
<GD> ::= (at end <GD>)
   | (always <GD>)
   | (sometime <GD>)
   | (within <num> <GD>)
   | (at-most-once <GD>)
   | (sometime-after <GD> <GD>)
   | (sometime-before <GD> <GD>)
   | (always-within <num> <GD> <GD>)
   | (hold-during <num> <num> <GD> | …
```

# Temporal Operators

| Operator | | PDDL3 |
|---|---|---|
| ne**X**t | **X**ρ | `(within 1 ρ)` |
| **U**ntil | ρ**U**ω | `(always-until ρ ω)` |
| **F**uture | ρ**F**ω | `(sometime-after ρ ω)` |
| **G**lobally | **G**ρ | `(always ρ)` |
| **R**elease | ρ**R**ω | `(or (always ω) (always-until ω ρ))` |

# Expressing Temporal Logic in PDDL

- The traffic light will turn red in the next state




(:goal (within 1 (turn red)))

**Command Syntax**

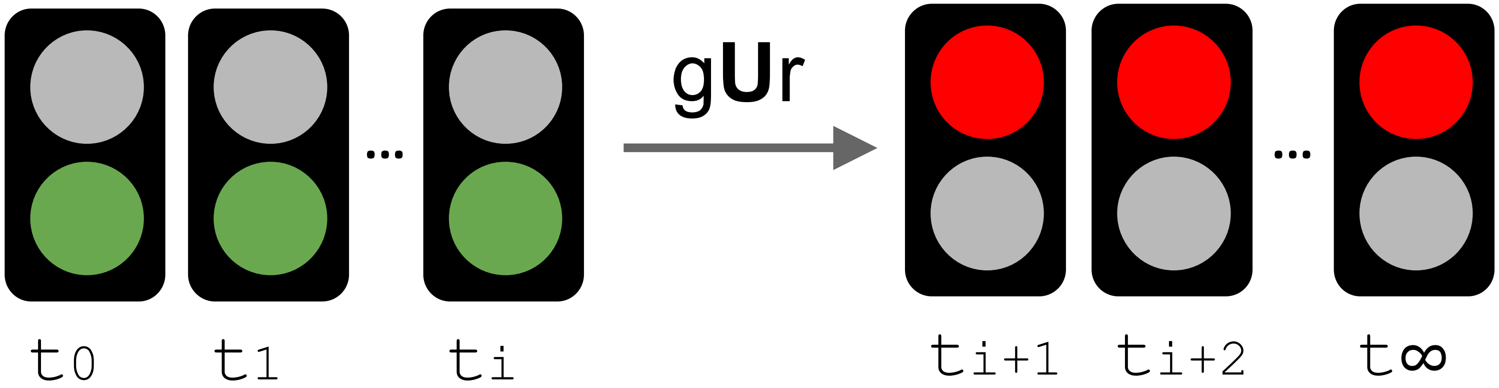
(within <num> <GD>)

(within <num> φ) would mean that φ must hold within <num> happenings

# Expressing Temporal Logic in PDDL

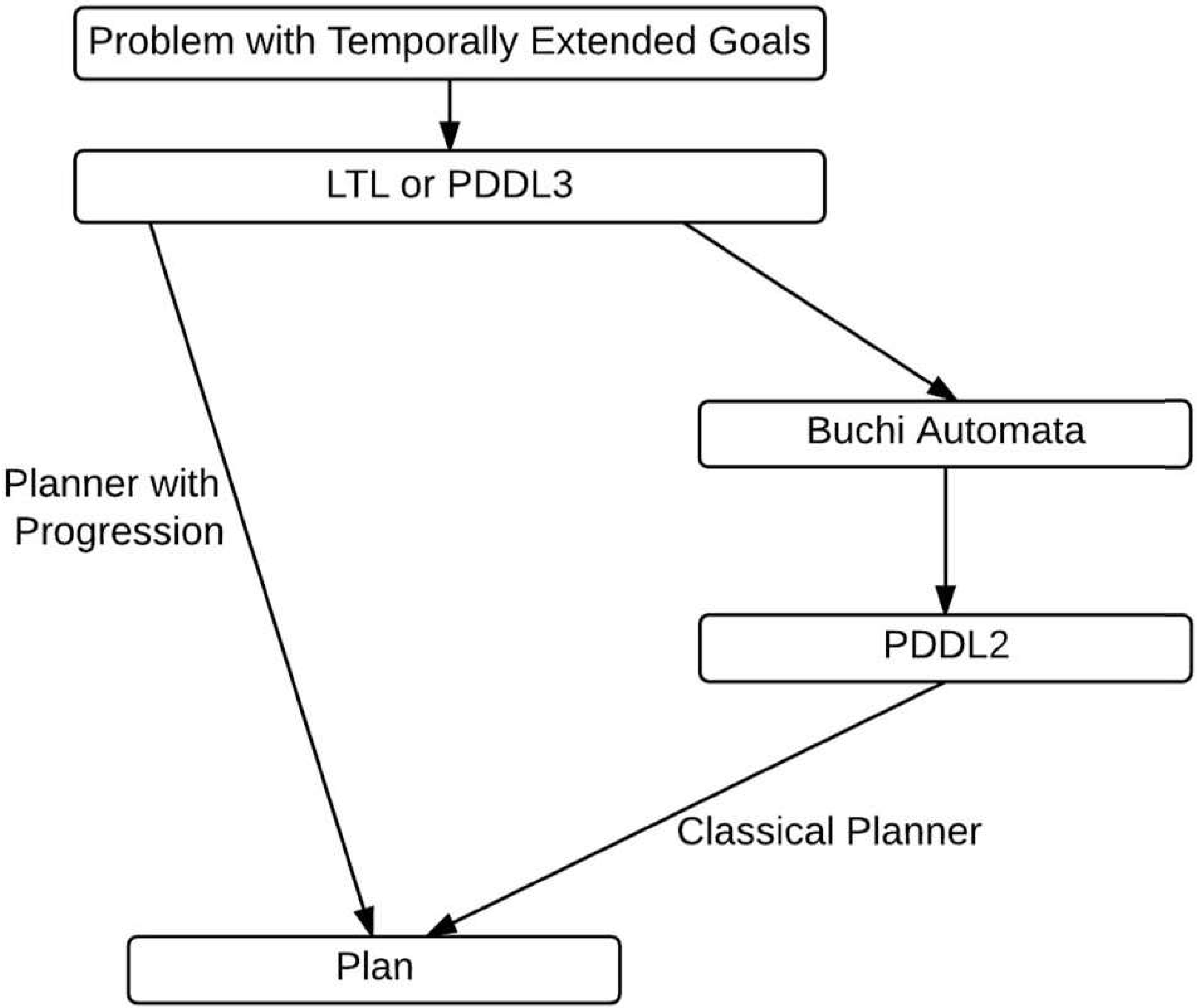
- The traffic light will be green until it turns red at which point it will be red forever

$$(g\ \mathbf{U}\ r) \wedge (r \rightarrow \mathbf{G}r)$$




