Oblig 1

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i)

We are asked to verify the analytical solution for flow in ducts of different shapes.

$$\mu \frac{d^2u}{dy^2} = \frac{d\hat{p}}{dx} = \text{constant} \tag{1}$$

Transforming it into its variational form we get

$$\begin{aligned} (u'', v) &= (\frac{d\hat{p}}{dx}, v) \\ u'v|_{d\Omega} - \int_\Omega u'v'dy &= \int_\Omega \frac{d\hat{p}}{dx} v dy \\ -\int_\Omega u'v'dy &= \int_\Omega \frac{d\hat{p}}{dx} v dy \end{aligned} \tag{2}$$

Implemented in fenics as

```
-mu*inner(grad(u),grad(v))*dx == dpdx * v * dx
```

Ellipse

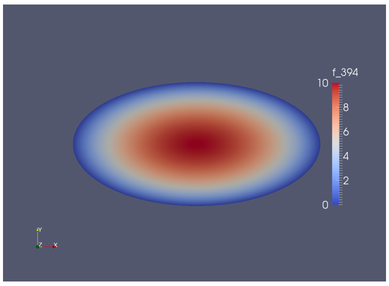
```
meshsize: 0.0591501982186
P 1 : error: 0.198612991237
      Qdif:  0.297069394929
P 2 : error: 0.210289258749
      Qdif:  0.262444333081
P 3 : error: 0.209726175255
      Qdif:  0.261816464212

meshsize: 0.0286080707886
P 1 : error: 0.0490186164831
      Qdif:  0.0742216200784
P 2 : error: 0.0523161573248
      Qdif:  0.0653951481331
P 3 : error: 0.0522316313421
      Qdif:  0.0653017153615

meshsize: 0.0139846868179
P 1 : error: 0.012118367695
      Qdif:  0.0183633262084
P 2 : error: 0.0129971279264
      Qdif:  0.016265804744
P 3 : error: 0.0129865902472
      Qdif:  0.0162541180776
```

The mesh refinement does make the errornorm and flowrate-difference better, for all chosen 'CG' elements. Comparing P1, P2 and P3 at the same meshsize shows that there isn't much of a difference.

Figure 1: numerical velocity field. Fine mesh



Triangle

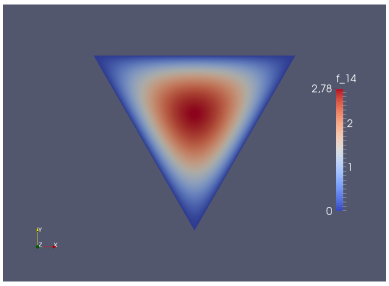
```
meshsize: 0.0350377086466
P 1 : error: 0.00171802938519
      Qdif:  0.00516271982613
P 2 : error: 1.5715194884e-05
      Qdif:  1.57157366643e-06
P 3 : error: 3.77584863776e-07
      Qdif:  1.82520665248e-13

meshsize: 0.0172032083643
P 1 : error: 0.000457344678688
      Qdif:  0.0012954806937
P 2 : error: 2.09535710775e-06
      Qdif:  9.82156328488e-08
P 3 : error: 3.56580191718e-07
      Qdif:  7.22977233636e-13

meshsize: 0.00840202867406
P 1 : error: 0.000121216065957
      Qdif:  0.000325354790669
P 2 : error: 2.73151840825e-07
      Qdif:  6.13381312409e-09
P 3 : error: 3.54823796132e-07
      Qdif:  2.97228908153e-12
```

Comparing the numerical solution with the analytical solution for the triangle gives us great results. Not much difference when we refine for P3. P1 gives us a convergence rate of 2 and P2 gives us a convergence rate of 3.

Figure 2: numerical velocity field. Fine mesh



Annulus

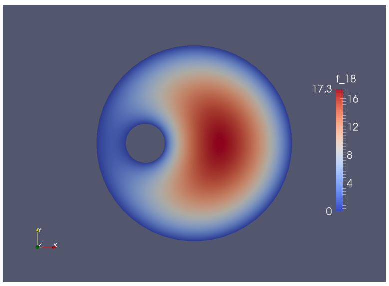
```
meshsize: 0.0588455214472
P 1 Qdif: 0.856400783432
P 2 Qdif: 0.696032328298
P 3 Qdif: 0.692098040142

meshsize: 0.0283145494553
P 1 Qdif: 0.214641713003
P 2 Qdif: 0.171826383154
P 3 Qdif: 0.17126495232

meshsize: 0.0136809542295
P 1 Qdif: 0.0535579359806
P 2 Qdif: 0.0425287440364
P 3 Qdif: 0.042454583381
```

For the annulus we can only compare the flow rate. We can see that we get better results when refining our mesh, even though the results aren't that good.

