Homework Set 5

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7.1 The set of possible models for the presence of pits and a wumpus in [1,3],[2,2], and [3,1] is:

	P in [1,3]	P in [2,2]	P in [3,1]	W in [1,3]	W in [2,2]	W in [3,1]	Notes
1	N	N	N	N	N	N	α_2
2	N	N	N	N	N	Y	α_2
3	N	N	N	N	Y	N	α_2
4	N	N	N	Y	N	N	α_2, α_3
5	N	N	Y	N	N	N	α_2
6	N	N	Y	N	N	Y	α_2
7	N	N	Y	N	Y	N	α_2
8	N	N	Y	Y	N	N	KB, α_2, α_3
9	N	Y	N	N	N	N	
10	N	Y	N	N	N	Y	
11	N	Y	N	N	Y	N	
12	N	Y	N	Y	N	N	α_3
13	N	Y	Y	N	N	N	
14	N	Y	Y	N	N	Y	
15	N	Y	Y	N	Y	N	
16	N	Y	Y	Y	N	N	α_3
17	Y	N	N	N	N	N	α_2
18	Y	N	N	N	N	Y	α_2
19	Y	N	N	N	Y	N	α_2
20	Y	N	N	Y	N	N	α_2, α_3
21	Y	N	Y	N	N	N	α_2
22	Y	N	Y	N	N	Y	α_2
23	Y	N	Y	N	Y	N	α_2
24	Y	N	Y	Y	N	N	α_2, α_3
25	Y	Y	N	N	N	N	
26	Y	Y	N	N	N	Y	
27	Y	Y	N	N	Y	N	
28	Y	Y	N	Y	N	N	α_3
29	Y	Y	Y	N	N	N	
30	Y	Y	Y	N	N	Y	
31	Y	Y	Y	N	Y	N	
32	Y	Y	Y	Y	N	N	α_3

Since in all models where KB is true (only model 8), α_2 and α_3 are also true, $KB \models \alpha_2$ and $KB \models \alpha_3$.

7.2 It is given that:

$1 : mythical \Rightarrow immortal$
$2 : \neg mythical \Rightarrow (mortal \wedge mammal)$
$3 : (immortal \vee mammal) \Rightarrow horned$
$4 : horned \Rightarrow magical$

To attempt to prove *mythical*, assume $\neg mythical$ and look for a contradiction. By 2, this leads to *mortal* and *mammal*, which leads to *horned* and *magical* by 3 and 4. Since no further inferences can be made, and since no contradiction is derived, it is not provable that the unicorn is mythical.

Since *mythical* is either true or false, by 1 and 2 $immortal \lor (mortal \land mammal)$. By 3, this implies *horned*, and then by 4 this implies *magical*. It is provable that the unicorn is magical and horned.

7.6 a) True. By the property of monotonicity for KB's, adding to a KB cannot reduce the set of statements that it entails. It is given that $\alpha \models \gamma$ or $\beta \models \gamma$. If $\alpha \models \gamma$, then adding β to α does not reduce its power to entail γ. Similarly, if $\beta \models \gamma$, then adding α does not reduce β's power.

b) True. By the truth table of *and*, knowing $(\beta \land \gamma)$ implies separately that β is true and γ is true. So any KB that entails $(\beta \land \gamma)$ also entails them separately.

c) False. Given any arbitrary knowledge base, it can be said that the price of tea in China is either some arbitrary value p, or it is not. But if the knowledge base has nothing to do with tea whatsoever, than it does not entail that the price of tea in China is p, and it does not entail that it isn't either.

7.10 a) Valid because implications cannot be false when their left and right sides have the same truth value.

b) Neither. Not valid because if *Smoke* is true and *Fire* is false, the sentence is false. Not unsatisfiable because the sentence is satisfied if for example *Smoke* and *Fire* are both true.

c) Neither. Not valid because the sentence is false if *Smoke* is false and *Fire* is true. Not unsatisfiable because the sentence is satisfied if for example both *Smoke* and *Fire* are true.

d) Valid because $Fire \lor \neg Fire$ is always true, and anything or'ed with that is true also.

e) Valid because the sentence is true for all combinations of values of the three variables.

f) Valid because the sentence is true for all combinations of values of the three variables.

g) Valid because the sentence is true if any of *Big*, *Dumb*, or $Big \Rightarrow Dumb$ are true, and $Big \Rightarrow Dumb$ can only be false when *Big* is true.

7.22 a) $(X_{1,2} \land X_{2,1}) \lor (X_{1,2} \land X_{2,2}) \lor (X_{2,1} \land X_{2,2})$

b) Make a disjunctive clause for each of the n choose k combinations of squares, and *or* them together. Each combination represents one of the possible placements of k mines into the n neighbors. To turn this disjunctive form into CNF, use De Morgan's Laws and/or the distributive law.

c) Use the process described in b) for every currently visible, numbered square on the board. Put all CNF clauses generated into DPLL. If DPLL returns a definitive value for a given square, than that square for certain does or does not contain a mine depending on the value returned. If it does not, then it is not provable that the square contains or does not contain a mine

d) Since there are N choose M ways to place M mines into N squares, the number of clauses at any give point is $O(NchooseM)$.

e) If DPLL in c) does not return a definitive value for a square, it may still be provable that a square does not contain a mine due to the global constraint.

f)

7.26 $FacingEast^{t+1} = (FacingEast^t \wedge (\neg TurnRight^t \wedge \neg TurnLeft^t))$
$\vee (FacingSouth^t \wedge TurnLeft^t)$
$\vee (FacingNorth^t \wedge TurnRight^t)$

$WumpusAlive^{t+1} = WumpusAlive^t \wedge \neg (Shoot^t \wedge Scream^{t+1})$