```
(:goal
   (and
          (always-until (turn green) (turn
  red))
      (implies (turn red) (always (turn red)))
))
```

# Application to Planning



# Büchi Automata

Büchi Automata - *extension of finite automaton to infinite inputs (words)*

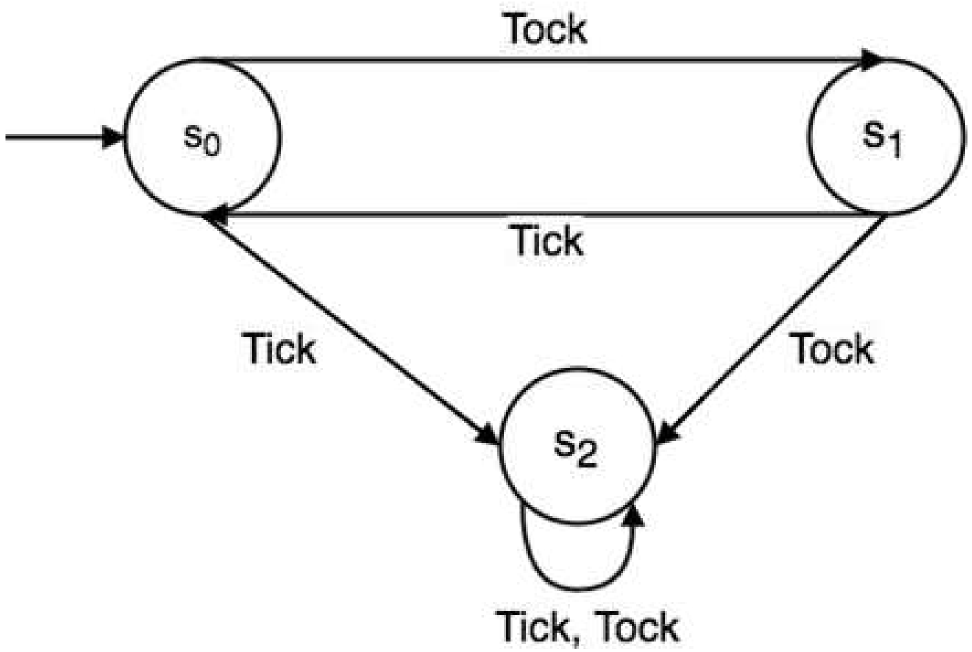
A Büchi automaton is 5-tuple $<S, s_0, T, F, \Sigma>$

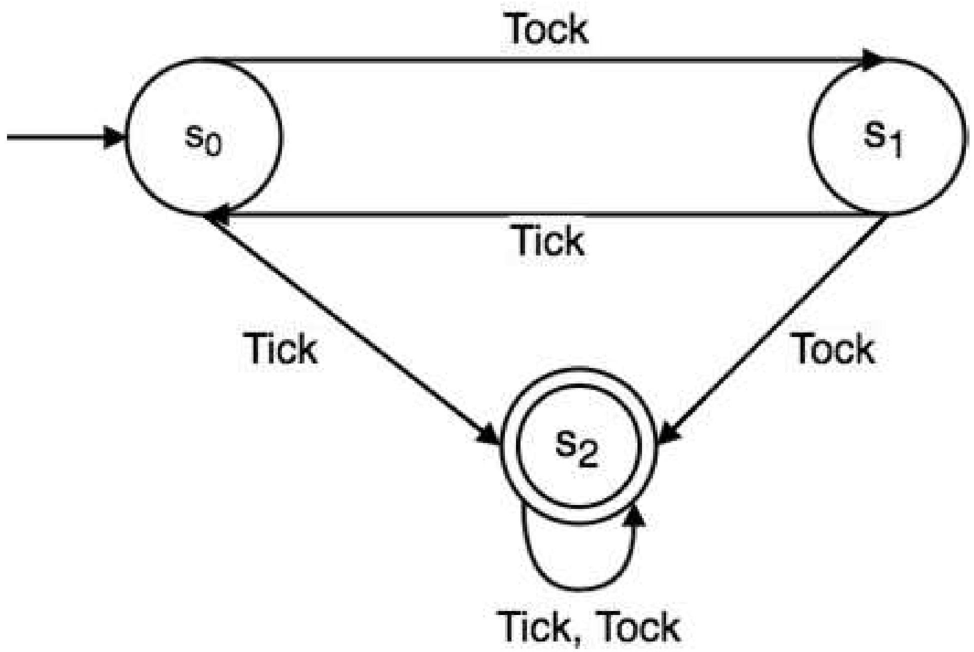
- S is a finite set of states
- $s_0 \in S$ is an initial state
- $T \subseteq S \times \Sigma \rightarrow S$ is a transition relation