Figure 3: numerical velocity field. Fine mesh



ii)

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$$F''' + FF'' + 1 - (F')^2 = 0 \tag{3}$$

By using the hint, $F' = H$, we get

$$H'' + FH' + 1 - H^2 = 0 \tag{4}$$

Then the boundary conditions become

$$\begin{aligned} F(0) = F'(0) = 0 \rightarrow F(0) = H(0) = 0 \\ F'(\infty) = 1 \rightarrow H(\infty) = 1 \end{aligned} \tag{5}$$

Using two test functions, vh and vf, we may display our problem as follows

$$(H, vf) = (F', vf) \tag{6}$$

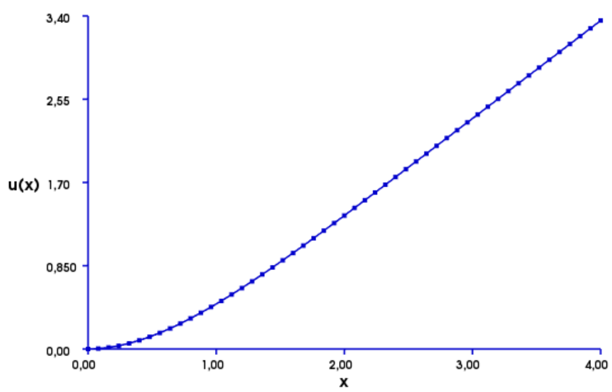
$$-(H', vh') + (FH', vh) + (1, vh) - (H^2, vh) = 0 \tag{7}$$

We then use Newton- and Picard iteration because of the non linearity.

Newton

Newton solver finished in 6 iterations and 6 linear solver iterations.

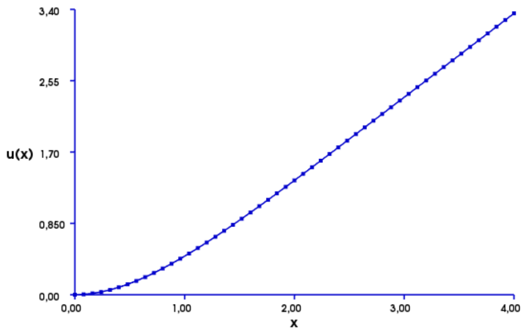
Figure 4: plot of F, using Newtons method



Picard

```
iter=1: norm=9.1337
iter=2: norm=7.14
iter=3: norm=3.97771
        .
        .
        .
iter=64: norm=2.85202e-12
iter=65: norm=1.92189e-12
iter=66: norm=4.34496e-13
```

Figure 5: plot of F, using Picard iteration



Both methods give similar result. Newtons method was however much faster

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$$F''' + 2FF'' + 1 - (F')^2 = 0 \tag{8}$$

As above we substitute, get two equations, and use two test functions. We end up with

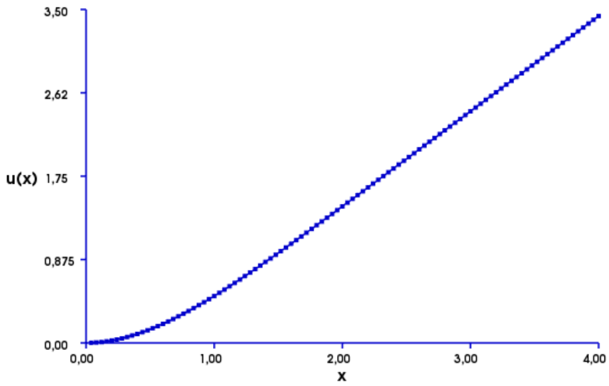
$$(H, vf) = (F', vf) \tag{9}$$

$$-(H', vh') + 2(FH', vh) + (1, vh) - (H^2, vh) = 0 \tag{10}$$

Newton

Newton solver finished in 6 iterations and 6 linear solver iterations

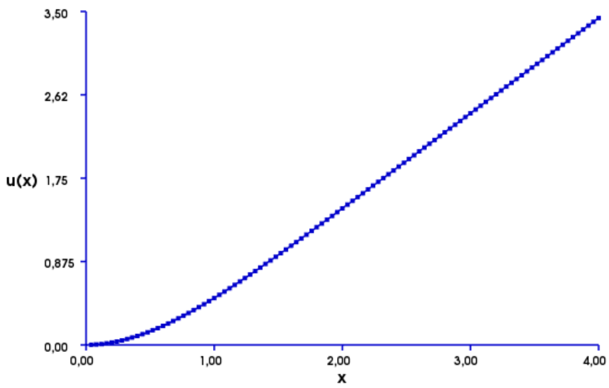
Figure 6: plot of F, using Newtons method