- $F \subseteq S$ is a set of accepting states
- $\Sigma$ is a finite set of symbols ('alphabet')

An infinite sequence of states is accepted iff it visits the accepting state(s) infinitely often

# Example Büchi Automata

## Example: Model a clock



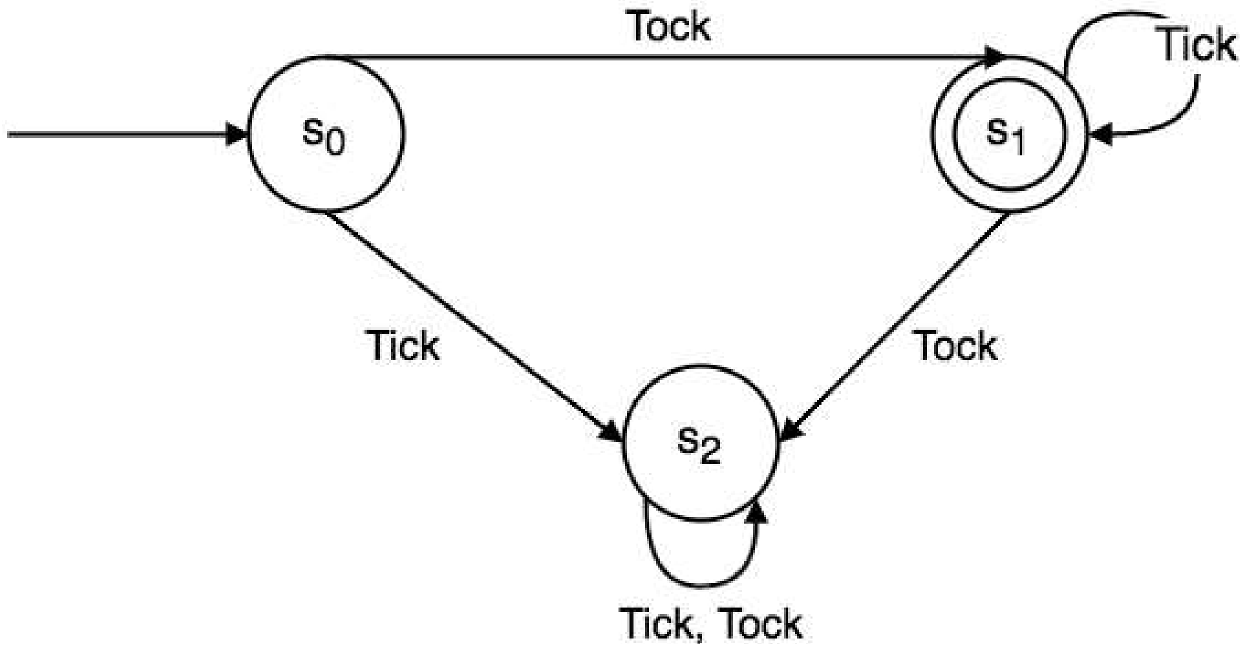

*Accepted words:*

TickTockTockTickTockTickTickTickTock...

TockTickTockTickTickTockTockTickTock...

# Example Büchi Automata

## Example: Model a clock

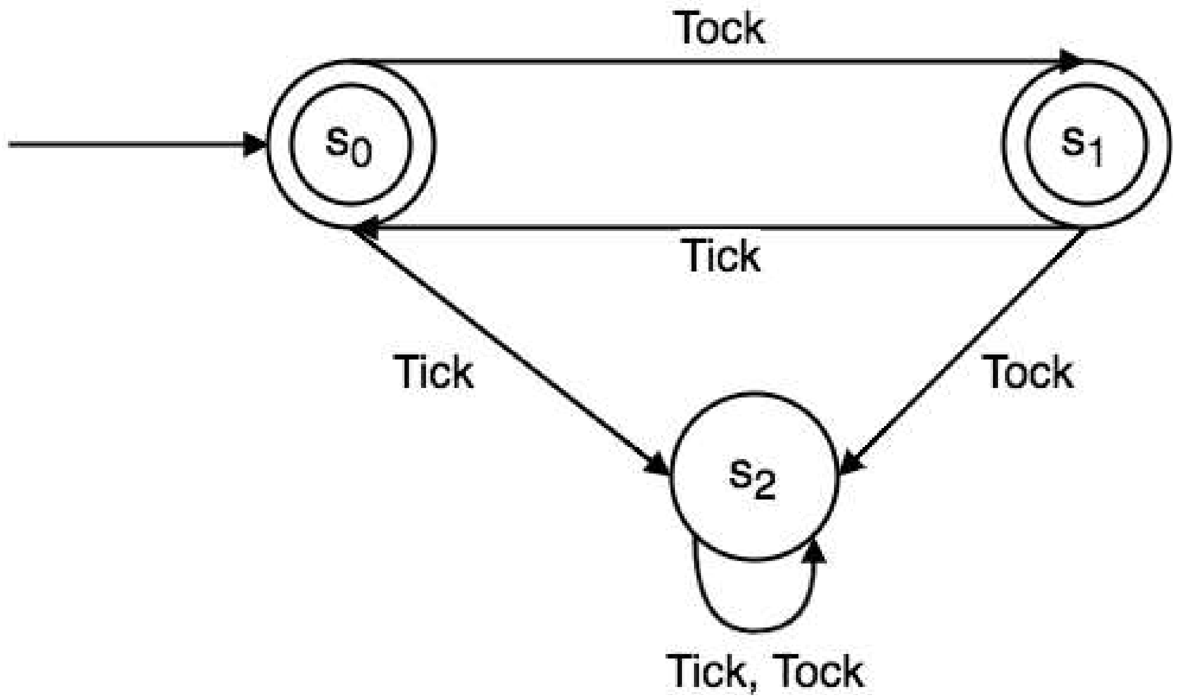



*Accepted words:*

TockTickTickTickTickTickTickTick...

# Example Büchi Automata

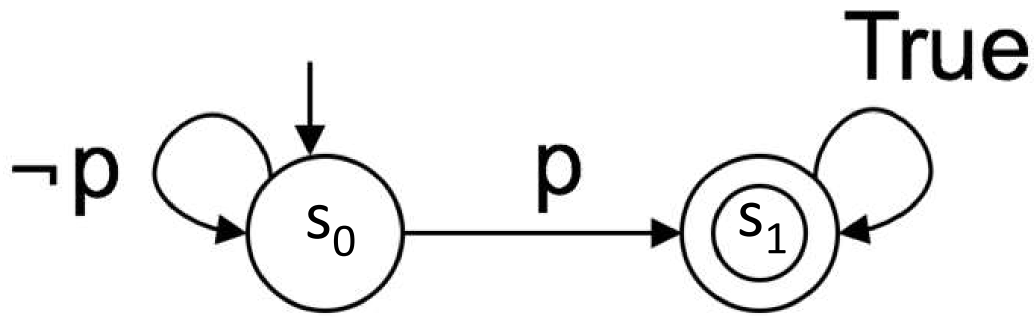
## Example: Model a clock




*Accepted words:*

TockTickTockTickTockTickTockTick...

# LTL to Büchi Automata



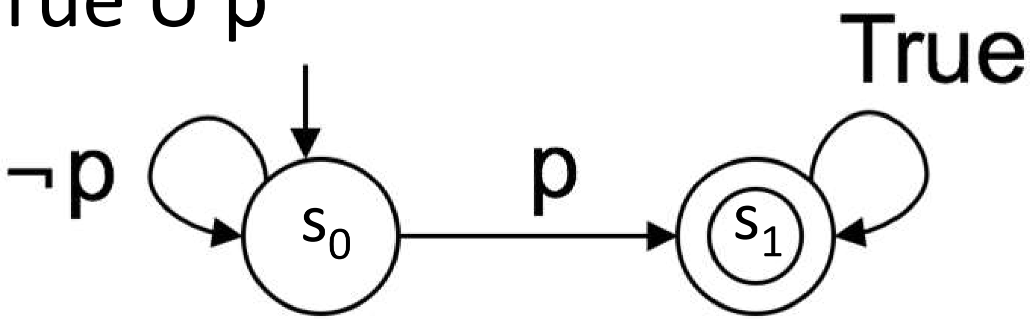

neXt?

Future/Eventually?

Globally?

# LTL to Büchi Automata

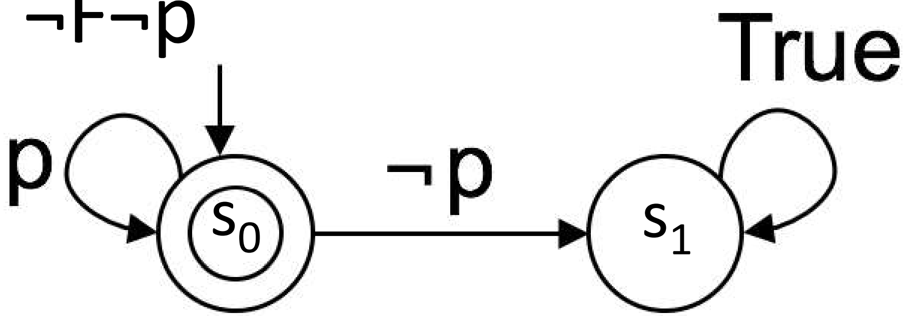
**F**uture - **F**p ≡ True U p




*Accepted word:* ¬p ¬p ¬p p p ¬p …

*Sequence of states:* $s_0$ $s_0$ $s_0$ $s_1$ $s_1$ $s_1$...