Picard

```
iter=1: norm=2.59437
iter=2: norm=2.20915
iter=3: norm=0.272425
        .
        .
        .
iter=16: norm=1.75043e-11
iter=17: norm=2.37214e-12
iter=18: norm=3.51235e-13
```

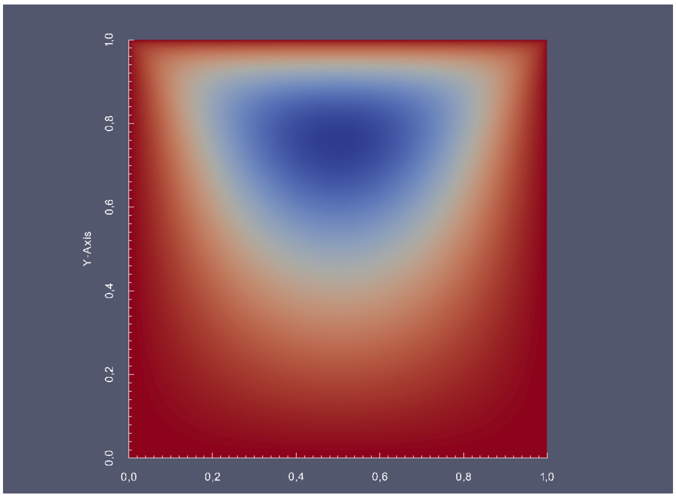
Figure 7: plot of F, using Picard iteration



iii)

```
Meshsize: 0.0141421356236
Vortex center at x: 0.5 and y: 0.76
```

Figure 8: Plot of stream function



iv)

a)

```
meshsize: 0.111027, minvalue: -0.0579006617083
Vortex center at x: 0.0 and y: 0.2

meshsize: 0.055514, minvalue: -0.0547620168121
Vortex center at x: 0.0 and y: 0.2

meshsize: 0.027757, minvalue: -0.0537767674796
Vortex center at x: 1.06551379861 and y: 0.0778896071092

meshsize: 0.013878, minvalue: -0.0533669982984
Vortex center at x: 1.07097328183 and y: 0.077088741035

meshsize: 0.006939, minvalue: -0.0531659519101
Vortex center at x: 1.06824354022 and y: 0.0806141740721
```

b)

Figure 9: Figure 3-37b in White shows the recirculation bubble in flow past a step

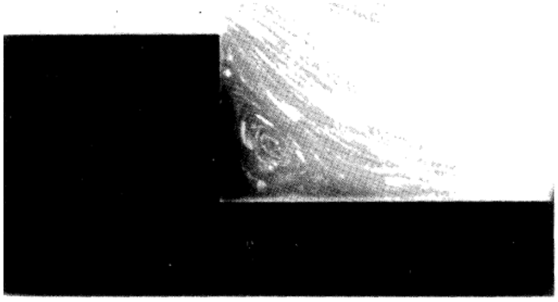
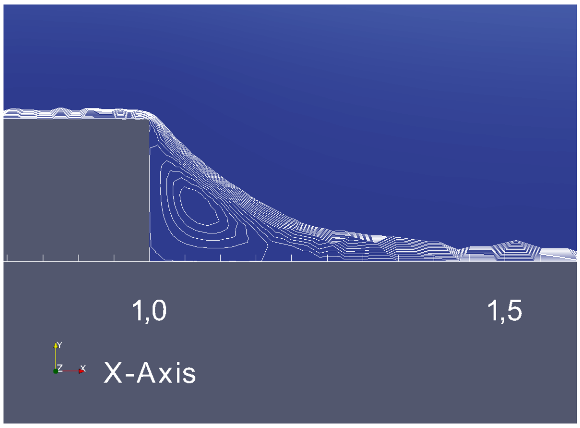


Figure 10: recirculation bubble at step



c)

```
Flux inn: 0.425219807767
Flux out: 0.425219807767
Flux diff: 6.66133814775e-16
```

d)

The vortex center is at exactly the same point when direction of the flow is reversed

e)

Normal stress on the bottom wall for flow i positive x-direction:

```
normal stress: 243.204770541
```

Normal stress on bottom wall for flow i negative x-direction:

```
normal stress: -243.204770541
```