**G**lobally - **G**p ≡ ¬F¬p




*Accepted word:* p p p p p....

*Sequence of states:* $s_0$ $s_0$ $s_0$ $s_0$ $s_0$...

# LTL to Büchi Algorithm

N – Node object

- N.curr – LTL formulas to be processed
- N.old – LTL formulas already processed
- N.next – LTL formulas to be processed in next node
- N.incoming – Incoming transitions from predecessor nodes

$N_s$ – List of processed Nodes

$N_i$ – Arbitrary node from $N_s$

**expand** ($N, N_s$)

    if N.curr is empty
        if N.curr = N_i.curr
            Append N.curr to N_i.curr
        else
            Append N to N_s
            Create new node N_new with N_new.curr = N.next
            expand (N_new, N_s)
    else
        Remove an LTL formula f from N.curr and append to N.old
        Perform Progression on f
        Call expand on result of Progression

The result of this algorithm is a generalized Buchi automata which is then transformed into a simple Buchi automata.

# Progression Algorithm

**progress**(f,N, Δt = 1)  *#Δt is time between successive states*

if  f  contains no temporal qualities:

 if  N.curr entails f:

  f' = True

 else

  f' = False

if  f  =  $f_1$  ∧  $f_2$:

 **progress**($f_1$, N, Δt) ∧ **progress**($f_2$, N, Δt)

if  f  =  **X**$f_1$:

 N.next.append($f_1$)

if  f  =  $f_1$ $\mathbf{U}_{[a,b]}$ $f_2$:    *#[a,b] is a time interval that could be infinite*

 if  *b < a:*

  f' = False

 else if 0 ∈ [a,b]:

  **progress**($f_2$, N, Δt) ∨ (**progress**($f_1$,N, Δt) ∧ N.next.append($f_1$ $\mathbf{U}_{[a,b] - \Delta t}$ f))

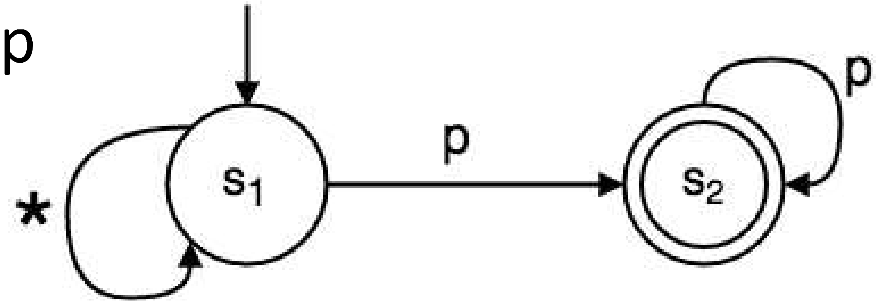
 else

  **progress**($f_1$,N, Δt) ∧ N.next.append($f_1$ $\mathbf{U}_{[a,b] - \Delta t}$ f)

# Büchi Automata to PDDL2

Büchi states are not equivalent to PDDL2 states. Consider:

**F**uture**G**lobally - **FG**p




Two ways to transform temporally extended goals to PDDL2:

- Create new actions that encapsulate the allowable transitions in each state
- Introduce derived predicates
  - Do not depend on the actions
  - Used to determine which state the planner is in
  - Goal of the planner is to move from initial state to any accepting state

# Planning with Preferences

# Preference Based Planning

## Classical Planning Problem

$$problem := (S, s_0, A, G)$$

*S - set of states* *$s_0$ - initial state* *A - set of operators* *G - set of goal states*

## Preference-based Planning Problem

$$problem := (S, s_0, A, G, R)$$

*R is a partial or total relation expressing preferences ($\preccurlyeq$) between plans*

**Preferences express properties of the plan that are desired but not required**

# Preference Expression Languages

- **Quantitative** - *assign numeric values to plans to compare them*
  - Markov Decision Processes (MDP's)
    - Find preferred policy using a reward function over conditional plans
  - PDDL3
    - Preferences expressed through reward function based on satisfying/violating logical formulas on the plan
- **Qualitative** - *relations compare plans based on properties of the plans that need not be numeric*
  - Ranked Knowledge Bases
    - Plan properties are ranked with preferred formulas ranked higher
  - Temporally Extended Preferences
    - Use LTL to express plan properties that are then ranked

**Quantitative languages imply total comparibility while qualitative languages may allow incomparability**

# Expressing Preferences in PDDL3

## Syntax for modeling preferences:

`(preference [name] <GD>)` - *label for fluents that represent preferences*

`is-violated` - *function that returns the number of times the preference was not satisfied in the plan*

## Example:

*Traffic light is green until it turns red*

```
(preference gUr
        (always-until(turn green) (turn
red)))
```

*Plan tries to not violate any preferred fluents*

```
(metric minimize (is-violated gUr))
```

# LPP Language Overview

- LPP is a quantitative language to express temporal preferences for planning
  - Preferences between different temporal goals can be expressed along with the strength of preference
    - i.e. Goal A is preferred twice as much as Goal B
- LPP is an extension of an older language PP
- Preference formulas in LPP are constructed hierarchically

See Bienvenu, Meghyn, Christian Fritz, and Sheila A. McIlraith. "Planning with Qualitative Temporal Preferences." KR 6 (2006): 134-144.

# Constructing a Preference Formula

***Basic Desire Formula (BDFs)***

*express temporally extended propositions*

- At some point, will cook
  - $b_1 = \mathbf{F}(cook)$
- At some point, will order takeout
  - $b_2 = \mathbf{F}(orderTakeout)$
- At some point, will eat spaghetti
  - $b_3 = \mathbf{F}(eatSpaghetti)$
- At some point, will eat pizza
  - $b_4 = \mathbf{F}(eatPizza)$

# Constructing a Preference Formula

***Atomic Preference Formulas (APFs)***

*express preferences between BDFs*

- In this example, weights associated with each BDF define preferences
  - Lower weight is preferred

- Prefer to cook over ordering takeout
  - $a_1 = b_1[0.2] \gg b_2[0.4]$
- Prefer eating spaghetti over eating pizza
  - $a_2 = b_3[0.3] \gg b_4[0.9]$

# Constructing a Preference Formula

***General Preference Formulas (GPFs)***

*allow conjunctions or disjunctions of APFs or qualification of BDFs with conditionals*

- Satisfy the most preferred option among the APFs (satisfy APF with lowest weight)
  - $g_1 = a_1 \mid a_2$
- Choose the most preferred option that satisfies both APFs (minimize the maximum weight across both APFs)
  - $g_2 = a_1 \ \& \ a_2$

# Constructing a Preference Formula

***Aggregated Preferences Formulas (APFs)***
*define the order in which preferences should be relaxed*

- Prefer that if both $g_1$ and $g_2$ from previous slide can't be met, that $g_2$ from previous slide is met
  - $g_1 \wedge g_2 \preccurlyeq g_2 \preccurlyeq g_1$
- Situations that aren't distinguished any other way can be sorted lexicographically (alphabetically)

# LPP Formula Hierarchy Review

- **Basic Desire Formula (BDF)**
  - Express temporally extended propositions
- **Atomic Preference Formula (APF)**
  - Express preferences between BDFs
- **General Preference Formula (GPF)**
  - Allow conjunctions or disjunctions of APFs or qualification of BDFs with conditionals
- **Aggregated Preference Formula (APF)**
  - Define the order in which preferences should be relaxed

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# Appendix

# Solving Planning Problems with Preferences

- PPLAN
  - implemented by Meghyn Bienvenu, Christian Fritz, and Sheila A. McIlraith
- Solves planning problems with preferences expressed in LPP via bounded best-first search forward chaining planner
  - use of *progression* efficiently evaluates how well partial plans satisfy $\Phi$ (a general preference formula)
  - use of admissible *evaluation function* ensures best-first search is optimal